Black holes, a centenary after the birth of general relativity

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Outline

1. Black holes: a century-old history
2. Some recent developments
3. Black holes in the sky
4. Observing black holes via gravitational waves: a dream come true
5. Testing general relativity with black holes
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A two centuries-old prehistory...

$$V_{\text{esc}} > c \iff \frac{2GM}{R} > c^2 \iff \frac{2G}{R} \times \frac{4}{3} \pi R^3 \rho > c^2 \iff \sqrt[3]{\frac{3c^2}{8\pi G \rho}}$$

John Michell (1784)

"If there should really exist in nature any bodies, whose density is not less than that of the sun, and whose diameters are more than 500 times the diameter of the sun, since their light could not arrive at us, ..., we could have no information from sight"

[Phil. Trans. R. Soc. Lond. 74, 35 (1784)]

Pierre Simon de Laplace (1796)

"Un astre lumineux, de la même densité que la Terre, et dont le diamètre serait 250 fois plus grand que le Soleil, ne permettrait, en vertu de son attraction, à aucun de ses rayons de parvenir jusqu'à nous. Il est dès lors possible que les plus grands corps lumineux de l'univers puissent, par cette cause, être invisibles."

[Exposition du système du monde (1796)]
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\[ V_{\text{esc}} > c \iff \frac{2GM}{R} > c^2 \iff \frac{2G}{R} \times \frac{4}{3} \pi R^3 \rho > c^2 \iff R > \sqrt[3]{\frac{3c^2}{8\pi G \rho}} \]

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No privileged role of the velocity of light in Newtonian theory: nothing forbids $V > c$: the “dark stars” are not causally disconnected from the rest of the Universe.
Limits of the Newtonian concept of a black hole

- No privileged role of the velocity of light in Newtonian theory: nothing forbids $V > c$: the "dark stars" are not causally disconnected from the rest of the Universe.

- $V_{\text{esc}} \sim c \implies$ gravitational potential energy $\sim$ mass energy $Mc^2 
\implies$ a relativistic theory of gravitation is necessary!
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- $V_{\text{esc}} \sim c \implies$ gravitational potential energy $\sim$ mass energy $Mc^2$
  $\implies$ a relativistic theory of gravitation is necessary!
- No clear action of the gravitation field on electromagnetic waves in Newtonian gravity
101 years ago: a relativistic theory of gravitation

$R - \frac{1}{2} R g = \frac{8\pi G}{c^4} T$

Karl Schwarzschild (letter to Einstein 22 Dec. 1915; publ. submitted 13 Jan 1916)
Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie,

⇒ First exact non-trivial solution of Einstein equation:

\[
ds^2 = - \left(1 - \frac{2m}{r}\right) c^2 dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2 \theta \, d\varphi^2\right) \tag{1}
\]

with

- coordinates \((t, \bar{r}, \theta, \varphi)\)
- “auxiliary quantity” : \(r := (\bar{r}^3 + 8m^3)^{1/3}\)
- parameter \(m = GM/c^2\), with \(M\) gravitational mass of the “mass point”

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The Schwarzschild solution (1915)

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The “center”

Origin of coordinates : \(\bar{r} = 0 \iff r = 2m\)

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Johannes Droste (communication 27 May 1916)


Derives the Schwarzschild solution (independently of Schwarzschild) via some coordinates \((t, r', \theta, \varphi)\) such that \(g_{r'r'} = 1\); presents the result in the standard form (1) via a change of coordinates leading to the areal radius \(r\)

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**Apparent barrier at } r = 2m\]

A particle falling from infinity never reaches \(r = 2m\) within a finite amount of “time” \(t\).

