Black holes in the centenary year of general relativity

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Outline

1. A century-old history
2. Black holes in the sky
3. Testing general relativity with black holes
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A century-old history

A two centuries-old prehistory...

\[ V_{\text{esc}} > c \iff \frac{2GM}{R} > c^2 \iff \frac{2G}{R} \times \frac{4}{3} \pi R^3 \rho > c^2 \iff R > \sqrt{\frac{3c^2}{8\pi G \rho}} \]
John Michell (1784)

“If there should really exist in nature any bodies, whose density is not less than that of the sun, and whose diameters are more than 500 times the diameter of the sun, since their light could not arrive at us, ..., we could have no information from sight”

[Phil. Trans. R. Soc. Lond. 74, 35 (1784)]
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Pierre Simon de Laplace (1796)

“Un astre lumineux, de la même densité que la Terre, et dont le diamètre serait 250 fois plus grand que le Soleil, ne permettrait, en vertu de son attraction, à aucun de ses rayons de parvenir jusqu’à nous. Il est dès lors possible que les plus grands corps lumineux de l’univers puissent, par cette cause, être invisibles.”

[Exposition du système du monde (1796)]
Limits of the Newtonian concept of a black hole

- No privileged role of the velocity of light in Newtonian theory: nothing forbids $V > c$: the “dark stars” are not causally disconnected from the rest of the Universe.

\[ V_{\text{esc}} \sim c = \Rightarrow \text{gravitational potential energy} \sim \text{mass energy} M c^2 = \Rightarrow \text{a relativistic theory of gravitation is necessary!} \]

No clear action of the gravitation field on electromagnetic waves in Newtonian gravity.

[R. Taillet]
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[R. Taillet]
100 years ago: a relativistic theory of gravitation

\[ R - \frac{1}{2} R g = \frac{8\pi G}{c^4} T \]

The Schwarzschild solution

- Nov-Dec. 1915: Karl Schwarzschild: first exact non-trivial solution of Einstein equation $\Rightarrow$ spacetime metric outside a spherical body of mass $M$

\[g_{\alpha\beta}dx^\alpha dx^\beta = -\left(1 - \frac{2GM}{c^2r}\right)c^2dt^2 + \left(1 - \frac{2GM}{c^2r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\varphi^2)\]

- 1916: Johannes Drostes: circular orbit of photons at $r = 3GM/c^2$
- 1920: Alexander Anderson: light cannot emerge from the region $r < R_S := \frac{2GM}{c^2}$ ("shrouded in darkness")
- 1923: George Birkhoff: outside any spherical body, the metric is Schwarzschild metric
- 1932: Georges Lemaître: the singularity at $r = R_S$ is a coordinate singularity
- 1939: Robert Oppenheimer & Hartland Snyder: first solution describing a gravitational collapse $\Rightarrow$ for a external observer, $R \rightarrow R_S$ as $t \rightarrow +\infty$
1960: Martin Kruskal, John A. Wheeler: complete mathematical description of Schwarzschild spacetime ($\mathbb{R}^2 \times S^2$ manifold)

Schwarzschild-Droste coordinates $(t, r)$
The Schwarzschild solution: the complete picture

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Schwarzschild-Droste coordinates $(t, r)$

figure: [http://sagemanifolds.obspm.fr](http://sagemanifolds.obspm.fr)
The Schwarzschild spacetime: Carter-Penrose diagram

figure: http://sagemanifolds.obspm.fr
Roy Kerr (1963)

\[
g_{\alpha\beta} \, dx^\alpha \, dx^\beta = - \left( 1 - \frac{2GMr}{c^2 \rho^2} \right) c^2 dt^2 - \frac{4GMar \sin^2 \theta}{c^2 \rho^2} \, c \, dt \, d\varphi + \frac{\rho^2}{\Delta} \, dr^2
\]
\[+ \rho^2 d\theta^2 + \left( r^2 + a^2 + \frac{2GMar^2 \sin^2 \theta}{c^2 \rho^2} \right) \sin^2 \theta \, d\varphi^2 \]

where

\[
\rho^2 := r^2 + a^2 \cos^2 \theta, \quad \Delta := r^2 - \frac{2GM}{c^2} r + a^2, \quad a := \frac{J}{cM}
\]

