Black hole physics entering a new observational era

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Laboratoire de Mathématiques et Physique Théorique
Tours, 29 November 2012
Part 1
- What is a black hole?
- Overview of the black hole theory
- The current observational status of black holes
- The near-future observations of black holes

Part 2
- Tests of gravitation
- The Gyoto tool
- Ray-tracing in numerical spacetimes
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2. Overview of the black hole theory
3. The current observational status of black holes
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2. The Gyoto tool
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What is a black hole?

... for the layman:

A black hole is a region of spacetime from which nothing, not even light, can escape.

The (immaterial) boundary between the black hole interior and the rest of the Universe is called the event horizon.

[Alain Riazuelo, 2007]
Part 1

What is a black hole?

... for the mathematical physicist:

black hole: \( \mathcal{B} := \mathcal{M} - J^- (\mathcal{I}^+) \)

i.e. the region of spacetime where light rays cannot escape to infinity

- \((\mathcal{M}, g) =\) asymptotically flat manifold
- \(\mathcal{I}^+ =\) future null infinity
- \(J^- (\mathcal{I}^+) =\) causal past of \(\mathcal{I}^+\)

event horizon: \( \mathcal{H} := \partial J^- (\mathcal{I}^+) \)

(boundary of \(J^- (\mathcal{I}^+)\))

\(\mathcal{H}\) smooth \(\implies\) \(\mathcal{H}\) null hypersurface
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What is a black hole?

... for the astrophysicist: a very deep gravitational potential well

[J.A. Marck, CQG 13, 393 (1996)]
What is a black hole?

... for the astrophysicist: a very deep gravitational potential well

Binary BH in galaxy NGC 6240

\[ d = 1.4 \text{ kpc} \]


Binary BH in radio galaxy 0402+379

\[ d = 7.3 \text{ pc} \]

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A short history of the black hole concept

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- Georges Lemaître (1932): the singularity at $r = R_s$ is not physical (coordinate singularity)
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- **Albert Einstein (1939)**: wrong article to prove that a body cannot have a size smaller than $R_s$
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- **John A. Wheeler (1967)**: coined the name **black hole**
Main properties of black holes (1/3)

- In general relativity, a black hole contains a region where the spacetime curvature diverges: the singularity *(NB: this is not the primary definition of a black hole)*. The singularity is inaccessible to observations, being hidden by the event horizon.
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A black hole **is not an infinitely dense object**: on the contrary it is made of vacuum (except maybe at the singularity); black holes can form in spacetimes empty of any matter, by collapse of gravitational wave packets.
Main properties of black holes (2/3)

Uniqueness theorem

A black hole in equilibrium is necessarily a Kerr-Newmann black hole, which is a vacuum solution of Einstein described by only three parameters:

- the total mass $M$
- the total angular momentum $J$
- the total electric charge $Q$

$⇒$ “a black hole has no hair” (John A. Wheeler)

- $Q = 0$ and $J = 0$ : Schwarzschild solution (1916)
- $Q = 0$ : Kerr solution (1963)
The mass $M$ is not a measure of the “matter amount” inside the black hole, but rather a parameter characterizing the external gravitational field; it is measurable from the orbital period of a test particle in circular orbit around the black hole and far from it (Kepler’s third law).
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The radius of a black hole is not a well defined concept: it \textit{does not} correspond to some distance between the black hole “centre” (the singularity) and the event horizon. A well defined quantity is the area of the event horizon, $A$.

The radius can be then defined from it: for a Schwarzschild black hole:

$$R := \sqrt{\frac{A}{4\pi}} = \frac{2GM}{c^2} \simeq 3 \left(\frac{M}{M_\odot}\right) \text{ km}$$
Massive stars end their lives as **supernova**: the explosion is triggered by the **gravitational collapse** of the stellar iron core, via some bounce.