The *Schwarzschild radius*: \(R_S := 2m = \frac{2GM}{c^2}\)
The “barrier” at $r = R_S$

Radial null geodesics of Schwarzschild spacetime in term of Schwarzschild-Droste coordinates $(t, r)$. Solid (resp. dashed) lines correspond to outgoing (resp. ingoing) geodesics. The interiors of some future light cones are depicted in yellow.
The Schwarzschild solution: early discussions

- **1920**: Alexander Anderson: light cannot emerge from the region
  \[ r < R_S := 2m = \frac{2GM}{c^2} \]
  (region “shrouded in darkness”)
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- **1923** : George Birkhoff : outside any *spherical* body, the metric is Schwarzschild metric

- **1924** : Arthur Eddington introduced the coord. \[ t' := t - \frac{2m}{c} \ln \left( \frac{r}{2m} - 1 \right), \]
  leading to

  \[
  ds^2 = -c^2 dt'^2 + dr^2 + r^2 (d\theta^2 + \sin^2 \theta \, d\varphi^2) + \frac{2m}{r} (cdt' - dr)^2 \quad (2)
  \]

  but did not noticed that the metric components w.r.t. coordinates \((t', r, \theta, \varphi)\) are regular at \(r = 2m\)!

Actually, Eddington’s aim was elsewhere : comparing Whitehead theory (1922) to general relativity
The Schwarzschild solution: Lemaître breakthrough

Georges Lemaître (1932)


The singularity at $r = R_S$ is a mere coordinate singularity: the metric components are regular in Lemaître coordinates $(\tau, \chi, \theta, \varphi)$:

$$ds^2 = -c^2 d\tau^2 + \frac{R_S}{r} d\chi^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$  \hspace{1cm} (3)

$$r = r(\tau, \chi) := \left[ \frac{3}{2} \sqrt{R_S (c\tau - \chi)} \right]^{2/3}$$  \hspace{1cm} (4)
No longer any barrier at $r = R_S$

Radial null geodesics of Schwarzschild spacetime in term of ingoing Eddington-Finkelstein coordinates $(\tilde{t}, r)$

$$\tilde{t} = t + \frac{2m}{c} \ln \left| \frac{r}{2m} - 1 \right|$$

The ingoing null geodesics (dashed lines) do enter the region $r < R_S$. 

$\tilde{t}/m$

$r/m$
Hypersurfaces of constant Schwarzschild-Droste coordinate $t$ in term of the ingoing Eddington-Finkelstein coordinates $(\tilde{t}, r)$
1932: Georges Lemaître: general solutions of Einstein equation for spherically symmetric pressureless fluids (dust) $\Rightarrow$ gravitational collapse
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• 1939: Robert Oppenheimer & Hartland Snyder: gravitational collapse of a homogeneous dust ball of radius $R$ (special case of Lemaître’s general solution)

$\Rightarrow$ for an external observer, $R \rightarrow R_S$ as $t \rightarrow +\infty$

$\Rightarrow$ “frozen star”
Black holes: a century-old history

The Schwarzschild solution: the complete picture

John L. Synge (1950), Martin Kruskal (1960), George Szekeres (1960): complete mathematical description of Schwarzschild spacetime ($\mathbb{R}^2 \times S^2$ manifold)

Schwarzschild-Droste coordinates $(t, r)$
Carter-Penrose diagram of Schwarzschild spacetime based on Frolov-Novikov coordinates

Figure drawn with SageMath: [http://sagemanifolds.obspm.fr](http://sagemanifolds.obspm.fr)
Connecting the asymptotically flat regions $\mathcal{M}_1$ and $\mathcal{M}_{III}$ by hypersurfaces $T = T_0 \equiv \text{const}$ (blue horizontal lines).

$\Longrightarrow$ isometric embedding of equatorial sections $(T = T_0, \theta = \pi/2)$ in the Euclidean 3-space

*Rem* : for $|T_0| > 1$, the dotted parts cannot be embedded isometrically in Euclidean space.
Evolving Einstein-Rosen bridge

$T_0 = 0$ (Flamm paraboloid)  $T_0 = 0.5$  $T_0 = 0.9$

$T_0 = 1$  $T_0 = 1.5$  $T_0 = 2$
Rotation enters the game: the Kerr solution

Almost 50 years after Schwarzschild: Roy Kerr (1963)

\[ ds^2 = -\left(1 - \frac{2mr}{\rho^2}\right) dv^2 + 2dv \, dr - \frac{4amr \sin^2 \theta}{\rho^2} \, dv \, d\tilde{\varphi} \]

\[ -2a \sin^2 \theta \, dr \, d\tilde{\varphi} + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2a^2mr \sin^2 \theta}{\rho^2}\right) \sin^2 \theta \, d\tilde{\varphi}^2. \]

