Compact objects and strange quark stars

Eric Gourgoulhon

Laboratoire Univers et Théories (LUTH)
CNRS / Observatoire de Paris / Université Paris Diderot
F-92190 Meudon, France
eric.gourgoulhon@obspm.fr
http://www.luth.obspm.fr/~luthier/gourgoulhon/

Journées de la division Physique Nucléaire, SFP

Du plasma de quarks et de gluons aux étoiles à neutrons

Nantes, 13-14 May 2008
Plan

1. Compact stars in general relativity
2. Confronting theoretical models and observations: astrophysics as a lab
3. The search for strange stars
4. Gravitational wave observations
1. Compact stars in general relativity

2. Confronting theoretical models and observations: astrophysics as a lab

3. The search for strange stars

4. Gravitational wave observations
Density and compactness

Spherical object of mass $M$ and radius $R$

- **density**: $\rho = \frac{M}{\frac{4}{3} \pi R^3}$

- **compactness**: $\Xi := \frac{GM}{c^2 R} \sim \frac{|E_{\text{grav}}|}{M c^2} \sim \frac{\Phi_{\text{surf}}}{c^2} \sim \frac{V_{\text{esc}}^2}{c^2} \sim \frac{R_S}{R}$

$E_{\text{grav}}$ = gravitational potential energy; $\Phi_{\text{surf}}$ = gravitational potential at the surface; $V_{\text{esc}}$ = escape velocity from surf.; $R_S$ = Schwarzschild radius

Remark:

- $\Xi_{\text{Earth}} \sim 10^{-10}$
- $\Xi_{\text{Sun}} \sim 10^{-6}$
- $\Xi_{\text{white dwarf}} \sim 10^{-4}$
- $\Xi_{\text{black hole}} \sim 1$
Spherical object of mass $M$ and radius $R$

- **Density**: $\rho = \frac{M}{\frac{4}{3}\pi R^3}$

- **Compactness**: $\Xi := \frac{GM}{c^2 R} \sim \left| \frac{E_{\text{grav}}}{Mc^2} \right| \sim \left| \frac{\Phi_{\text{surf}}}{c^2} \right| \sim \frac{V_{\text{esc}}^2}{c^2} \sim \frac{R_S}{R}$

$E_{\text{grav}} =$ gravitational potential energy; $\Phi_{\text{surf}} =$ gravitational potential at the surface; $V_{\text{esc}} =$ escape velocity from surf.; $R_S =$ Schwarzschild radius

$\Xi = \frac{4\pi G}{3c^2} \rho R^2 \Rightarrow \Xi = 6 \times 10^{-10} \left( \frac{R}{\text{m}} \right)^2$ for $\rho = \rho_{\text{nuc}} = 2 \times 10^{17} \text{ kg m}^{-3}$
Density and compactness

Spherical object of mass $M$ and radius $R$

- **density**: $\rho = \frac{M}{\frac{4}{3}\pi R^3}$

- **compactness**: $\Xi := \frac{GM}{c^2 R} \sim \frac{|E_{\text{grav}}|}{Mc^2} \sim \frac{|\Phi_{\text{surf}}|}{c^2} \sim \frac{V_{\text{esc}}^2}{c^2} \sim \frac{R_S}{R}$

$E_{\text{grav}} = \text{gravitational potential energy}; \ \Phi_{\text{surf}} = \text{gravitational potential at the surface}; \ V_{\text{esc}} = \text{escape velocity from surf.}; \ R_S = \text{Schwarzschild radius}$

$\Xi = \frac{4\pi G}{3c^2} \rho R^2 \Rightarrow \Xi = 6 \times 10^{-10} \left(\frac{R}{\text{m}}\right)^2$ for $\rho = \rho_{\text{nuc}} = 2 \times 10^{17} \text{ kg m}^{-3}$

**proton**

$R = 10^{-15} \text{ m} \Rightarrow \Xi \sim 10^{-39}$

no need for general relativity
Density and compactness

Spherical object of mass $M$ and radius $R$

- **density**: 
  \[ \rho = \frac{M}{\frac{4}{3} \pi R^3} \]