Depending on the initial conditions, the collapse can be stopped by the *strong interaction* (the residual is then a *neutron star*) or be complete, leading to a black hole.
Other theoretical aspects

- The four laws of black hole dynamics
- Quantum properties (Bekenstein entropy, Hawking radiation)
- Black holes and gravitational waves
- Quasi-local approaches: trapping horizons, dynamical horizons, isolated horizons
- Black holes in higher dimensions
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Astrophysical black holes

There are three kinds of black holes in the Universe:

- **Stellar black holes**: supernova remnants:
  \[ M \sim 10 - 30 \, M_\odot \text{ and } R \sim 30 - 90 \, \text{km} \]
  example: Cyg X-1: \( M = 15 \, M_\odot \text{ and } R = 45 \, \text{km} \)
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- **Supermassive black holes**, in galactic nuclei:
  \[ M \sim 10^5 - 10^{10} \, M_\odot \text{ and } R \sim 3 \times 10^5 \, \text{km} - 200 \, \text{UA} \]
  
  example: Sgr A* : \( M = 4,3 \times 10^6 \, M_\odot \text{ and } 
  R = 13 \times 10^6 \, \text{km} = 18 \, R_\odot = 0,09 \, \text{UA} = \frac{1}{4} \text{radius of Mercury’s orbit} \)
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- **Intermediate mass black holes**, as ultra-luminous X-ray sources (?):
  \[ M \sim 10^2 - 10^4 \, M_\odot \text{ and } R \sim 300 \, \text{km} - 3 \times 10^4 \, \text{km} \]
  
  example: ESO 243-49 HLX-1 : \( M > 500 \, M_\odot \) and \( R > 1500 \, \text{km} \)
What we do not see yet...

[Alain Riazuelo, 2007]
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The black hole: a fantastic source of energy!

Release of potential gravitational energy by accretion on a black hole: up to 42% of the mass-energy $mc^2$ of accreted matter!

NB: thermonuclear reaction release less than 1% $mc^2$

Matter falling in a black hole forms an accretion disk [Donald Lynden-Bell (1969), Nicolaï Shakura & Rachid Sunayev (1973)]

[J.-A. Marck (1996)]
The accretion disk as a spacetime probe

X-ray spectrum of the accretion disk around the supermassive black hole in the nucleus of the galaxy MCG-6-30-15:

\[K\alpha\text{ line: } X\text{ fluorescence line of Fe atoms in the accretion disk (the Fe atoms are excited by the X-ray emitted from the plasma corona surrounding the disk).}\]

K\(\alpha\) line observed by the satellites XMM-Newton (red) and Suzaku (black) (adapted from [Miller (2007)])

Redshift \(\Rightarrow\) time dilatation
Part 1  The current observational status of black holes

**Black holes in the core of quasars**

**Quasar 3C 273**

HST - WFPC2, ACS

WFPC2

NASA, A. Martel (JHU), the ACS Science Team, J. Bahcall (IAS) and ESA

ACS/HRC

STScI-PRC03-03
Jet emitted by the nucleus of the giant elliptic galaxy M87, at the centre of Virgo cluster [HST]

\[ M_{BH} = 3 \times 10^9 \, M_{\odot} \]

\[ V_{\text{jet}} \simeq 0.99 \, c \]
Black holes in X-ray binaries

~ 20 identified stellar black holes in our galaxy
Detection of a black hole in a X-ray binary

Kepler’s third law: \( f := \frac{M_1^3 \sin^3 i}{(M_1 + M_2)^2} = \frac{K_2^3 P}{2\pi G} \)

\( f \) is a lower bound on \( M_1 : M_1 > f \)

Mass criterion: \( M_1 > M_{\text{max}}(\text{neutron star}) \simeq 3 M_\odot \)

\[ V_{\text{rad}}(t) = K_2 \cos(2\pi t/P) + V_0 \Rightarrow K_2, P \]
The first black hole identified via the mass criterion was *Cygnus X-1* in 1972. Since then, around 20 black holes have been identified in this way.