→ 2 parameters : \( M \) : gravitational mass; \( J \) : angular momentum
Rotation enters the game: the Kerr solution

Roy Kerr (1963)

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Schwarzschild as the subcase \( a = 0 \):

\[ g_{\alpha\beta} \, dx^\alpha \, dx^\beta = - \left( 1 - \frac{2GM}{c^2r} \right) c^2 dt^2 + \left( 1 - \frac{2GM}{c^2r} \right)^{-1} \, dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta \, d\varphi^2 \right) \]
Physical meaning of the parameters $M$ and $J$

- **mass $M$**: *not* a measure of the “amount of matter” inside the black hole, but rather a *characteristic of the external gravitational field*
  → measurable from the orbital period of a test particle in far circular orbit around the black hole (*Kepler’s third law*)

$$J = a M c$$
characterizes the gravito-magnetic part of the gravitational field

→ measurable from the precession of a gyroscope orbiting the black hole (*Lense-Thirring effect*)

Remark: the radius of a black hole is not a well defined concept: it does not correspond to some distance between the black hole “centre” and the event horizon. A well defined quantity is the area of the event horizon, $A$.

The radius can be then defined from it: for a Schwarzschild black hole:

$$R = \sqrt{\frac{A}{4 \pi}} = \frac{2GM}{c^2} \approx 3 \left( \frac{M}{M_\odot} \right) \text{ km}$$
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Slice $t = \text{const}$ and $\theta = \pi/2$ of the Kerr spacetime
A century-old history

The Golden Age of black hole theory

- 1964 : Edwin Salpeter, Yakov Zeldovich : quasars (just discovered!) shine thanks to accretion onto a supermassive black hole
- 1965 : Roger Penrose : if a trapped surface is formed in a gravitational collapse and matter obeys some energy condition, then a singularity will appear
- 1967 : John A. Wheeler coined the word *black hole*
- 1969 : Roger Penrose : energy can be extracted from a rotating black hole
- 1972 : Stephen Hawking : law of area increase $\implies$ BH thermodynamics
- 1975 : Stephen Hawking : Hawking radiation
- 1965-1972 : the no-hair theorem
Within 4-dimensional general relativity, a stationary black hole in an otherwise empty universe is necessarily a Kerr-Newmann black hole, which is an electro-vacuum solution of Einstein equation described by only 3 parameters:

- the total mass $M$
- the total specific angular momentum $a = J/(Mc)$
- the total electric charge $Q$

$\Rightarrow \text{“a black hole has no hair” (John A. Wheeler)}$
The no-hair theorem

Dorochkevitch, Novikov & Zeldovitch (1965), Israel (1967), Carter (1971), Hawking (1972)

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Astrophysical black holes have to be electrically neutral:

- $Q = 0$ : Kerr solution (1963)

Other special cases:

- $a = 0$ : Reissner-Nordström solution (1916, 1918)
- $a = 0$ and $Q = 0$ : Schwarzschild solution (1916)
- $a = 0$, $Q = 0$ and $M = 0$ : Minkowski metric (1907)
General definition of a black hole

The textbook definition

[Hawking & Ellis (1973)]

black hole : \[ \mathcal{B} := \mathcal{M} - J^- (I^+) \]

where

\((\mathcal{M}, g) = \) asymptotically flat manifold

\(I^+ = \) future null infinity

\(J^- (I^+) = \) causal past of \(I^+\)

i.e. black hole = region of spacetime from which light rays cannot escape to infinity

event horizon : \( \mathcal{H} := \partial J^- (I^+) \)

(boundary of \(J^- (I^+)\))

\(\mathcal{H} \) smooth \(\implies\) \(\mathcal{H} \) null hypersurface
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The singularity marks the limit of validity of general relativity: to describe it, a quantum theory of gravitation would be required.
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The singularity marks the \textit{limit of validity of general relativity}: to describe it, a quantum theory of gravitation would be required.