- **compactness**: 
  \[ \Xi := \frac{GM}{c^2 R} \sim \frac{|E_{\text{grav}}|}{Mc^2} \sim \frac{\Phi_{\text{surf}}}{c^2} \sim \frac{V_{\text{esc}}^2}{c^2} \sim \frac{R_S}{R} \]

$E_{\text{grav}}$ = gravitational potential energy; $\Phi_{\text{surf}}$ = gravitational potential at the surface; $V_{\text{esc}}$ = escape velocity from surf.; $R_S$ = Schwarzschild radius

\[ \Xi = \frac{4\pi G}{3c^2} \rho R^2 \Rightarrow \Xi = 6 \times 10^{-10} \left( \frac{R}{\text{m}} \right)^2 \text{ for } \rho = \rho_{\text{nuc}} = 2 \times 10^{17} \text{ kg m}^{-3} \]

<table>
<thead>
<tr>
<th>proton</th>
<th>neutron star</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R = 10^{-15}$ m $\Rightarrow \Xi \sim 10^{-39}$</td>
<td>$R = 10^4$ m $\Rightarrow \Xi \sim 0.1$</td>
</tr>
<tr>
<td>no need for general relativity</td>
<td>general relativity required !</td>
</tr>
</tbody>
</table>
Density and compactness

Spherical object of mass $M$ and radius $R$

- **density**: $\rho = \frac{M}{\frac{4}{3} \pi R^3}$

- **compactness**: $\Xi := \frac{GM}{c^2 R} \sim \frac{|E_{\text{grav}}|}{M c^2} \sim \frac{\phi_{\text{surf}}}{c^2} \sim \frac{V_{\text{esc}}^2}{c^2} \sim \frac{R_S}{R}$

$E_{\text{grav}}$ = gravitational potential energy; $\phi_{\text{surf}}$ = gravitational potential at the surface; $V_{\text{esc}}$ = escape velocity from surf.; $R_S$ = Schwarzschild radius

$\Xi = \frac{4 \pi G}{3 c^2} \rho R^2 \implies \Xi = 6 \times 10^{-10} \left(\frac{R}{\text{m}}\right)^2$ for $\rho = \rho_{\text{nuc}} = 2 \times 10^{17} \text{ kg m}^{-3}$

**proton**

$R = 10^{-15} \text{ m} \Rightarrow \Xi \sim 10^{-39}$

no need for general relativity

**neutron star**

$R = 10^4 \text{ m} \Rightarrow \Xi \sim 0.1$

general relativity required!

**Remark**: $\Xi_{\text{Earth}} \sim 10^{-10}$ $\Xi_{\text{Sun}} \sim 10^{-6}$ $\Xi_{\text{white dwarf}} \sim 10^{-4}$ $\Xi_{\text{black hole}} \sim 1$
Einstein equation

$$R - \frac{1}{2} Rg = \frac{8\pi G}{c^4} T$$

- $g$ = metric tensor on the 4-dimensional spacetime manifold $\mathcal{M}$
- $R$ = Ricci tensor; $R = \text{tr}_g R$
- $T$ = matter stress-energy tensor
  perfect fluid: $T = (e + p)u \otimes u + pg$ ($e$ = proper energy density, $p$ = pressure, $u$ = fluid 4-velocity)
Einstein equation

\[ R - \frac{1}{2} Rg = \frac{8\pi G}{c^4} T \]

- \( g \) = metric tensor on the 4-dimensional spacetime manifold \( \mathcal{M} \)
- \( R \) = Ricci tensor ; \( R = \text{tr}_g R \)
- \( T \) = matter stress-energy tensor
  - perfect fluid : \( T = (e + p)u \otimes u + pg \) (\( e \) = proper energy density, \( p \) = pressure, \( u \) = fluid 4-velocity)

