LISA: Gravitational Wave Detection in Space
Notes to Lectures for the General Relativity Trimester
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Outline of topics to be covered:

1. **Overview of gravitational wave sources.** The gravitational wave spectrum. Ground-based detector sources overview. LISA sources: supermassive black holes; extreme mass-ratio inspiral; galactic binaries; the big bang.

2. **Gravitational wave detectors.** History and present status of detector development; interaction of gravitational waves with beam detectors; physics of interferometers; LISA; LISA-PF.

3. **Data Analysis.** Matched filtering; parameter searches; burst searches; special challenges of LISA data analysis.

4. **Future Prospects.** Current status of LISA; development of data analysis systems; synergies with other possible missions; science return from LISA.

Lecture 1 – Overview of gravitational wave sources

The gravitational wave spectrum

Gravitational waves can of course have any frequency, but the frequencies of interest depend on the physical processes that generate them and on our ability to detect them, directly or indirectly. There are at least two regimes in which interesting gravitational waves can be produced: waves generated by cosmological processes in the early universe, and waves generated by isolated astrophysical systems formed after the universe developed structure.

In the early universe, which we will consider in detail below, waves are produced either by quantum fluctuations in the gravitational field or by inhomogeneities in highly relativistic matter. In either case the relevant length scale is the so-called particle horizon size, which is essentially the size of the causally-connected region of the universe at that time. This makes it relatively easy to estimate the wavelengths of the waves generated at a particular epoch, but not of course their amplitudes. All these processes produce a *cosmological background* of radiation with Gaussian (normally-distributed) random wave amplitudes. The spectrum depends on the physics, as we shall see below.

Waves produced by isolated systems, which are the most likely waves that LISA will detect, can usually be estimated to a first approximation by using the quadrupole formula. (There are exceptions, such as black hole vibrations and cosmic strings.) This is suitable for estimates of their detectability, but in most cases much more detailed computations are necessary in order to predict the waveforms well enough to recognize them against detector noise, or to infer from detected waves what the properties of the source are (mass, spin, size).

If a source is near enough or strong enough, then it may be detectable as an isolated source. On the other hand, if it is part of a large population of sources of similar strength whose waves all superpose, then they form a *confusion background*. If there are sufficiently many sources, they are said to form an *astrophysical background* of gravitational radiation. It will be an incoherent sum of all of the sources and, by the central limit theorem, it will also be a random normally distributed wave field, whose spectrum depends on the physics of the sources.

The frequency of radiation from an isolated source of course also depends on its physics. However, when the source's motions are determined by self-gravity (most of the time this is true) then there is a characteristic angular frequency of motion:

$$\omega^2 = \pi G\rho,$$

where ρ is the characteristic or mean mass density of the source. This gives correctly the orbital frequencies for systems covering a vast range of density: of a satellite skimming the surface of the Earth, the resonant frequency of the Earth due to seismic waves, the fundamental mode frequency of oscillation of a star, the dominant frequency of quasi-normal mode vibration of a disturbed black hole. All have characteristic frequencies given by this formula, to within factors of two or so! The frequency of the radiated gravitational wave is usually twice this frequency.

An exception to this rule is when the frequency of the waves comes from the spin of a body rather than its overall dynamics: radiation from, say, a spinning neutron star is emitted at a frequency that is related to its spin, which is a memory of its formation, a function of its conserved angular momentum. The characteristic frequency above is then an upper limit on the spin frequency, but realistic gravitational waves could come out at lower frequencies.

This remarkable circumstance allows one to understand much about the kinds of sources different detectors can see.

The rule of thumb for frequency allows us to understand much about the kinds of sources different detectors can see. Ground-based detectors are confined to frequencies above 10 Hz, because below this (and certainly below about 1 Hz) local gravity disturbances on the Earth are stronger than the expected wave amplitudes. These disturbances arise from density changes associated with seismic waves, weather systems, ocean waves, and so on, and cannot be screened out because they are changes in the gravitational field, not vibrations or other mechanical disturbances. The only way to observe below 1 Hz is to get away from the Earth into spacel.

At the relatively high frequencies available to ground-based detectors, gravitational wave sources must be highly relativistic and not too massive: a black hole of mass $1000M_\odot$ has a characteristic frequency of 10 Hz, and larger holes have *lower* frequencies because they are effectively less dense. Nothing of mass above $1000M_\odot$ can radiate at frequencies above 10 Hz. LISA will operate in the low-frequency regime, around 1 mHz. We can therefore expect to detect radiation from very massive black holes or more widely-separated binary stars. We shall look at these two frequency ranges separately.