Boyer & Lindquist (1967) coordinate change \((v, r, \theta, \tilde{\varphi}) \rightarrow (t, r, \theta, \varphi)\):

\[ ds^2 = -\left(1 - \frac{2mr}{\rho^2}\right) dt^2 - \frac{4amr \sin^2 \theta}{\rho^2} \, dt \, d\varphi + \frac{\rho^2}{\Delta} \, dr^2 \]

\[ + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2a^2mr \sin^2 \theta}{\rho^2}\right) \sin^2 \theta \, d\varphi^2, \]

where \(\rho^2 := r^2 + a^2 \cos^2 \theta\), \(\Delta := r^2 - 2mr + a^2\) and \(r \in (-\infty, \infty)\)

\(\rightarrow\) spacetime manifold \(\mathcal{M} = \mathbb{R}^2 \times S^2 \setminus \{r = 0 \& \theta = \pi/2\}\)

\(\rightarrow\) 2 parameters: \(m = \frac{GM}{c^2}\) and \(a = \frac{J}{cM}\); black hole \(\iff 0 \leq a \leq m\)

\(\rightarrow\) Schwarzschild metric for \(a = 0\)
mass $M$ : not a measure of the “amount of matter” inside the black hole, but rather a characteristic of the external gravitational field
→ measurable from the orbital period of a test particle in far circular orbit around the black hole (Kepler’s third law)
Physical meaning of the parameters $M$ and $J$

- **mass $M$**: *not* a measure of the “amount of matter” inside the black hole, but rather a *characteristic of the external gravitational field* → measurable from the orbital period of a test particle in far circular orbit around the black hole (*Kepler’s third law*)

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**Remark**: the *radius* of a black hole is not a well defined concept: it *does not* correspond to some distance between the black hole “centre” and the event horizon. A well defined quantity is the *area* of the event horizon, $A$. The “radius” can be defined from it: for a Schwarzschild black hole:

$$ R := \sqrt{\frac{A}{4\pi}} = \frac{2GM}{c^2} \simeq 3 \left( \frac{M}{M_\odot} \right) \text{ km} $$
Slice $t = \text{const}$ of the Kerr spacetime viewed in O’Neill coordinates $(R, \theta, \varphi)$, with $R := e^r$, $r \in (-\infty, +\infty)$.
Meridional view of a section $t = \text{const}$ of Kerr spacetime with $a/m = 0.90$
Maximal analytic extension of Kerr spacetime
1964: Edwin Salpeter, Yakov Zeldovich: quasars (just discovered!) shine thanks to accretion onto a supermassive black hole.
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The Golden Age of black hole theory

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1965-1972: the no-hair theorem
The no-hair theorem

Dorochkevitch, Novikov & Zeldovitch (1965), Israel (1967), Carter (1971), Hawking (1972)

**Within 4-dimensional general relativity, a stationary black hole in an otherwise empty universe is necessarily a **Kerr-Newmann black hole, which is an electro-vacuum solution of Einstein equation described by only 3 parameters:

- the total mass $M$
- the total specific angular momentum $a = J / (Mc)$
- the total electric charge $Q$

$\Rightarrow \text{“a black hole has no hair” (John A. Wheeler)}$
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$\implies$ “a black hole has no hair” (John A. Wheeler)

Astrophysical black holes have to be electrically neutral:

- $Q = 0$: Kerr solution (1963)

Other special cases:

- $a = 0$: Reissner-Nordström solution (1916, 1918)
- $a = 0$ and $Q = 0$: Schwarzschild solution (1916)
- $a = 0$, $Q = 0$ and $M = 0$: Minkowski metric (1907)
General definition of a black hole

The textbook definition

[Hawking & Ellis (1973)]

black hole: \( B := \mathcal{M} - J^- (\mathcal{I}^+) \)

where

- \((\mathcal{M}, g)\) = asymptotically flat manifold
- \(\mathcal{I}^+\) = future null infinity
- \(J^- (\mathcal{I}^+)\) = causal past of \(\mathcal{I}^+\)

i.e. black hole = region of spacetime from which light rays cannot escape to infinity

event horizon: \( \mathcal{H} := \partial J^- (\mathcal{I}^+) \)

(boundary of \( J^- (\mathcal{I}^+) \))

\( \mathcal{H} \) smooth \( \implies \) \( \mathcal{H} \) null hypersurface
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The event horizon $\mathcal{H}$ is a global structure of spacetime: no physical experiment whatsoever can detect the crossing of $\mathcal{H}$.
Viewed by a distant observer, the horizon approach is perceived with an infinite redshift, or equivalently, by an infinite time dilation.

