A history of black holes
from a physicist perspective

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Outline

1. The prehistory
2. Schwarzschild black hole
3. Kerr black hole
4. The Golden Age of black hole theory
5. Some recent developments
6. Testing general relativity with black holes
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A two centuries-old prehistory...

\[ V_{\text{esc}} > c \iff \frac{2GM}{R} > c^2 \iff \frac{2G}{R} \times \frac{4}{3} \pi R^3 \rho > c^2 \iff R > \sqrt{\frac{3c^2}{8\pi G \rho}} \]
The prehistory

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John Michell (1784)

“If there should really exist in nature any bodies, whose density is not less than that of the sun, and whose diameters are more than 500 times the diameter of the sun, since their light could not arrive at us, …, we could have no information from sight”

[Phil. Trans. R. Soc. Lond. 74, 35 (1784)]
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Pierre Simon de Laplace (1796)

“Un astre lumineux, de la même densité que la Terre, et dont le diamètre serait 250 fois plus grand que le Soleil, ne permettrait, en vertu de son attraction, à aucun de ses rayons de parvenir jusqu’à nous. Il est dès lors possible que les plus grands corps lumineux de l’univers puissent, par cette cause, être invisibles.”

[Exposition du système du monde (1796)]
No privileged role of the velocity of light in Newtonian theory: nothing forbids $V > c$: the “dark stars” are not causally disconnected from the rest of the Universe.
Limits of the Newtonian concept of a black hole

- No privileged role of the velocity of light in Newtonian theory: nothing forbids $V > c$: the “dark stars” are not causally disconnected from the rest of the Universe.
- $V_{\text{esc}} \sim c \implies$ gravitational potential energy $\sim$ mass energy $Mc^2$  
  $\implies$ a *relativistic* theory of gravitation is necessary!
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- $V_{\text{esc}} \sim c \Rightarrow$ gravitational potential energy $\sim$ mass energy $Mc^2$ $\Rightarrow$ a relativistic theory of gravitation is necessary!

- No clear action of the gravitation field on electromagnetic waves in Newtonian gravity.

[R. Taillet]
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101 years ago: a relativistic theory of gravitation

\[ R - \frac{1}{2} \nabla^2 g = 8\pi G \frac{T}{c^4} \]


Die Feldgleichungen der Gravitation.
Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen\(^1\) habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariablen gegenüber kovariant sind.
The Schwarzschild solution (1915)

Karl Schwarzschild (letter to Einstein 22 Dec. 1915; publ. submitted 13 Jan 1916)

Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie,

⇒ First exact non-trivial solution of Einstein equation:

\[ ds^2 = - \left( 1 - \frac{2m}{r} \right) c^2 dt^2 + \left( 1 - \frac{2m}{r} \right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta \, d\varphi^2) \tag{1} \]

with

- coordinates \((t, \bar{r}, \theta, \varphi)\)
- “auxiliary quantity” : \(r := (\bar{r}^3 + 8m^3)^{1/3}\)
- parameter \(m = GM/c^2\), with \(M\) gravitational mass of the “mass point”

1. Schwarzschild’s notations : \(r = \bar{r}, \quad R = r, \quad \alpha = 2m\)
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The “center”

Origin of coordinates : \( \bar{r} = 0 \iff r = 2m \)

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Johannes Droste (communication 27 May 1916)


⇒ derives the Schwarzschild solution (independently of Schwarzschild) via some coordinates \((t, r', \theta, \varphi)\) such that \(g_{r'r'} = 1\); presents the result in the standard form (1) via a change of coordinates leading to the areal radius \(r\)

⇒ makes a detailed study of timelike geodesics in the obtained geometry
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**Apparent barrier at** \(r = 2m\)

A particle falling from infinity never reaches \(r = 2m\) within a finite amount of “time” \(t\).