Coordinate system \((x^\alpha)\) on \( \mathcal{M} \) \( \Rightarrow \) Einstein equation results in a system of 10 **coupled non-linear second order partial differential equations** for the 10 coefficients \( g_{\alpha\beta} \) of the metric tensor

\[
R_{\alpha\beta} = -\frac{1}{2} g^{\mu\nu} \left( \frac{\partial^2 g_{\alpha\beta}}{\partial x^\mu \partial x^\nu} + \frac{\partial^2 g_{\mu\nu}}{\partial x^\alpha \partial x^\beta} - \frac{\partial^2 g_{\nu\beta}}{\partial x^\alpha \partial x^\mu} - \frac{\partial^2 g_{\alpha\nu}}{\partial x^\beta \partial x^\mu} \right) + Q_{\alpha\beta} \left( g_{\mu\nu}, \frac{\partial g_{\mu\nu}}{\partial x^\rho} \right)
\]

\[
R = g^{\mu\nu} R_{\mu\nu} \quad T_{\alpha\beta} = (e + p)u_\alpha u_\beta + pg_{\alpha\beta}
\]
Stationary rotating star

- **stationarity**: ∃ coordinates \( (x^\alpha) \) on \( \mathcal{M} \) such that \( \frac{\partial g_{\alpha\beta}}{\partial x^0} = 0 \) and \( k := \frac{\partial}{\partial x^0} \) asymptotically timelike

- **axisymmetry**: ∃ coordinates \( (x^\alpha) \) on \( \mathcal{M} \) such that \( \frac{\partial g_{\alpha\beta}}{\partial x^3} = 0 \), \( m := \frac{\partial}{\partial x^3} \) spacelike, vanishes on a 2-surface (rotation axis) and has closed orbits \( k \) and \( m \) are called **Killing vectors** associated with resp. stationarity and axisymmetry

Stationarity + axisymmetry \( \Rightarrow \exists \) coord. \( (x^\alpha) = (t, r, \theta, \varphi) \) on \( \mathcal{M} \) such that

\[
g_{\alpha\beta} = g_{\alpha\beta}(r, \theta)
\]
Stationary rotating star

- **stationarity**: ∃ coordinates \((x^\alpha)\) on \(\mathcal{M}\) such that \(\frac{\partial g_{\alpha\beta}}{\partial x^0} = 0\) and \(k := \frac{\partial}{\partial x^0}\) asymptotically timelike

- **axisymmetry**: ∃ coordinates \((x^\alpha)\) on \(\mathcal{M}\) such that \(\frac{\partial g_{\alpha\beta}}{\partial x^3} = 0\), \(m := \frac{\partial}{\partial x^3}\) spacelike, vanishes on a 2-surface (rotation axis) and has closed orbits \(k\) and \(m\) are called **Killing vectors** associated with resp. stationarity and axisymmetry

Stationarity + axisymmetry \(\Rightarrow\) ∃ coord. \((x^\alpha) = (t, r, \theta, \varphi)\) on \(\mathcal{M}\) such that

\[
g_{\alpha\beta} = g_{\alpha\beta}(r, \theta)
\]

Another important simplification: **Papapetrou theorem**:

if \(u = u^0 k + u^\varphi m\) (circular motion), then ∃ coordinates \((x^\alpha) = (t, r, \theta, \varphi)\) on \(\mathcal{M}\) such that \(g_{tr} = 0, g_{t\theta} = 0, g_{r\theta} = 0, g_{r\varphi} = 0, g_{\theta\varphi} = 0\), i.e.

\[
g_{\alpha\beta} dx^\alpha dx^\beta = -N^2 dt^2 + A^2 (dr^2 + r^2 d\theta^2) + B^2 r^2 \sin^2 \theta (d\varphi + \beta^\varphi dt)^2
\]

\[
N = N(r, \theta), \quad \beta^\varphi = \beta^\varphi(r, \theta), \quad A = A(r, \theta), \quad B = B(r, \theta)
\]