The event horizon \(\mathcal{H}\) is a \textit{global structure} of spacetime: no physical experiment whatsoever can detect the crossing of \(\mathcal{H}\).
Viewed by a distant observer, the horizon approach is perceived with an infinite redshift, or equivalently, by an infinite time dilation.

A black hole is not an infinitely dense object: on the contrary it is made of vacuum (except maybe at the singularity); if one defines its “mean density” by $\bar{\rho} = \frac{M}{(4/3\pi R^3)}$, then

- for the Galactic centre BH (Sgr A*): $\bar{\rho} \sim 10^6$ kg m$^{-3}$ $\sim$ 2 $10^{-4}$ $\rho_{\text{white dwarf}}$
- for the BH at the centre of M87: $\bar{\rho} \sim 2$ kg m$^{-3}$ $\sim$ 2 $10^{-3}$ $\rho_{\text{water}}$!

$\implies$ a black hole is a compact object: $\frac{M}{R}$ large, not $\frac{M}{R^3}$!

Due to the non-linearity of general relativity, black holes can form in spacetimes empty of any matter, by collapse of gravitational wave packets.
The standard definition of a black hole is highly non-local: determination of $\dot{J}^-(\mathcal{I}^+)$ requires the knowledge of the entire future null infinity. Moreover this is not locally linked with the notion of strong gravitational field:

Example of event horizon in a flat region of spacetime:

Vaidya metric, describing incoming radiation from infinity:

$$ds^2 = -\left(1 - \frac{2m(v)}{r}\right) dv^2 + 2dv dr + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

with $m(v) = 0$ for $v < 0$

$dm/dv > 0$ for $0 \leq v \leq v_0$

$m(v) = M_0$ for $v > v_0$

[Ashtekar & Krishnan, LRR 7, 10 (2004)]
Teleological nature of event horizons

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$⇒$ no local physical experiment can locate the event horizon

[Ashtekar & Krishnan, LRR 7, 10 (2004)]
New paradigm for the theoretical approach to black holes: instead of event horizons, black holes are described by

- trapping horizons (Hayward 1994)
- isolated horizons (Ashtekar et al. 1999)
- dynamical horizons (Ashtekar and Krishnan 2002)
- slowly evolving horizons (Booth and Fairhurst 2004)

All these concepts are local and are based on the notion of trapped surfaces.
The 2000’s: the triumph of numerical relativity

[Caltech/Cornell SXS]
[Scheel et al., PRD 79, 024003 (2009)]
Black holes in the sky

Outline

1. A century-old history
2. Black holes in the sky
3. Testing general relativity with black holes
Known black holes

Three kinds of black holes are known in the Universe:

- **Stellar black holes**: supernova remnants:
  \[ M \sim 10 - 30 \, M_\odot \text{ and } R \sim 30 - 90 \, \text{km} \]
  example: Cyg X-1: \[ M = 15 \, M_\odot \text{ and } R = 45 \, \text{km} \]

- **Supermassive black holes**, in galactic nuclei:
  \[ M \sim 10^5 - 10^{10} \, M_\odot \text{ and } R \sim 3 \times 10^5 - 200 \, \text{UA} \]
  example: Sgr A*: \[ M = 4.3 \times 10^6 \, M_\odot \text{ and } R = 13 \times 10^6 \, \text{km} = 18 \, R_\odot = 0.09 \, \text{UA} = \frac{14}{5} \times \text{radius of Mercury's orbit} \]