A black hole is not an infinitely dense object: on the contrary it is made of vacuum (except maybe at the singularity); if one defines its “mean density” by $\bar{\rho} = M/(4/3\pi R^3)$, then:

- for the Galactic centre BH (Sgr A*): $\bar{\rho} \sim 10^6 \text{ kg m}^{-3} \sim 2 \times 10^{-4} \rho_{\text{white dwarf}}$
- for the BH at the centre of M87: $\bar{\rho} \sim 2 \text{ kg m}^{-3} \sim 2 \times 10^{-3} \rho_{\text{water}}$

$\Rightarrow$ a black hole is a compact object: $\frac{M}{R}$ large, not $\frac{M}{R^3}$!

Due to the non-linearity of general relativity, black holes can form in spacetimes without any matter, by collapse of gravitational wave packets.
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Example of event horizon in a flat region of spacetime: Vaidya metric, describing incoming radiation from infinity:

$$ds^2 = -\left(1 - \frac{2m(v)}{r}\right) dv^2 + 2dv dr + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

with

- $m(v) = 0$ for $v < 0$
- $dm/dv > 0$ for $0 \leq v \leq v_0$
- $m(v) = M_0$ for $v > v_0$

[Ashtekar & Krishnan, LRR 7, 10 (2004)]
Some recent developments

The quasi-local approach: motivation

The standard definition of a black hole is **highly non-local**: determination of $\partial J^-(\mathcal{I}^+)$ requires the knowledge of the entire future null infinity. Moreover this is **not locally linked with the notion of strong gravitational field**.

Example of event horizon in a **flat** region of spacetime: Vaidya metric, describing incoming radiation from infinity:

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⇒ no local physical experiment can locate the event horizon

[Ashtekar & Krishnan, LRR 7, 10 (2004)]
New paradigm for the theoretical approach to black holes, motivated by quantum gravity and numerical relativity: instead of event horizons, black holes are described by

- trapping horizons (Hayward 1994)
- isolated horizons (Ashtekar et al. 1999)
- dynamical horizons (Ashtekar and Krishnan 2002)
- slowly evolving horizons (Booth and Fairhurst 2004)

All these concepts are quasi-local and are based on hypersurfaces foliated by marginally trapped surfaces
Some recent developments

The 2000’s : the triumph of numerical relativity

[Caltech/Cornell SXS]
[Scheel et al., PRD 79, 024003 (2009)]
A recent hot topic: black holes and gauge/gravity duality

**Gauge/gravity duality ("holographic principle")**

4D strongly-coupled gauge theory ≡ 5D gravitation

*Prototype*: AdS/CFT correspondence

**Spacetime diagram of a heavy-ion collision (LHC)**

\[ \tau_0 \approx 0.2 \text{ fm}/c = 6 \times 10^{-25} \text{ s} \]

\[ \tau_1 \approx 10 \tau_0 \]
Some recent developments

A recent hot topic: black holes and gauge/gravity duality

Gauge/gravity duality ("holographic principle")

4D strongly-coupled gauge theory $\equiv$ 5D gravitation

Prototype: AdS/CFT correspondence

Example: Quark-gluon plasma (QGP) in heavy-ion collisions: low-viscosity fluid with anisotropic pressure ($p_x < p_y$)

Thermalization of QGP $\equiv$ 5D black hole formation

Gauge theory: QCD

Gravity: 5D Lifshitz-like spacetime (anisotropic generalization of AdS$_5$) with formation of a black brane (Vaidya-type collapse)

Results: faster thermalization in the transversal direction; evolution of the entanglement entropy
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Known black holes