### Selection of 5 black holes in X-ray binaries:

<table>
<thead>
<tr>
<th>Nom</th>
<th>Masse [M(_\odot)]</th>
<th>Spin (a = cJ / (GM^2))</th>
<th>Distance [1000 al]</th>
<th>Période orbitale [j]</th>
<th>Fonction de masse [M(_\odot)]</th>
<th>Masse du compagnon [M(_\odot)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cyg X-1</td>
<td>14,8 ± 1,0</td>
<td>&gt; 0,97 (?)</td>
<td>6,1 ± 0,3</td>
<td>5,6</td>
<td>0,24</td>
<td>19,2 ± 1,9</td>
</tr>
<tr>
<td>HDE 226868</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A 0620-00</td>
<td>6,6 ± 0,25</td>
<td>0,12 ± 0,18 (?)</td>
<td>3,4 ± 0,4</td>
<td>0,32</td>
<td>2,76 ± 0,01</td>
<td>0,40 ± 0,03</td>
</tr>
<tr>
<td>V 404 Cyg</td>
<td>12 ± 2</td>
<td>?</td>
<td>7,8 ± 0,4</td>
<td>6,5</td>
<td>6,08 ± 0,06</td>
<td>0,70 ± 0,05</td>
</tr>
<tr>
<td>GS 2023+338</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GRS 1915+105</td>
<td>14,4 ± 4,4</td>
<td>&gt; 0,98 (?)</td>
<td>32 ± 12</td>
<td>30,8</td>
<td>9,5 ± 3,0</td>
<td>1,2 ± 0,2</td>
</tr>
<tr>
<td>V1487 Aql</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GRO J1655-40</td>
<td>6,3 ± 0,3</td>
<td>0,70 ± 0,05 (?)</td>
<td>10 ± 2</td>
<td>2,6</td>
<td>2,73 ± 0,09</td>
<td>2,50 ± 0,15</td>
</tr>
<tr>
<td>XN Sco 94</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>
Part 1  The current observational status of black holes

Black holes in X-ray binaries

[McClintock et al. (2011)]
The black hole at the centre of our galaxy

Orbit of the star S2 around the black hole Sgr A*

\[ M_{\text{BH}} = 4.3 \times 10^6 M_\odot \]
Supermassive black holes

Selection of 6 supermassive black holes:

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<th>Distance $[10^6 \text{ al}]$</th>
<th>Diamètre apparent $[10^{-6} \text{ } \text{&quot;}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sgr A*</td>
<td>$4.3 \pm 0.3 \ 10^6$</td>
<td>?</td>
<td>0.027</td>
<td>53</td>
</tr>
<tr>
<td>M31</td>
<td>$1.6 \pm 0.5 \ 10^8$</td>
<td>?</td>
<td>2.5</td>
<td>20</td>
</tr>
<tr>
<td>M81</td>
<td>$8 \pm 2 \ 10^7$</td>
<td>?</td>
<td>13</td>
<td>2</td>
</tr>
<tr>
<td>NGC 4258</td>
<td>$3.78 \pm 0.01 \ 10^7$</td>
<td>?</td>
<td>23</td>
<td>0.5</td>
</tr>
<tr>
<td>M87</td>
<td>$3.6 \pm 1.0 \ 10^9$</td>
<td>?</td>
<td>55</td>
<td>21</td>
</tr>
<tr>
<td>MCG-6-30-15</td>
<td>$4 \pm 2 \ 10^6$</td>
<td>$0.989 \pm 0.009$</td>
<td>120</td>
<td>0.01</td>
</tr>
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</table>
Better than the mass criterion: clues for a horizon!