The *Schwarzschild radius*: \(R_S := 2m = \frac{2GM}{c^2}\)
Radial null geodesics of Schwarzschild spacetime in term of Schwarzschild-Droste coordinates \((t, r)\). Solid (resp. dashed) lines correspond to outgoing (resp. ingoing) geodesics. The interiors of some future light cones are depicted in yellow.
The Schwarzschild solution: early discussions

- **1920**: Alexander Anderson: light cannot emerge from the region
  \[ r < R_S := 2m = \frac{2GM}{c^2} \]
  (region “shrouded in darkness”)

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1924: Arthur Eddington introduced the coord. \( t' := t - \frac{2m}{c} \ln \left( \frac{r}{2m} - 1 \right) \), leading to

\[
\mathrm{d}s^2 = -c^2 \mathrm{d}t'^2 + \mathrm{d}r^2 + r^2 \left( \mathrm{d}\theta^2 + \sin^2 \theta \, \mathrm{d}\varphi^2 \right) + \frac{2m}{r} \left( c \mathrm{d}t' - \mathrm{d}r \right)^2 \tag{2}
\]

but did not noticed that the metric components w.r.t. coordinates \( (t', r, \theta, \varphi) \) are regular at \( r = 2m \)!

Actually, Eddington’s aim was elsewhere: comparing Whitehead theory (1922) to general relativity.
The singularity at \( r = R_S \) is a mere coordinate singularity: the metric components are regular in Lemaître coordinates \((\tau, \chi, \theta, \varphi)\):

\[
\begin{align*}
 ds^2 &= -c^2 d\tau^2 + \frac{R_S}{r} d\chi^2 + r^2 (d\theta^2 + \sin^2 \theta \, d\varphi^2) \quad (3) \\
 r &= r(\tau, \chi) := \left[ \frac{3}{2} \sqrt{R_S (c\tau - \chi)} \right]^{2/3} \quad (4)
\end{align*}
\]
Schwarzschild black hole

No longer any barrier at $r = R_S$

Radial null geodesics of Schwarzschild spacetime in term of ingoing Eddington-Finkelstein coordinates $(\tilde{t}, r)$

$$\tilde{t} = t + \frac{2m}{c} \ln \left| \frac{r}{2m} - 1 \right|$$

The ingoing null geodesics (dashed lines) do enter the region $r < R_S$. 
Hypersurfaces of constant Schwarzschild-Droste coordinate $t$ in term of the ingoing Eddington-Finkelstein coordinates $(\tilde{t}, r)$
1932: Georges Lemaître: general solution of Einstein equation for a spherically symmetric pressureless fluid $\Rightarrow$ gravitational collapse
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1939: Robert Oppenheimer & Hartland Snyder: gravitational collapse of a homogeneous dust ball of radius $R$ (special case of Lemaître’s general solution)

$\Rightarrow$ for an external observer, $R \rightarrow R_S$ as $t \rightarrow +\infty$

$\Rightarrow$ “frozen star”
**The Schwarzschild solution: the complete picture**

John L. Synge (1950), Martin Kruskal (1960), George Szekeres (1960): complete mathematical description of Schwarzschild spacetime ($\mathbb{R}^2 \times S^2$ manifold)

Schwarzschild-Droste coordinates $(t, r)$
Carter-Penrose diagram of Schwarzschild spacetime

Figure drawn with SageMath: http://sagemanifolds.obspm.fr
Connecting the asymptotically flat regions $\mathcal{M}_1$ and $\mathcal{M}_{III}$ by hypersurfaces $T = T_0 = \text{const}$ (blue horizontal lines).

$\Longrightarrow$ isometric embedding of equatorial sections $(T = T_0, \theta = \pi/2)$ in the Euclidean 3-space

$\text{Rem : for } |T_0| > 1$, the dotted parts cannot be embedded isometrically in Euclidean space.
Schwarzschild black hole

Evolving Einstein-Rosen bridge

$T_0 = 0$ (Flamm paraboloid)  $T_0 = 0.5$  $T_0 = 0.9$

$T_0 = 1$  $T_0 = 1.5$  $T_0 = 2$
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Rotation enters the game: the Kerr solution