Three kinds of black holes are known in the Universe:

- **Stellar black holes**: supernova remnants:
  \[ M \sim 10 - 30 \, M_\odot \text{ and } R \sim 30 - 90 \, \text{km} \]
  
  example: Cyg X-1: \( M = 15 \, M_\odot \) and \( R = 45 \, \text{km} \)
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- **Supermassive black holes**, in galactic nuclei:
  \[ M \sim 10^5 - 10^{10} \, M_\odot \text{ and } R \sim 3 \times 10^5 \, \text{km} - 200 \, \text{UA} \]
  example: Sgr A*: \( M = 4.3 \times 10^6 \, M_\odot \) and
  \( R = 13 \times 10^6 \, \text{km} = 18 \, R_\odot = 0.09 \, \text{UA} = \frac{1}{4} \times \text{radius of Mercury’s orbit} \)
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  example: Cyg X-1: \( M = 15 \, M_\odot \) and \( R = 45 \, \text{km} \)

- **Supermassive black holes**, in galactic nuclei:
  \[ M \sim 10^5 - 10^{10} \, M_\odot \text{ and } R \sim 3 \times 10^5 \, \text{km} - 200 \, \text{UA} \]
  example: Sgr A*: \( M = 4.3 \times 10^6 \, M_\odot \) and \( R = 13 \times 10^6 \, \text{km} = 18 \, R_\odot = 0.09 \, \text{UA} = \frac{1}{4} \times \text{radius of Mercury’s orbit} \)

- **Intermediate mass black holes**, as ultra-luminous X-ray sources:
  \[ M \sim 10^2 - 10^4 \, M_\odot \text{ and } R \sim 300 \, \text{km} - 3 \times 10^4 \, \text{km} \]
  example: ESO 243-49 HLX-1: \( M > 500 \, M_\odot \) and \( R > 1500 \, \text{km} \)
Stellar black holes in X-ray binaries

[McClintock et al. (2011)]

Black holes in the sky
Black holes in the sky

Stellar black holes in X-ray binaries

Dynamically measured masses of black holes in transient low-mass X-ray binaries (right), compared with measured masses of neutron stars (left)

[Corral-Santana et al., A&A 587, A61 (2016)]
Jet emitted by the nucleus of the giant elliptic galaxy M87, at the centre of Virgo cluster [HST]

\[ M_{\text{BH}} = 3 \times 10^9 M_\odot \]

\[ V_{\text{jet}} \approx 0.99 c \]
Black holes in the sky

The black hole at the centre of our galaxy: Sgr A*

Measure of the mass of Sgr A* black hole by stellar dynamics:

\[ M_{\text{BH}} = 4.3 \times 10^6 \, M_\odot \]

\( \leftarrow \) Orbit of the star S2 around Sgr A*

\[ P = 16 \, \text{yr}, \quad r_{\text{per}} = 120 \, \text{UA} = 1400 \, R_S, \quad V_{\text{per}} = 0.02 \, c \]

[Genzel, Eisenhauer & Gillessen, RMP 82, 3121 (2010)]
Can we see a black hole from the Earth?

Angular diameter of the event horizon of a Schwarzschild BH of mass $M$ seen from a distance $d$:

$$\Theta = 6\sqrt{3} \frac{GM}{c^2 d} \simeq 2.60 \frac{2R_S}{d}$$

Image of a thin accretion disk around a Schwarzschild BH

[Vincent, Paumard, Gourgoulhon & Perrin, CQG 28, 225011 (2011)]
Can we see a black hole from the Earth?