Luminosity of X-ray binaries during quiescent stages: the systems with a black hole (●) are \( \sim 100 \) times less luminous than those with a neutron star (○)

\[ \log [L_{\text{min}} \text{ (erg s}^{-1})] \]\n
\[ P_{\text{orb}} \text{ (hr)} \]

Beyond the mass: measuring the spin

Innermost Stable Circular Orbit (ISCO):

\[ R_{\text{ISCO}}(a = 0) = \frac{6GM}{c^2} \quad \text{and} \quad R_{\text{ISCO}}(a = 1) = \frac{GM}{c^2} \]

The internal edge of the accretion disk is located at the ISCO

Comparison of the X-ray spectrum to an emission model \(\Rightarrow\) estimation of \(a\)

[NASA/CXC/M. Weiss]
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Seeing the black hole shadow

Largest black-hole apparent sizes in the Earth’s sky:

\textbf{Sgr A* : } $D = 53 \, \mu\text{as}$

\textbf{M87 : } $D = 21 \, \mu\text{as}$

\textbf{M31 : } $D = 20 \, \mu\text{as}$

\textit{Rem. 1:} black holes in X-ray binaries are $\sim 10^5$ times smaller, for $D \propto M/d$

\textit{Rem. 2:} HST angular resolution: $D_{\text{min}} \sim 10^5 \, \mu\text{as} !$

Thin accretion disk

[Vincent, Paumard, Gourgoulhon & Perrin, CQG 28, 225011 (2011)]
Part 1  The near-future observations of black holes

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Rem. 2: HST angular resolution: $D_{\text{min}} \sim 10^5 \mu\text{as}$
The solution to reach the \( \mu \text{as} \) regime: interferometry!

**Existing American VLBI network** [Doeleman et al. 2011]

**Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves**
The near-future observations of black holes

The solution to reach the $\mu$as regime: interferometry!

Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

The best result so far: VLBI observations at 1.3 mm have shown that the size of the emitting region in Sgr A* is only 37 $\mu$as.

Existing American VLBI network [Doeleman et al. 2011]
Atacama Large Millimeter Array (ALMA)
part of the Event Horizon Telescope (EHT) to be completed by 2020
The near future: the Event Horizon Telescope

Simulations of VLBI observations of Sgr A* at $\lambda = 0.8$ mm

*left:* perfect image, *centre:* 7 stations ($\sim 2015$), *right:* 13 stations ($\sim 2020$)

$a = 0$, $i = 30^\circ$

[Fish & Doeleman, arXiv:0906.4040 (2009)]
The near future: the Event Horizon Telescope

Simulations of VLBI observations of Sgr A* at $\lambda = 0.8$ mm

*left*: perfect image, *centre*: 7 stations ($\sim 2015$), *right*: 13 stations ($\sim 2020$)

*top*: $a = 0.5$, $i = 85^\circ$; *bottom*: $a = 0$, $i = 60^\circ$

[Doeleman et al. (2009)]
Near-infrared optical interferometry

[Image of VLT telescopes with GRAVITY instrument indicated]

GRAVITY instrument at VLT (2014)

Beam combiner (the four 8 m telescopes + four auxiliary telescopes) \( \implies \) astrometric precision of \( 10 \mu \text{as} \)

[Gillessen et al. 2010]
Simulations of GRAVITY observations

[Simulated observation of 3 stars, m =15, whole night integration]

[Associated CLEANed image]

Observation of 3 stars of magnitude 15 during a whole night.

[Vincent et al., MNRAS 412, 2653 (2011)]
Testing the no-hair theorem

GRAVITY is expected to observe stars on relativistic orbits (closer than S2).

Measure of relativistic effects:
- periastron advance
- Lense-Thirring precession

\[ \Rightarrow \text{constraints on the spacetime metric in the vicinity of the central object} \]
\[ \Rightarrow \text{is it really the Kerr metric } (a, M) ? \]
Another future observational mean: gravitational waves

Gravitational waves = perturbations in the spacetime curvature
- reveal the spacetime dynamics
- generated by matter or black hole acceleration
- far from sources, are propagating at the speed of light
- NB: electromagnetic waves are perturbation of the electromagnetic field propagating within spacetime, whereas gravitational waves are waves of spacetime itself

[Baker et al., 2006]
Detection of gravitational waves

Interferometric detector VIRGO at Cascina, near Pisa [CNRS/INFN]
Outline

Part 1
- What is a black hole?
- Overview of the black hole theory
- The current observational status of black holes
- The near-future observations of black holes

Part 2
- Tests of gravitation
- The Gyoto tool
- Ray-tracing in numerical spacetimes
Part 1
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### Theoretical alternatives to the Kerr black hole

#### Within general relativity
- boson stars
- gravastar
- Q-star
- dark stars
- ...

#### Beyond general relativity
“hairy” black holes
- in Einstein-Yang-Mills
- in Einstein-Gauss-Bonnet with dilaton
- in Chern-Simons gravity
- ...