Almost 50 years after Schwarzschild: Roy Kerr (1963)

\[
ds^2 = - \left(1 - \frac{2mr}{\rho^2}\right) dv^2 + 2dv dr - \frac{4amr \sin^2 \theta}{\rho^2} dv d\tilde{\phi} \\
-2a \sin^2 \theta dr d\tilde{\phi} + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2a^2mr \sin^2 \theta}{\rho^2}\right) \sin^2 \theta d\tilde{\phi}^2.
\]

Boyer & Lindquist (1967) coordinate change \((v, r, \theta, \tilde{\phi}) \rightarrow (t, r, \theta, \varphi)\):

\[
ds^2 = - \left(1 - \frac{2mr}{\rho^2}\right) dt^2 - \frac{4amr \sin^2 \theta}{\rho^2} dt d\varphi + \frac{\rho^2}{\Delta} dr^2 \\
+ \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2a^2mr \sin^2 \theta}{\rho^2}\right) \sin^2 \theta d\varphi^2,
\]

where \(\rho^2 := r^2 + a^2 \cos^2 \theta\), \(\Delta := r^2 - 2mr + a^2\) and \(r \in (-\infty, \infty)\)

→ spacetime manifold \(\mathcal{M} = \mathbb{R}^2 \times S^2 \setminus \{r = 0 \& \theta = \pi/2\}\)

→ 2 parameters: \(m = \frac{GM}{c^2}\) and \(a = \frac{J}{cM}\); black hole \(\iff\) \(0 \leq a \leq m\)

→ Schwarzschild metric for \(a = 0\)
Kerr black hole

Physical meaning of the parameters $M$ and $J$

- **mass $M$**: *not* a measure of the “amount of matter” inside the black hole, but rather a *characteristic of the external gravitational field*
  → measurable from the orbital period of a test particle in far circular orbit around the black hole (*Kepler’s third law*)
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**Remark :** the *radius* of a black hole is not a well defined concept : it *does not* correspond to some distance between the black hole “centre” and the event horizon. A well defined quantity is the *area* of the event horizon, $A$. The “radius” can be defined from it : for a Schwarzschild black hole :

$$R := \sqrt{\frac{A}{4\pi}} = \frac{2GM}{c^2} \simeq 3 \left( \frac{M}{M_\odot} \right) \text{ km}$$
Slice $t = \text{const}$ of the Kerr spacetime viewed in O’Neill coordinates $(R, \theta, \varphi)$, with $R := e^r$, $r \in (-\infty, +\infty)$.
Kerr black hole

Kerr spacetime: ergoregion and Carter time machine

Meridional view of a section $t = \text{const}$ of Kerr spacetime with $a/m = 0.90$
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1965-1972 : the no-hair theorem
Dorochkevitch, Novikov & Zeldovitch (1965), Israel (1967), Carter (1971), Hawking (1972)

*Within 4-dimensional general relativity, a stationary black hole in an otherwise empty universe is necessarily a Kerr-Newmann black hole, which is an electro-vacuum solution of Einstein equation described by only 3 parameters:*

- the total mass $M$
- the total specific angular momentum $a = J/(Mc)$
- the total electric charge $Q$

$\implies$ “a black hole has no hair” (John A. Wheeler)
The no-hair theorem

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Astrophysical black holes have to be electrically neutral:
- $Q = 0$ : Kerr solution (1963)

Other special cases:
- $a = 0$ : Reissner-Nordström solution (1916, 1918)
- $a = 0$ and $Q = 0$ : Schwarzschild solution (1916)
- $a = 0$, $Q = 0$ and $M = 0$ : Minkowski metric (1907)
General definition of a black hole

The textbook definition

[Hawking & Ellis (1973)]

black hole : \( B := \mathcal{M} - J^- (\mathcal{I}^+) \)

where

- \((\mathcal{M}, g)\) = asymptotically flat manifold
- \(\mathcal{I}^+\) = future null infinity
- \( J^- (\mathcal{I}^+) \) = causal past of \( \mathcal{I}^+ \)

i.e. black hole = region of spacetime from which light rays cannot escape to infinity

event horizon : \( \mathcal{H} := \partial J^- (\mathcal{I}^+) \)