Angular diameter of the event horizon of a Schwarzschild BH of mass $M$ seen from a distance $d$:

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Largest black holes in the Earth’s sky:

- Sgr A*: $\Theta = 53 \mu\text{as}$
- M87: $\Theta = 21 \mu\text{as}$
- M31: $\Theta = 20 \mu\text{as}$

Remark: black holes in X-ray binaries are $\sim 10^5$ times smaller, for $\Theta \propto M/d$
Black holes in the sky

Reaching the \( \mu \)as resolution with VLBI

Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

Existing American VLBI network [Doeleman et al. 2011]
Reaching the $\mu$as resolution with VLBI

Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

The best result so far: VLBI observations at 1.3 mm have shown that the size of the emitting region in Sgr A* is only 37 $\mu$as

[Doeleman et al., Nature 455, 78 (2008)]

Existing American VLBI network [Doeleman et al. 2011]
To go further:
- shorten the wavelength: $1.3 \text{ mm} \rightarrow 0.8 \text{ mm}$
- increase the number of stations; in particular add ALMA

Atacama Large Millimeter Array (ALMA)
part of the Event Horizon Telescope (EHT) to be completed by 2020
August 2015: VLBI observations involving ALMA and VLBA
Black holes in the sky

VLBA and EHT observations of M87

40 μas

EHT beam

optically-thick region (≥ 21 μas)

optically-thin region (40 μas)

jet base of M87 (VLBA at 43GHz)

Near-infrared optical interferometry: GRAVITY

GRAVITY instrument at VLTI (2016)
Beam combiner (the four 8 m telescopes + four auxiliary telescopes)

astrometric precision on orbits: 10 μas

[Gillessen et al. 2010]
Near-infrared optical interferometry: GRAVITY

July 2015: GRAVITY shipped to Chile and successfully assembled at the Paranal Observatory.

Fall 2016: observations have started!

[MPE/GRAVITY team]
Observing black holes at high energy: the Athena mission

X-ray observatory Athena: the future L2 mission of ESA (launch ~ 2028)

Among the scientific objectives:

- Determine the formation and early growth of supermassive black holes, via the observation of a large sample of AGN at $z \sim 6 - 8$
- Measure the spins of supermassive black holes
- Measure the spins of stellar black holes
Outline

1. Black holes: a century-old history
2. Some recent developments
3. Black holes in the sky
4. Observing black holes via gravitational waves: a dream come true
5. Testing general relativity with black holes
Gravitational waves

Linearization of Einstein equation in weak field: \( g = \eta + h \),
\( \eta = \) Minkowski metric

\[ \Box \bar{h} = -\frac{16\pi G}{c^4} T \]
(Lorenz gauge)

with \( \Box = -\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \), \( \bar{h} = h - \frac{1}{2} h \eta \) and \( h = \text{Trace}(h) \).
Link between black holes and gravitational waves:
Both are spacetime distortions:
- extreme distortions (black holes)
- small distortions (gravitational waves)

In particular, black holes and gravitational waves are both vacuum solutions of Einstein equation.
Emission of gravitational waves by the neutron star binary system PSR B1913+16 (binary pulsar)

\[ P = 7 \text{ h} 45 \text{ min} \]

\( \Rightarrow \) coalescence in 140 millions year.

[Weisber & Taylor (2002)]
Emission of gravitational waves by the neutron star binary system PSR B1913+16 (*binary pulsar*)

\[ P = 7 \text{ h } 45 \text{ min } \]

Produced by the reaction to gravitational radiation \( \Rightarrow \) coalescence in 140 millions year.

Nobel Prize in Physics to R. Hulse & J. Taylor (1993)
Measurable effects of a gravitational wave passage

Measure of the distance $L$ between two free masses by a “radar” method:

$$L = \frac{1}{2} c(t_2 - t_1)$$

Variation of length $L$ when a gravitational wave passes by:

$$\delta L \sim h L$$

$h = \text{amplitude of the gravitational wave}$

In practice, $h$ is so small that our senses are not sensitive to it:

for the most important astrophysical sources:

$h \sim 10^{-21}$
Observing black holes via gravitational waves: a dream come true

Advanced LIGO detectors

Éric Gourgoulhon (LUTH)

Black holes, 100 yr after the birth of GR

MIAN, Moscow, 22 May 2017

[Abbott et al., PRL 116, 061102 (2016)]
Observing black holes via gravitational waves: a dream come true

Advanced ground-based GW detectors

- **Adv. LIGO**: started Sept. 2015
- **Adv. Virgo**: will start in spring 2017
- **KAGRA (Japan)**: 2018

Gravitational wave detector VIRGO in Cascina, near Pisa (Italy) [CNRS/INFN]
Observing black holes via gravitational waves: a dream come true