- **Intermediate mass black holes**, as ultra-luminous X-ray sources (?):
  \[ M \sim 10^2 - 10^4 \, M_\odot \text{ and } R \sim 300 \, \text{km} - 3 \times 10^4 \, \text{km} \]
  example: ESO 243-49 HLX-1: \[ M > 500 \, M_\odot \text{ and } R > 1500 \, \text{km} \]
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Stellar black holes in X-ray binaries

[McClintock et al. (2011)]
Stellar black holes in X-ray binaries

Dynamically measured masses of black holes in transient low-mass X-ray binaries (right), compared with measured masses of neutron stars (left)

Jet emitted by the nucleus of the giant elliptic galaxy M87, at the centre of Virgo cluster [HST]

\[ M_{\text{BH}} = 3 \times 10^9 M_\odot \]

\[ V_{\text{jet}} \simeq 0.99 c \]
The black hole at the centre of our galaxy: Sgr A*

Measure of the mass of Sgr A* black hole by stellar dynamics:

$$M_{BH} = 4.3 \times 10^6 M_\odot$$

Orbit of the star S2 around Sgr A*

$$P = 16 \text{ yr}, \quad r_{\text{per}} = 120 \text{ UA} = 1400 R_S, \quad V_{\text{per}} = 0.02 c$$

[Genzel, Eisenhauer & Gillessen, RMP 82, 3121 (2010)]
Supermassive black hole formation

[Volonteri et al., arXiv:1511.02588]
Can we see a black hole from the Earth?

Image of a thin accretion disk around a Schwarzschild BH

[Vincent, Paumard, Gourgoulhon & Perrin, CQG 28, 225011 (2011)]

Angular diameter of the event horizon of a Schwarzschild BH of mass $M$ seen from a distance $d$:

$$\Theta = 6\sqrt{3} \frac{GM}{c^2d} \simeq 2.60 \frac{2R_S}{d}$$
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Largest black holes in the Earth’s sky:

Sgr A* : $\Theta = 53 \mu$as
M87 : $\Theta = 21 \mu$as
M31 : $\Theta = 20 \mu$as

Remark: black holes in X-ray binaries are $\sim 10^5$ times smaller, for $\Theta \propto M/d$
Reaching the $\mu\text{as}$ resolution with VLBI

Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

Existing American VLBI network [Doeleman et al. 2011]
Reaching the $\mu$as resolution with VLBI

Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

The best result so far: VLBI observations at 1.3 mm have shown that the size of the emitting region in Sgr A* is only 37 $\mu$as.

[Doeleman et al., Nature 455, 78 (2008)]

Existing American VLBI network [Doeleman et al. 2011]
The near future: the Event Horizon Telescope

To go further:
- shorten the wavelength: $1.3 \text{ mm} \rightarrow 0.8 \text{ mm}$
- increase the number of stations; in particular add ALMA

Atacama Large Millimeter Array (ALMA)

part of the Event Horizon Telescope (EHT) to be completed by 2020 August

2015: VLBI observations involving ALMA and VLBA
VLBA and EHT observations of M87

- Possible BH shadow
- Optically-thick region ($\geq 21 \mu\text{as}$)
- Optically-thin region (40 $\mu\text{as}$)
- EHT beam
- Jet base of M87 (VLBA at 43GHz)

Near-infrared optical interferometry: GRAVITY

GRAVITY instrument at VLTI (2016)
Beam combiner (the four 8 m telescopes + four auxiliary telescopes)

astrometric precision on orbits: 10 $\mu$as

[Gillessen et al. 2010]
Near-infrared optical interferometry : GRAVITY

July 2015 : GRAVITY shipped to Chile and successfully assembled at the Paranal Observatory Commissioning with the four 8-m VLT Unit Telescope : first half 2016.

[MPE/GRAVITY team]
Another way of observing BH: gravitational waves!