(boundary of \( J^- (\mathcal{I}^+) \))

\( \mathcal{H} \) smooth \( \implies \mathcal{H} \) null hypersurface
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The singularity marks the limit of validity of general relativity: to describe it, a quantum theory of gravitation would be required.
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The event horizon $\mathcal{H}$ is a global structure of spacetime: no physical experiment whatsoever can detect the crossing of $\mathcal{H}$. 
Viewed by a distant observer, the horizon approach is perceived with an infinite redshift, or equivalently, by an infinite time dilation.

A black hole is not an infinitely dense object: on the contrary it is made of vacuum (except maybe at the singularity); if one defines its “mean density” by \( \bar{\rho} = \frac{M}{(4/3 \pi R^3)} \), then

- for the Galactic centre BH (Sgr A*): \( \bar{\rho} \sim 10^6 \text{ kg m}^{-3} \sim 2 \times 10^{-4} \rho_{\text{white dwarf}} \)
- for the BH at the centre of M87: \( \bar{\rho} \sim 2 \text{ kg m}^{-3} \sim 2 \times 10^{-3} \rho_{\text{water}} \)

\[ \implies \text{a black hole is a compact object: } \frac{M}{R} \text{ large, not } \frac{M}{R^3} ! \]

Due to the non-linearity of general relativity, black holes can form in spacetimes without any matter, by collapse of gravitational wave packets.
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The quasi-local approach: motivation

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Example of event horizon in a flat region of spacetime: Vaidya metric, describing incoming radiation from infinity:

\[
ds^2 = - \left( 1 - \frac{2m(v)}{r} \right) dv^2 + 2dv dr + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)
\]

with \( m(v) = 0 \) for \( v < 0 \)
\( dm/dv > 0 \) for \( 0 \leq v \leq v_0 \)
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[Ashtekar & Krishnan, LRR 7, 10 (2004)]
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\[\Rightarrow\text{ no local physical experiment can locate the event horizon}\]
**New paradigm** for the theoretical approach to black holes, motivated by quantum gravity and numerical relativity, instead of *event horizons*, black holes are described by

- trapping horizons (Hayward 1994)
- isolated horizons (Ashtekar et al. 1999)
- dynamical horizons (Ashtekar and Krishnan 2002)
- slowly evolving horizons (Booth and Fairhurst 2004)

All these concepts are **local** and are based on the notion of **trapped surfaces**
Some recent developments

The 2000’s : the triumph of numerical relativity

[Caltech/Cornell SXS]

[Scheel et al., PRD 79, 024003 (2009)]
A recent hot topic: black holes and gauge/gravity duality

Gauge/gravity duality ("holographic principle")

4D strongly-coupled gauge theory $\equiv$ 5D gravitation

**Example:** AdS/CFT correspondence

Quark-gluon plasma (QGP) in heavy-ion collisions: low-viscosity fluid with *anisotropic* pressure ($p_x < p_y$)

Thermalization of QGP $\equiv$ 5D black hole formation

Gauge theory: QCD

Gravity: 5D Lifshitz-like spacetime (*anisotropic* generalization of AdS$_5$) with formation of a black brane (Vaidya-type collapse); new exact solutions with the help of SageManifolds

Results: faster thermalization in the transversal direction; evolution of the entanglement entropy

\[ \tau_1 \sim 10 \tau_0 \]

\[ \tau_0 \approx 0.2 \text{ fm}/c = 6 \times 10^{-25} \text{ s} \]

Spacetime diagram of a heavy-ion collision (LHC)

\[ t = t_2 \]

\[ t = t_1 \]

\[ \eta = -1 \]

\[ \eta = -0.25 \]

\[ \eta = 0 \]

\[ \eta = 0.25 \]

\[ \eta = 0.5 \]

\[ \eta = 1 \]