September 14, 2015, 09:50:45 UTC

Éric Gourgoulhon (LUTH)

Hanford, Washington (H1)  Livingston, Louisiana (L1)

Strain (10^{-21})

H1 observed
L1 observed
H1 observed (shifted, inverted)
L1 observed (shifted, inverted)

Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)

Residual

Frequency (Hz)

Time (s)

Normalized amplitude

512
256
128
64
32
0.30 0.35 0.40 0.45

Time (s)
Observing black holes via gravitational waves: a dream come true

GW150914 event

Signal:
\[ \Delta t = 0.2 \text{s} \]
\[ f : 35 \rightarrow 250 \text{ Hz} \]
\[ h_{\text{max}} = 1.0 \times 10^{-21} \]

Matched filter:
\[ S/N = 24 \]
\[ F_{\text{false}} = 1/203,000 \text{ yr} \]
\[ M_1 = 36 \pm 5 M_{\odot} \]
\[ M_2 = 29 \pm 4 M_{\odot} \]
\[ d = 410 \pm 180 \text{ Mpc} \]
\[ z = 0.09 \pm 0.04 \]
\[ M_{\text{final}} = 62 \pm 4 M_{\odot} \]
\[ \Rightarrow E_{\text{rad}}^{GW} = 3.0 \pm 0.5 M_{\odot} c^2 \]
\[ a_1 < 0.7, \quad a_2 < 0.9 \]
\[ a_{\text{final}} = 0.67 \pm 0.07 \]

[Abbott et al., PRL 116, 061102 (2016)]
Observing black holes via gravitational waves: a dream come true

LIGO events in run O1 (12 Sept. 2015 - 19 Jan. 2016)

[Abbott et al., PRX 6, 041015 (2016)]

NB: LVT = LIGO-Virgo Trigger (not significant enough to be a detection)
Observing black holes via gravitational waves: a dream come true

LIGO events in run O1 (12 Sept. 2015 - 19 Jan. 2016)

[Abbott et al., PRX 6, 041015 (2016)]
Observing black holes via gravitational waves: a dream come true

GW detectors in different bandwidths

- Stochastic background
- eLISA
- Massive binaries
- Supermassive binaries
- Resolvable galactic binaries
- Extreme mass ratio inspirals
- Compact binary inspirals
- Core collapse supernovae
- Pulsars

Characteristic Strain vs Frequency / Hz
Observing black holes via gravitational waves: a dream come true

Space detector LISA (ESA)

Interferometric gravitational wave detector in solar orbit

- theme selected by ESA in 2013 for the L3 mission
- launch ~ 2034
- technology demonstrator LISA Pathfinder launched on 3 December 2015; successful results announced in June 2016!

[elISA / NGO]
Observing black holes via gravitational waves: a dream come true

LISA observations of massive binary BH mergers

Signal-to-noise ratio for gravitational waves from the inspiral of a BH binary at $z = 0.5$

Observing black holes via gravitational waves: a dream come true

The promise of multi-band gravitational wave astronomy

[Sesana, PRL 116, 231102 (2016)]

Eric Gourgoulhon (LUTH)

Black holes, 100 yr after the birth of GR

MIAN, Moscow, 22 May 2017
Observing black holes via gravitational waves: a dream come true

Detecting gravitational waves by pulsar timing

Le grand radiotélescope de Nançay fête ses 50 ans. © Observatoire de Paris
**EPTA results on supermassive BH binaries**

**EPTA** : European Pulsar Timing Array

![Graph](image)

[ Babak et al., MNRAS 455, 1665 (2016) ]
Is general relativity unique?

Yes if we assume

- a 4-dimensional spacetime
- gravitation only described by a metric tensor $g$
- field equation involving only derivatives of $g$ up to second order
- diffeomorphism invariance
- $\nabla \cdot T = 0$ ($\implies$ weak equivalence principle)

The above is a consequence of Lovelock theorem (1972).
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The above is a consequence of Lovelock theorem (1972).