(quasi-isotropic coordinates)
Stationary rotating star in QI coordinates

\[ g_{\alpha\beta}dx^\alpha dx^\beta = -N^2 c^2 dt^2 + A^2 (dr^2 + r^2 d\theta^2) + B^2 r^2 \sin^2 \theta (d\varphi + \beta\varphi dt)^2 \]

\[ N = N(r, \theta), \quad \beta\varphi = \beta\varphi(r, \theta), \quad A = A(r, \theta), \quad B = B(r, \theta) \]

Important limits:

- Vanishing gravitational field: \( N \to 1, \quad \beta\varphi \to 0, \quad A \to 1, \quad B \to 1 \)

\[ g_{\alpha\beta}dx^\alpha dx^\beta = -c^2 dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2 d\varphi^2 \]

(Minkowski metric in spherical coordinates)
Stationary rotating star in QI coordinates

\[ g_{\alpha\beta}dx^\alpha dx^\beta = -N^2 c^2 dt^2 + A^2 \left( dr^2 + r^2 d\theta^2 \right) + B^2 r^2 \sin^2 \theta \left( d\varphi + \beta^\varphi dt \right)^2 \]

\[ N = N(r, \theta), \quad \beta^\varphi = \beta^\varphi(r, \theta), \quad A = A(r, \theta), \quad B = B(r, \theta) \]

Important limits:

- **Vanishing gravitational field**: \( N \to 1, \beta^\varphi \to 0, A \to 1, B \to 1 \)

\[ g_{\alpha\beta}dx^\alpha dx^\beta = -c^2 dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \]

(Minkowski metric in spherical coordinates)

- **Spherical symmetry**: \( \beta^\varphi \to 0, A - B \to 0 \)

\[ g_{\alpha\beta}dx^\alpha dx^\beta = -N^2 c^2 dt^2 + A^2 \left( dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right) \]
Stationary rotating star in QI coordinates

\[ g_{\alpha\beta} dx^{\alpha} dx^{\beta} = -N^2 c^2 dt^2 + A^2 \left( dr^2 + r^2 d\theta^2 \right) + B^2 r^2 \sin^2 \theta \left( d\varphi + \beta^\varphi dt \right)^2 \]

\[ N = N(r, \theta), \quad \beta^\varphi = \beta^\varphi(r, \theta), \quad A = A(r, \theta), \quad B = B(r, \theta) \]

Important limits:

- Vanishing gravitational field: \( N \to 1, \beta^\varphi \to 0, A \to 1, B \to 1 \)

\[ g_{\alpha\beta} dx^{\alpha} dx^{\beta} = -c^2 dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \]

(Minkowski metric in spherical coordinates)

- Spherical symmetry: \( \beta^\varphi \to 0, A - B \to 0 \)

\[ g_{\alpha\beta} dx^{\alpha} dx^{\beta} = -N^2 c^2 dt^2 + A^2 \left( dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right) \]

- Weak gravitational field:

\[ g_{\alpha\beta} dx^{\alpha} dx^{\beta} = - \left( 1 + 2 \frac{\Phi}{c^2} \right) c^2 dt^2 + \left( 1 - 2 \frac{\Phi}{c^2} \right) \left( dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right) \]

\[ \Phi \sim \text{Newtonian gravitational potential}; \quad |\Phi_{\text{surf}}|/c^2 = \Xi \text{ (compactness)} \]
In QI coord., the Einstein equations reduce to 4 elliptic equations:

\[
\Delta_3 \nu = 4\pi A^2 (E + S^i_i) + \frac{B^2 r^2 \sin^2 \theta}{2N^2} (\partial \beta \varphi)^2 - \partial \nu \partial (\nu + \ln B)
\]

\[
\tilde{\Delta}_3 (\beta \varphi r \sin \theta) = 16\pi \frac{NA^2}{B^2} \frac{J_\varphi}{r \sin \theta} - r \sin \theta \partial \beta \varphi \partial (3 \ln B - \nu)
\]

\[
\Delta_2 [(NB - 1) r \sin \theta] = 8\pi NA^2 B (S^r_r + S^\theta_\theta) r \sin \theta
\]