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Éric Gourgoulhon (LUTH)  
Black holes physics entering a new observational era  
FDP, Tours, 29 November 2012
Search for

- stellar orbits deviating from Kerr timelike geodesics (GRAVITY)
- accretion disk spectra different from those arising in Kerr metric (X-ray observatories)
- images of the black hole shadow different from that of a Kerr black hole (EHT)
How to test the alternatives?

Search for
- stellar orbits deviating from Kerr timelike geodesics (GRAVITY)
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Need for a good and versatile geodesic integrator
to compute timelike geodesics (orbits) and null geodesics (ray-tracing) in any kind of metric
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Integration of geodesics in Kerr metric
Integration of geodesics in any numerically computed 3+1 metric
Radiative transfer included in optically thin media
Very modular code (C++)
Yorick interface
Free software (GPL):
http://gyoto.obspm.fr/

[Vincent, Paumard, Gourgoulhon & Perrin, CQG 28, 225011 (2011)]
[Vincent, Gourgoulhon & Novak, CQG 29, 245005 (2012)]
Computed images of a thin accretion disk around a Schwarzschild black hole
Measuring the spin from the black hole silhouette

Spin parameter of a Kerr black hole: \( a = \frac{J}{M} \)

Accretion structure around Sgr A* modeled as a ion torus, derived from the polish doughnut class [Abramowicz, Jaroszynski & Sikora (1978)]

Radiative transfer included (thermal synchrotron, bremsstrahlung, inverse Compton)

\( \text{Image of an ion torus computed with Gyoto for the inclination angle } i = 80^\circ: \)

- black: \( a = 0.5M \)
- red: \( a = 0.9M \)

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Numerical spacetimes are generally computed within the 3+1 formalism for general relativity.

4-dimensional spacetime \((\mathcal{M}, g)\) foliated by spacelike hypersurfaces \((\Sigma_t)_{t \in \mathbb{R}}\)

Unit timelike normal: \(\mathbf{n} = -N \nabla t\)

Induced metric: \(\gamma = g + \mathbf{n} \otimes \mathbf{n}\)

Shift vector of adapted coordinates \((t, x^i)\): vector \(\mathbf{\beta}\) tangent to \(\Sigma_t\) such that \(\partial/\partial t = N \mathbf{n} + \mathbf{\beta}\)

\[
g_{\mu\nu} \, dx^\mu \, dx^\nu = -N^2 dt^2 + \gamma_{ij} (dx^i + \beta^i dt)(dx^j + \beta^j dt)
\]
A particle $\mathcal{P}$ of 4-momentum vector $p$ follows a geodesic iff $\nabla_pp = 0$

3+1 decomposition of $p$: $p = E(n + V)$, with

- $E$: particle’s energy with respect to the Eulerian observer (4-velocity $n$)
- $V$: vector tangent to $\Sigma_t$, representing the particle’s 3-velocity with respect to the Eulerian observer
3+1 decomposition of the geodesic equation (2/2)

Equation of \( \mathcal{P} \)'s worldline in terms of the 3+1 coordinates: \( x^i = X^i(t) \)

The physical 3-velocity \( \mathbf{V} \) is related to the coordinate velocity \( \dot{X}^i := \frac{dx^i}{dt} \) by

\[
V^i = \frac{1}{N} \left( \dot{X}^i + \beta^i \right)
\]

Orth. projection of \( \nabla_p p = 0 \) along \( n \):

\[
\frac{dE}{dt} = E \left( NK_{jk} V^j V^k - V^j \partial_j N \right)
\]