[Janssen et al., PoS(AASKA14)037 (2014)]
Advanced ground-based GW detectors

- Adv. LIGO: started Sept. 2015
- KAGRA: 2018

Gravitational wave detector VIRGO in Cascina, near Pisa (Italy) [CNRS/INFN]
Black holes in the sky

eLISA space detector

LISA Pathfinder in Kourou, getting ready for the launch on 2 Dec. 2015!

eLISA scientific theme selected in 2013 for ESA L3 mission → launch ~ 2028
eLISA space detector

Signal-to-noise ratio for gravitational waves from the inspiral of a BH binary at $z = 0.5$

Detecting gravitational waves by pulsar timing
EPTA results on supermassive BH binaries

[Babak et al., arXiv:1509.02165]

EPTA : European Pulsar Timing Array
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Is general relativity unique?

Yes if we assume

- a 4-dimensional spacetime
- gravitation only described by a metric tensor $g$
- field equation involving only derivatives of $g$ up to second order
- diffeomorphism invariance
- $\nabla \cdot T = 0$ (⇒ weak equivalence principle)

The above is a consequence of Lovelock theorem (1972).
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However, GR is certainly not the ultimate theory of gravitation:

- it is not a quantum theory
- cosmological constant / dark energy problem

GR is generally considered as a low-energy limit of a more fundamental theory:

- string theory
- loop quantum gravity
- ...
Testing general relativity with black holes

Extensions of general relativity

[Berti et al., CGQ in press, arXiv:1501.07274]
Test: are astrophysical black holes Kerr black holes?

- GR $\implies$ Kerr BH (no-hair theorem)
- Extension of GR $\implies$ BH may deviate from Kerr
Testing general relativity with black holes

Test: are astrophysical black holes Kerr black holes?

- $\text{GR} \implies \text{Kerr BH (no-hair theorem)}$
- extension of GR $\implies$ BH may deviate from Kerr

Observational tests

Search for

- stellar orbits deviating from Kerr timelike geodesics (**GRAVITY**)
- accretion disk spectra different from those arising in Kerr metric (**X-ray observatories, e.g. Athena**)
- images of the black hole silhouette different from that of a Kerr BH (**EHT**)

Eric Gourgoulhon (LUTH)
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Need for a good and versatile geodesic integrator
to compute timelike geodesics (orbits) and null geodesics (ray-tracing) in any kind of metric
Alain Riazuelo code

[A. Riazuelo, arXiv:1511.06025]
Integration of geodesics in Kerr metric
Integration of geodesics in any numerically computed 3+1 metric
Radiative transfer included in optically thin media
Very modular code (C++)
Yorick and Python interfaces
Free software (GPL): http://gyoto.obspm.fr/

[Vincent, Paumard, Gourgoulhon & Perrin, CQG 28, 225011 (2011)]
[Vincent, Gourgoulhon & Novak, CQG 29, 245005 (2012)]
Boson star = localized configurations of a self-gravitating massive complex scalar field \( \Phi \equiv "\text{Klein-Gordon geons}" \)
[Bonazzola & Pacini (1966), Kaup (1968)]

Boson stars may behave as black-hole mimickers

- Solutions of the Einstein-Klein-Gordon system computed by means of Kadath
  [Grandclément, JCP 229, 3334 (2010)]

- Timelike geodesics computed by means of Gyoto

Zero-angular-momentum orbit around a rotating boson star based on a free scalar field \( \Phi = \phi(r, \theta)e^{i(\omega t + 2\varphi)} \)
with \( \omega = 0.75 \text{ m/h} \).
[Granclément, Somé & Gourgoulhon, PRD 90, 024068 (2014)]
Kerr BH \[ a/M = 0.9 \]

Boson star \[ k = 1, \omega = 0.70 \text{ } m/\hbar \]

[Vincent, Meliani, Grandclément, Gourgoulhon & Straub, arXiv:1510.04170]
Conclusion

After a century marked by the Golden Age (1965-1975), the first astronomical discoveries and the ubiquity of black holes in high-energy astrophysics, black hole physics is very much alive.

It is entering a new observational era, with the advent of high-angular-resolution telescopes and gravitational wave detectors, which will provide unique opportunities to test general relativity in the strong field regime.