\[
\Delta_2 \zeta = 8\pi A^2 S^\varphi \varphi + \frac{3B^2 r^2 \sin^2 \theta}{4N^2} (\partial \beta \varphi)^2 - (\partial \nu)^2
\]

with the abbreviations: \( \nu := \ln N \), \( \zeta := \ln(AN) \)

\[
\Delta_2 := \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}
\]

\[
\Delta_3 := \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{1}{r^2 \tan \theta} \frac{\partial}{\partial \theta}
\]

\[
\tilde{\Delta}_3 := \Delta_3 - \frac{1}{r^2 \sin^2 \theta}
\]

\[\partial a \partial b := \frac{\partial a}{\partial r} \frac{\partial b}{\partial r} + \frac{1}{r^2} \frac{\partial a}{\partial \theta} \frac{\partial b}{\partial \theta} \]
Fluid equations

Energy momentum conservation: \( \nabla \cdot T = 0 \)

Circular motion: \( u = \lambda \ell \) with \( \ell = k + \Omega m \), \( \Omega \) = angular rotation velocity

Rigid rotation: \( \Omega = \text{const} \Rightarrow \ell \) Killing vector \( \Rightarrow \exists \) a first integral to \( \nabla \cdot T = 0 \):

\[ \ell \cdot (hu) = \text{const} \]

with \( h := \frac{e + p}{m_B n c^2} \) (specific enthalpy).

Newtonian limit: \( (h - 1) + \Phi - \frac{1}{2}((\Omega \times r)^2 = \text{const} \)
Compact stars in general relativity

**Fluid equations**

Energy momentum conservation: \( \nabla \cdot T = 0 \)

**Circular motion** : \( u = \lambda \ell \) with \( \ell = k + \Omega m \), \( \Omega = \) angular rotation velocity

**Rigid rotation** : \( \Omega = \text{const} \) \( \Rightarrow \ell \) Killing vector \( \Rightarrow \exists \) a first integral to \( \nabla \cdot T = 0 \):
\[
\ell \cdot (hu) = \text{const}
\]

with \( h := \frac{e + p}{m_B n c^2} \) (specific enthalpy).

Newtonian limit: \( (h - 1) + \Phi - \frac{1}{2}(\Omega \times r)^2 = \text{const} \)

**Equation of state (EOS)**: cold dense matter: \( e = e(h), \quad p = p(h), \quad n = n(h) \)
Energy momentum conservation: \( \nabla \cdot T = 0 \)

Circular motion: \( u = \lambda \ell \) with \( \ell = k + \Omega m \), \( \Omega \) = angular rotation velocity

Rigid rotation: \( \Omega = \text{const} \) \( \Rightarrow \ell \) Killing vector \( \Rightarrow \exists \) a first integral to \( \nabla \cdot T = 0 \):
\[
\ell \cdot (hu) = \text{const}
\]

with \( h := \frac{e + p}{m_B n c^2} \) (specific enthalpy).

Newtonian limit: \( (h - 1) + \Phi - \frac{1}{2}(\Omega \times r)^2 = \text{const} \)

Equation of state (EOS): cold dense matter: \( e = e(h), \ p = p(h), \ n = n(h) \)

closed system of equations

At fixed EOS, a model is fully specified by two parameters, e.g. central density and \( \Omega \).
Framework: **RotStar** code based on **LORENE C++ library**

http://www.lorene.obspm.fr/

Resolution of Einstein equations for stationary axisymmetric rotating stars

*Numerical technique: spectral methods*
⇒ high accuracy

*Microphysics input: equation of state (EOS)*
Examples of numerical results

Rapidly rotating model

Enthalpy isocontours (f=1 kHz)

Isocontour of specific enthalpy $h$:

$h < 1 \iff$ stellar exterior (dashed lines);
$h = 1 \iff$ stellar surface (thick solid line);
$h > 1 \iff$ stellar interior (solid line)

Maximum rotation rate
(Keplerian limit)

Enthalpy isocontours Keplerian frequency

Eric Gourgoulhon  (LUTH)
Compact objects and strange quark stars
Nantes, 14 May 2008
Global quantities