Orth. projection of \( \nabla_p p = 0 \) onto \( \Sigma_t \):

\[
\begin{align*}
\frac{dX^i}{dt} &= NV^i - \beta^i \\
\frac{dV^i}{dt} &= NV^j \left[ V^i \left( \partial_j \ln N - K_{jk} V^k \right) + 2K^i_j - 3\Gamma_{jk}^i V^k \right] - \gamma^{ij} \partial_j N - V^j \partial_j \beta^i
\end{align*}
\]

[Vincent, Gourgoulhon & Novak, CQG 29, 245005 (2012)]
Metric in 3+1 form is obtained from

- analytic solution (e.g. Kerr) \( \Rightarrow \) tests
- rotating neutron star model code LORENE/nrotstar
- simulation of a neutron star collapsing to a black hole with CoCoNuT

Gravitational fields are computed using spectral methods and represented by a set of coefficients \( \{ c_{i\ell m} \}_{(i,\ell,m) \in [0 \ldots N]} \):

\[
f(r, \theta, \varphi) = \sum_{i,\ell,m} c_{i\ell m} T_i(r) Y_{\ell}^{m}(\theta, \varphi)
\]

\( \Rightarrow \) metric fields can be evaluated at any spatial point with this triple sum.

Integration of geodesics is done backward: from observer to the object, using a RK4, with adaptive step.
Test on Kerr spacetime

Integration of a null geodesic in the Kerr metric, using “numerical” (LORENE-prepared) metric fields in Boyer-Lindquist coordinates and 3+1 approach.

Comparison with integration using analytical expressions for the metric, with $a = 0.5M$.

Accuracy on $(r(t), \theta(t), \varphi(t))$ for:

- $t = 1000M, r = 100M \rightarrow t = 0, r = 865M$
- the smallest distance $r = 4.3M \, @\, t \sim 900M$. 
Stationary neutron star

Rapidly rotating neutron star generated by LORENE/nrotstar

- EOS of Akmal, Pandharipande & Ravenhall
- $1.4\,M_\odot$ gravitational mass
- static or rotating with $f = 716$ Hz
- optically thick, emitting as a blackbody at $10^6$ K

Map of specific intensity in $W\,m^{-2}\,ster^{-1}\,Hz^{-1}$

$\implies$ check of conservation of $p_t\,(10^{-6}), p_\varphi\,(10^{-4})$ and $p_\mu p^\mu\,(10^{-5})$ along the geodesics
Ray-tracing in dynamical spacetimes: collapse to a black hole

Spacetime generated by the CoCoNuT code

Initial data:

- spherically symmetric neutron star on the unstable branch
- polytropic EoS, $\gamma = 2$, $M_{\text{grav}} = 1.62M_\odot$, $M_{\text{bar}} = 1.77M_\odot$
- initial perturbation $\rho \rightarrow \rho \left[ 1 + 0.01 \sin \left( \frac{\pi r}{10 \text{ km}} \right) \right]$
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- initial perturbation $\rho \to \rho \left[1 + 0.01 \sin \left(\frac{\pi r}{10 \text{ km}}\right)\right]$ sent to CoCoNuT, run with 500 radial cells.

- at $t = 0.438$ ms, appearance of the apparent horizon
- at $t = 0.495$ ms, 99.99% of matter is inside the AH
- run is stopped when too strong gradients appear on metric (maximal slicing)

$\Longrightarrow$ 3+1 metric $(N, \beta^i, \gamma_{ij})$, $K_{ij}$, fluid velocity $u^\mu$, radius of the star and/or AH exported at every time-step to GYOTO

$\Longrightarrow$ 3rd-order interpolation in time to integrate geodesic equations
Ray-tracing in dynamical spacetimes: collapse to a black hole

Integration backward until reaching the star’s surface or the apparent horizon.

Surface of the star: blackbody at $10^6$ K. Intensity given in logarithmic scale.

- Coordinate radius of the star 7 km (left) → 2.9 km (right).
- Relativistic bending of light rays → apparent radius larger.
- Event horizon first appear at the centre, closer to the observer.