However, GR is certainly not the ultimate theory of gravitation:
- it is not a quantum theory
- cosmological constant / dark energy problem

GR is generally considered as a low-energy limit of a more fundamental theory:
- string theory
- loop quantum gravity
- ...
Testing general relativity with black holes

Extensions of general relativity

[Berti et al., CGQ 32, 243001 (2015)]
Test: are astrophysical black holes Kerr black holes?

- GR $\rightarrow$ Kerr BH \textit{(no-hair theorem)}
- extension of GR $\rightarrow$ BH may deviate from Kerr
Test: are astrophysical black holes Kerr black holes?

- GR $\implies$ Kerr BH (no-hair theorem)
- extension of GR $\implies$ BH may deviate from Kerr

Observational tests

Search for

- stellar orbits deviating from Kerr timelike geodesics (GRAVITY)
- accretion disk spectra different from those arising in Kerr metric (X-ray observatories, e.g. Athena)
- images of the black hole silhouette different from that of a Kerr BH (EHT)
Testing general relativity with black holes

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### Observational tests

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- **stellar orbits** deviating from Kerr timelike geodesics (**GRAVITY**)
- **accretion disk spectra** different from those arising in Kerr metric (**X-ray observatories, e.g. Athena**)
- **images of the black hole silhouette** different from that of a Kerr BH (**EHT**)

Need for a good and versatile geodesic integrator
to compute timelike geodesics (orbits) and null geodesics (ray-tracing) in any kind of metric
Gyoto code

Main developers: T. Paumard & F. Vincent

- Integration of geodesics in Kerr metric
- Integration of geodesics in any numerically computed 3+1 metric
- Radiative transfer included in optically thin media
- Very modular code (C++)
- Yorick and Python interfaces
- Free software (GPL): http://gyoto.obspm.fr/

[Vincent, Paumard, Gourgoulhon & Perrin, CQG 28, 225011 (2011)]
[Vincent, Gourgoulhon & Novak, CQG 29, 245005 (2012)]
**Boson star** = localized configurations of a self-gravitating massive complex scalar field \( \Phi \equiv “\text{Klein-Gordon geons}” \)

[Bonazzola & Pacini (1966), Kaup (1968)]

Boson stars may behave as black-hole mimickers

- Solutions of the *Einstein-Klein-Gordon* system computed by means of *Kadath*
  [Grandclément, JCP 229, 3334 (2010)]

- Timelike geodesics computed by means of *Gyoto*

Pointy petal orbit around a rotating boson star for a free scalar field

\[
\Phi = \phi(r, \theta) e^{i(\omega t + 2\varphi)}, \quad \omega = 0.75 \frac{m}{\hbar}
\]

[Granclément, Somé & Gourgoulhon, PRD 90, 024068 (2014)]
Kerr BH  \( a/M = 0.9 \)  

Boson star  \( k = 1, \omega = 0.70 \, m/\hbar \) 

[Vincent, Meliani, Grandclément, Gourgoulhon & Straub, CQG 33, 105015 (2016)]
Testing general relativity with black holes

Black holes with scalar hair

Kerr black hole

\[ \frac{a}{M} = 0.9 \]

boson star [1]

\[ k = 1, \omega = 0.7 \frac{m}{\hbar} \]

hairy black hole [2]

\[ \frac{a}{M} = 0.9 \]

Kadath \rightarrow metric

HR code \rightarrow metric

(via Lorene)

Gyoto \rightarrow ray-tracing

[1. Vincent, Meliani, Grandclément, Gourgoulhon & Straub, Class. Quantum Grav. 33, 105015 (2016)]

Conclusions

After a century marked by the Golden Age (1965-1975), the first astronomical discoveries and the ubiquity of black holes in high-energy astrophysics, black hole physics is very much alive.
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Conclusions

After a century marked by the Golden Age (1965-1975), the first astronomical discoveries and the ubiquity of black holes in high-energy astrophysics, black hole physics is very much alive.

From the theory side, a strong impulse is provided by the gauge/gravity duality.

Besides, a new observational era is opening, with the advent of high-angular-resolution telescopes and gravitational wave detectors, which provide unique opportunities to test general relativity in the strong field regime.

The GW150914 event was both the first direct detection of gravitational waves and the first observation of the merger of two black holes — the most dynamical event in relativistic gravity. The waveform was found consistent with general relativity.