- $M$ : gravitational mass (*)
- $M_B$ : baryon mass $= m_B \times$ total baryon number
- $J$ : total angular momentum (about the rotation axis) (*)
- $\Omega$ : angular rotation frequency (as seen from infinity)
- $R$ : circumferential radius of the equator
- $z_p, z_{\pm eq}$ : redshifts from pole, and equator (forward and backward) (*)
- $f_{\text{ISCO}}$ : orbital frequency at the innermost stable circular orbit (*)

(*) = measurable quantities
Outline

1. Compact stars in general relativity

2. Confronting theoretical models and observations: astrophysics as a lab

3. The search for strange stars

4. Gravitational wave observations
Our (poor) knowledge of matter at supernuclear densities

Internal structure of compact stars

Confronting theoretical models and observations: astrophysics as a lab

Confronting theoretical models and observations: astrophysics as a lab

Large discrepancies in theoretical models...

adiabatic index

non-rotating neutron star mass

[Haensel, Potekhin & Yakovlev (2007)]
Measured neutron star masses

Measured rotation velocity

\[ \dot{P} \text{ vs period } P \]

Fastest pulsar known to date:
PSR J1748-2446ad
\[ P = 1.396 \text{ ms} \quad f = 716 \text{ Hz} \]

[Hessels et al., Science 311, 1901 (2006)]
Measured rotation velocity

\[ \dot{P} \text{ vs } P \]

for radio pulsars

\[ \text{Fastest pulsar known to date:} \]
\[ \text{PSR J1748-2446ad} \]
\[ P = 1.396 \text{ ms} \quad f = 716 \text{ Hz} \]

\[ \text{Discovery of an oscillation frequency} \]
\[ f = 1122 \text{ Hz} \]
\[ \text{in a X-ray burst from X-ray transient XTE J1739-285} \]

\[ \text{Spin frequency? This would imply } P = 0.89 \text{ ms!} \]

\[ \text{Not confirmed yet!} \]
Confronting theoretical models and observations: astrophysics as a lab

Impact of the discovery of very high rotation frequencies

Mass-radius relation at $f = 1122$ Hz

Confronting theoretical models and observations: astrophysics as a lab

Impact of the discovery of very high rotation frequencies

Mass-radius relation at \( f = 1122 \) Hz

\[
\begin{align*}
M [M_\odot] & \quad R_{eq} [\text{km}] \\
2.4 & \quad 10 \\
2.2 & \quad 12 \\
2.0 & \quad 14 \\
1.8 & \quad 16 \\
1.6 & \quad 18 \\
1.4 & \quad 20
\end{align*}
\]

- APR
- DH
- GN3
- GNH3
- BBB2
- FPS
- BGN1H1
- BPAL12
- GMGS\(p\)
- GMGS\(m\)


Mass-radius relation at fixed \( f \)

\[
\begin{align*}
M [M_\odot] & \quad R_{eq} [\text{km}] \\
2.5 & \quad 8 \\
2.0 & \quad 10 \\
1.5 & \quad 12 \\
1.0 & \quad 14 \\
\end{align*}
\]

- 1000 Hz
- 1200 Hz
- 1400 Hz
- 1600 Hz

[Zdunik, Haensel, Bejger & Gourgoulhon, arXiv:0710.5010]
Measured compactness (M/R)

Observation (XMM-Newton) of iron and oxygen spectral lines from the compact star in the low-mass X-ray binary EXO 07748-676

\[ z = \frac{\lambda_{\text{obs}} - \lambda}{\lambda} = 0.35 \quad (\text{NB: } z_{\text{Doppler}} \sim 10^{-3}) \]

\[ z = (1 - 2\Xi)^{-1/2} - 1 = 0.35 \quad \Rightarrow \quad \Xi = \frac{GM}{c^2 R} = 0.23 \]

Unfortunately we know neither \( M \) nor \( R \) for this system...
Hyperon = baryon (i.e. hadron + fermion) made of 3 quarks, with at least one strange quark:
- $\Lambda_0 = uds$
- $\Sigma^{-} = dds$
- $\Xi^{0} =uss$
- etc...