[\text{Pb}] [\text{Pb}]

\[ \tau_0 \]

\[ \tau_1 \]

\[ \tau_2 \]

\[ \tau_3 \]

\[ \tau_4 \]

\[ \tau_5 \]

\[ \tau_6 \]

\[ \tau_7 \]

\[ \tau_8 \]

\[ \tau_9 \]

\[ \tau_{10} \]

\[ \tau_{11} \]

\[ \tau_{12} \]

\[ \tau_{13} \]

\[ \tau_{14} \]

\[ \tau_{15} \]

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\[ \tau_{30} \]

\[ \tau_{31} \]

\[ \tau_{32} \]

\[ \tau_{33} \]

\[ \tau_{34} \]

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Outline

1. The prehistory
2. Schwarzschild black hole
3. Kerr black hole
4. The Golden Age of black hole theory
5. Some recent developments
6. Testing general relativity with black holes
Is general relativity unique?

Yes if we assume

- a 4-dimensional spacetime
- gravitation only described by a metric tensor $g$
- field equation involving only derivatives of $g$ up to second order
- diffeomorphism invariance
- $\nabla \cdot T = 0$ (⇒ weak equivalence principle)

The above is a consequence of Lovelock theorem (1972).
Is general relativity unique?

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However, GR is certainly not the ultimate theory of gravitation:

- it is not a quantum theory
- cosmological constant / dark energy problem

GR is generally considered as a low-energy limit of a more fundamental theory:

- string theory
- loop quantum gravity
- ...
Extensions of general relativity

Higher dimensions

WEP violations

Extra fields

Diff-invar. violations

Lovelock theorem

Nondynamical fields

Dynamical fields (SEP violations)

Massive gravity

Lorentz-violations

Palatini f(R)
Eddington-Born-Infeld

Scalars

Vectors

Tensors

Scalar-tensor, Metric f(R)
Horndeski, galileons
Quadratic gravity, n-DBI

Einstein-Aether
Horava-Lifshitz

TeVeS
Bimetric gravity

dRGT theory
Massive bimetric
gravity

Einstein-Aether
Horava-Lifshitz
n-DBI

[Berti et al., CGQ 32, 243001 (2015)]
Test: are astrophysical black holes Kerr black holes?

- GR $\iff$ Kerr BH (no-hair theorem)
- extension of GR $\iff$ BH may deviate from Kerr
Testing general relativity with black holes

Test: are astrophysical black holes Kerr black holes?

- GR $\rightarrow$ Kerr BH (no-hair theorem)
- extension of GR $\rightarrow$ BH may deviate from Kerr

Observational tests

Search for

- stellar orbits deviating from Kerr timelike geodesics (GRAVITY)
- accretion disk spectra different from those arising in Kerr metric (X-ray observatories, e.g. Athena)
- images of the black hole silhouette different from that of a Kerr BH (EHT)
Testing general relativity with black holes

Test: are astrophysical black holes Kerr black holes?

- GR $\implies$ Kerr BH (no-hair theorem)
- extension of GR $\implies$ BH may deviate from Kerr

Observational tests

Search for
- stellar orbits deviating from Kerr timelike geodesics (GRAVITY)
- accretion disk spectra different from those arising in Kerr metric (X-ray observatories, e.g. Athena)
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Need for a good and versatile geodesic integrator
to compute timelike geodesics (orbits) and null geodesics (ray-tracing) in any kind of metric
Gyoto code

Main developers: T. Paumard & F. Vincent

- Integration of geodesics in Kerr metric
- Integration of geodesics in any numerically computed 3+1 metric
- Radiative transfer included in optically thin media
- Very modular code (C++)
- Yorick and Python interfaces
- Free software (GPL): http://gyoto.obspm.fr/

[Vincent, Paumard, Gourgoulhon & Perrin, CQG 28, 225011 (2011)]
[Vincent, Gourgoulhon & Novak, CQG 29, 245005 (2012)]
**Boson star** = localized configurations of a self-gravitating massive complex scalar field $\Phi \equiv "Klein-Gordon geons"$

[Bonazzola & Pacini (1966), Kaup (1968)]

Boson stars may behave as black-hole mimickers

- Solutions of the *Einstein-Klein-Gordon* system computed by means of *Kadath*
  [Grandclément, JCP 229, 3334 (2010)]

- Timelike geodesics computed by means of *Gyoto*

Zero-angular-momentum orbit around a rotating boson star based on a free scalar field $\Phi = \phi(r, \theta)e^{i(\omega t + 2\varphi)}$
with $\omega = 0.75 \, m/\hbar$.