Should appear at high density ($\rho > 2\rho_{\text{nuc}}$) 
$\Rightarrow$ EOS softening

$N_1 = np$, $N_1H_1, N_2H_1 = np\Lambda\Sigma$, $N_1H_2, N_2H_2 = np\Lambda\Sigma\Xi$

Balberg & Gal (1997)
Hyperon softening of the EOS $\Rightarrow$ back-bending: spin-up by angular momentum loss

Detectability: pulsar with $\dot{P} < 0$

The search for strange stars

Outline

1. Compact stars in general relativity
2. Confronting theoretical models and observations: astrophysics as a lab
3. The search for strange stars
4. Gravitational wave observations
The search for strange stars

A short history of strange stars

- **1970:** N. Itoh considered compact stars made of free degenerate Fermi gas of \(u, d\) and \(s\) quarks of equal mass \(m_q = 10\ \text{GeV}/\text{c}^2\) (not self-bound, vanishing density at the surface) \(\Rightarrow M_{\text{max}} = 10^{-3} M_{\odot}\).

- **1971:** A.R. Bodmer: the ground state of nuclear matter may be a state of **deconfined quarks**.

- **1984:** E. Witten reformulated (independently) this idea, and contemplated the possibility that neutron stars are in fact **strange quark stars**.

- **1986:** first detailed numerical models of static strange stars by P. Haensel, J.L. Zdunik & R. Schaeffer, as well as C. Alcock, E. Farhi & A.V. Olinto.

- **1989:** announcement of a half-millisecond pulsar in SN 1987A

- **1996:** discovery of high frequency QPO in low-mass X-ray binaries

- **2002:** NASA announcement of “discovery” of two strange stars
Basic properties of strange stars

Simplest models: improved MIT bag model

\[ 3\text{-parameter EOS for SQM matter:} \]

- \( B \): bag constant,
- \( m_s \): mass of \( s \) quark,
- \( \alpha_s \): QCD structure constant \((\alpha_s = \frac{g^2}{4\pi}, g: \text{QCD coupling constant})\)

- finite density at the surface (zero pressure)
- for small mass (weak gravity): almost constant density profile \( \varepsilon \sim 4B \)
- more compact than neutron stars

\[ \text{figures from [Glendenning (1997)]} \]
From strangelets to strange stars

The search for strange stars

Rapidly rotating strange stars

Enthalpy

\[ x \text{ [km]} \]


Minimal rotation period (for \( m_s = 0 \) and \( \alpha_s = 0 \)): \( P_{\text{min}} = 0.634 B_{60}^{-1/2} \) ms
The search for strange stars

Models with solid crust

EOS: $B = 56$ MeV fm$^{-3}$, $\alpha_s = 0.2$, $m_s = 200$ MeV $c^{-2}$

star: $M_B = 1.63 \, M_\odot$, $f = 1210$ Hz.

Recent studies of quark matter

- Study of quark matter \((u,d,s)\) in color-flavor-locked (CFL) states ⇒ No stable configuration with uniquely CFL matter
  
  [Buballa, Neumann, Oertel & Shovkoy, PLB 595, 36 (2004)]

- Goldstone bosons in CFL states
  

- Color superconductivity effects on the properties of a strange star: electron atmosphere, transport properties
  
  [Oertel & Urban, PRD 77, 074015 (2008)]
The search for strange stars

The case of RX J1856.5-3754

• Discovered as an X-ray source with ROSAT in 1996 [Walter et al., Nature 379, 233 (1996)]
  Best fit black body \( kT_\infty = 57 \pm 1 \text{ eV} \)
  \( \iff T_\infty \simeq 6.6 \times 10^5 \text{ K} \)
  In front of molecular cloud \( R \ Coronae Australis \)
  \( \Rightarrow d \lesssim 130 - 170 \text{ pc} \)

• Optical counterpart discovered in 1997 with HST [Walter & Matthews, Nature 389, 358 (1997)]
  magnitude \( V = 25.6 \)
  Optical flux 2 to 3 times larger than the tail of the 57 eV black body
RX J1856.5-3754 observed by VLT

VLT Kueyen + FORS2 (field: 80" × 80")

→ bowshock (heated interstellar gas by accelerated $e^-$ and $p$ from the star?)