[Granclément, Somé & Gourgoulhon, PRD 90, 024068 (2014)]
Testing general relativity with black holes

Images of accretion torus around alternatives to the Kerr black hole

Kerr black hole

\[ \frac{a}{M} = 0.9 \]

boson star [1]

\[ k = 1, \ \omega = 0.7 \frac{m}{\hbar} \]

hairy black hole [2]

\[ \frac{a}{M} = 0.9 \]


Can we see a black hole from the Earth?

Angular diameter of the event horizon of a Schwarzschild BH of mass $M$ seen from a distance $d$:

$$\Theta = 6\sqrt{3} \frac{GM}{c^2 d} \simeq 2.60 \frac{2R_S}{d}$$

Image of a thin accretion disk around a Schwarzschild BH

[Vincent, Paumard, Gourgoulhon & Perrin, CQG 28, 225011 (2011)]
Angular diameter of the event horizon of a Schwarzschild BH of mass $M$ seen from a distance $d$:

$$\Theta = 6\sqrt{3} \frac{GM}{c^2 d} \approx 2.60 \frac{2R_S}{d}$$

Largest black holes in the Earth’s sky:

- **Sgr A***: $\Theta = 53$ µas
- **M87**: $\Theta = 21$ µas
- **M31**: $\Theta = 20$ µas

**Remark**: black holes in X-ray binaries are $\sim 10^5$ times smaller, for $\Theta \propto M/d$
Reaching the \( \mu \text{as} \) resolution with VLBI

Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

Existing American VLBI network [Doeleman et al. 2011]
Testing general relativity with black holes

Reaching the $\mu$as resolution with VLBI

Existing American VLBI network [Doeleman et al. 2011]

Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

The best result so far: VLBI observations at 1.3 mm have shown that the size of the emitting region in Sgr A* is only 37 $\mu$as [Doeleman et al., Nature 455, 78 (2008)]
The near future: the Event Horizon Telescope

To go further:

- shorten the wavelength: $1.3 \text{ mm} \rightarrow 0.8 \text{ mm}$
- increase the number of stations; in particular add ALMA

Atacama Large Millimeter Array (ALMA) part of the Event Horizon Telescope (EHT) to be completed by 2020

August 2015: VLBI observations involving ALMA and VLBA
Near-infrared optical interferometry: GRAVITY

GRAVITY instrument at VLTI (2016)
Beam combiner (the four 8 m telescopes + four auxiliary telescopes)
astrometric precision on orbits: $10 \mu$as

[Gillessen et al. 2010]
July 2015 : GRAVITY shipped to Chile and successfully assembled at the Paranal Observatory
Fall 2016 : observations have started!

[MPE/GRAVITY team]
Testing general relativity with black holes

Observing black holes via gravitational waves
A dream come true on September 14, 2015, 09:50:45 UTC

[Abbott et al., PRL 116, 061102 (2016)]

Éric Gourgoulhon (LUTH)
A history of black holes
IHEB, Bruxelles, 26 Apr. 2017
Conclusions

After a century marked by the Golden Age (1965-1975), the first astronomical discoveries and the ubiquity of black holes in high-energy astrophysics, black hole physics is very much alive.
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It is entering a new observational era, with the advent of high-angular-resolution telescopes and gravitational wave detectors, which provide unique opportunities to test general relativity in the strong field regime.
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The GW150914 event was both the first direct detection of gravitational waves and the first observation of the merger of two black holes — the most dynamical event in relativistic gravity. The waveform was found consistent with general relativity.