[ESO 2000]
**The search for strange stars**

**Distance to RX J1856.5-3754**

  \[d = 61 \pm 9 \text{ pc}\]

\[\Rightarrow\text{ erroneous } d = 61 \pm 9 \text{ pc}\]

- Correct determinations of parallax:
  \[d = 140 \pm 40 \text{ pc} \ [\text{Kaplan, van Kerkwijk, Anderson, ApJ 571, 447 (2002)}]\]
  \[d = 117 \pm 12 \text{ pc} \ [\text{Walter & Lattimer, ApJ 576, L145 (2002)}]\]
The search for strange stars

RX J1856.5-3754 spectrum

Chandra image of RX J1856.5-3754

Spectrum from Chandra, EUVE and HST data:
- - - - : black body best fit to Chandra data $kT_\infty = 63$ eV
  [Burwitz et al., A&A 379, L35 (2001)]

.........: 63 eV black body + 15 eV black body with $R_\infty(15 \text{ eV}) = 5R_\infty(63 \text{ eV})$
The search for strange stars

Is RX J1856.5-3754 a strange star?

The small “radius” issue:

Assuming black body emission from the entire surface:

\[
R_\infty = \frac{d}{T_\infty^2} \left(\frac{f_\infty}{\sigma}\right)^{1/2}
\]

\[
R_\infty = \left(1 - \frac{2GM}{c^2R}\right)^{-1/2} R > R
\]

- Distance of Walter & Lattimer 2002: \(d = 117\) pc \(\Rightarrow R_\infty = 4.8\) km
- Distance of Kaplan et al. 2002: \(d = 140\) pc \(\Rightarrow R_\infty = 5.8\) km
Is RX J1856.5-3754 a strange star?

The small “radius” issue:
Assuming black body emission from the entire surface:

\[ R_\infty = \frac{d}{T_\infty^2} \left( \frac{f_\infty}{\sigma} \right)^{1/2} \]

\[ R_\infty = \left( 1 - \frac{2GM}{c^2R} \right)^{-1/2} \quad R > R \]

- Distance of Walter & Lattimer 2002: \( d = 117 \) pc \( \Rightarrow R_\infty = 4.8 \) km
- Distance of Kaplan et al. 2002: \( d = 140 \) pc \( \Rightarrow R_\infty = 5.8 \) km

Recent discovery of pulsations of period \( P = 7.055 \) s from XMM-Newton observations (with a pulsed fraction of only 1.2%)


\( \Rightarrow \) Hot spot model favored today
Outline

1. Compact stars in general relativity
2. Confronting theoretical models and observations: astrophysics as a lab
3. The search for strange stars
4. Gravitational wave observations
Gravitational wave observations

Observing compact stars via gravitational waves

LIGO: USA, Louisiana

LIGO: USA, Washington

VIRGO: France/Italy (Pisa)

Michelson-type lasers
VIRGO (3 km) and LIGO (4 km) ⇒ they are currently acquiring data.
Gravitational wave observations

Constraints on EOS from gravitational radiation (1/2)

GW from inspiraling binary neutrons stars
Primary target for VIRGO / LIGO

← Irrotational binary configurations close to mass-shedding limit for GlendNH3, AkmalPR and BPAL12 EOS

Constraints on EOS from gravitational radiation (2/2)

3 nuclear matter EOS
3 strange matter EOS

\[
\begin{array}{cccccc}
R [\text{km}] & 0.25 & 0.5 & 0.75 & 1 & 1.25 & 1.5 & 1.75 & 2 & 2.25 & 2.5 \\
\end{array}
\]

Inspiraling sequences