Overview of sources for ground-based detectors

Here is a brief list of sources that might lead to detectable gravitational waves for LIGO, VIRGO, GEO, and the other ground-based detectors.

- **Gravitational collapse.** The longest-sought and still the least-understood source, we still cannot do more than estimate the amplitude h that would be emitted if a source lost an assumed amount of energy to waves. A supernova releases a lot of energy, roughly the binding energy of a neutron star. The dominant emission is probably neutrinos, but some of this neutrino energy is transferred to the gas of the star, expelling it (what we see as the explosion) and inducing nuclear reactions in it (which produce many of the elements of which we are made). An unknown but probably small fraction of this energy goes into gravitational radiation. The problem is that it is still not possible to reliably estimate the amount of asymmetry in gravitational collapse. Numerical simulations show, however, that significant asymmetry only develops in rotating collapse, and even then only if the pre-collapse star is differentially rotating. It is difficult to know what initial conditions for collapse are reasonable.
- **Chirping binary stars.** Our best proof of the reliability of general relativity for gravitational waves is the behavior of double-neutron-star binary pulsar systems, especially the the Hulse-Taylor binary pulsar. These systems shrink due to the loss of energy to gravitational radiation. In some 10^8 yr, the Hulse-Taylor stars will coalesce. During the few minutes before coalescence it will emit radiation in the ground-based frequency band. Its frequency during this time will rise rapidly ("chirp") due to the enormous gravitational wave luminosity.

  These coalescing binaries are the most promising long-term source of radiation for LIGO and VIRGO. Their emitted radiation can be characterized rather well within the post-Newtonian approximation until they are close to coalescence. If the objects have significant spin, this complicates the orbital behaviour but it can still be modelled within the pN approximation.

  Black-hole binaries will also coalesce, and the numbers seen by ground-based detectors may be relatively high. There are two reasons for this. The first is because the formation of a black hole has less likelihood to break up a binary system than the formation of a neutron star, which involves a much larger loss of mass from the collapsing object. The second is that astronomers have come to realize that globular clusters are black-hole-binary factories, regularly producing and expelling compact binaries that may coalesce within a hubble time. Black hole binaries may therefore be among the first sources detected by ground-based detectors. If they are seen, they will not be accompanied by any optical event, and it will be difficult to convince ourselves beyond a reasonable doubt that we really have seen an event. But even the first event will show us the gravitational waves from the coalescence event itself, something we are trying to model on supercomputers today. The first observation will give us a strong hint about the correctness of our models. Subsequent observations will give better comparisons for numerical relativity simulations, and will help us learn about the distribution of masses of real black holes.

- **Pulsars and other spinning neutron stars.** There are a number of ways in which a neutron star may give off a continuous stream of gravitational waves: shape irregularities in a spinning star, unstable r-modes (also only spinning stars), pulsation modes excited during star formation or by transient events like starquakes (associated with glitches). Radiation from shape irregularities or r-modes will be weak but may be detectable if one observes for long periods of time, up to many months.

  The pulsation modes of neutron stars are at frequencies of 2 kHz or higher, where present detectors have poor sensitivity. These are interesting systems to observe because the exact frequency of such a mode would for the first time give us insight into the interior physics of a neutron star.

- **Random backgrounds.** Here is a brief list of ways in which cosmological or astrophysical backgrounds above 10 Hz could arise:

  1. Inflation. The earliest phase of the Big Bang is now thought to have been an enormously accelerated expansion called inflation, an acceleration driven by a large positive effective value of the cosmological constant. Quantum fluctuations in matter fields and in the gravitational field would have been amplified by this expansion (an example of parametric amplification, of which there are many examples in quantum optics), so that when inflation ended there were small perturbations in the universe. The density perturbations evolved into the galaxies and clusters we know today. The metric perturbations remain today as relic gravitational waves, but are predicted to be very weak, with $\Omega_{gw} < 10^{-14}$. It will be a major challenge to detect this

radiation, but the goal is an important one because it will bring us a direct view of the universe at the end of inflation. The cosmic microwave background observations, by contrast, show us the universe when it was some 3×10^5 years old.

2. Phase transitions. After inflation stopped, the universe contained a very hot plasma, and the laws of physics were quite different from what they are today. As the universe cooled by expansion, the physical world as we know it gradually took shape. Various phase transitions are thought to have occurred, during which previously symmetrical forces became distinct from one another (such as the weak from the electromagnetic). If such a transition involved density inhomogeneities, then it might have produced a background of random radiation on top of that from inflation. It is unlikely that they will exceed $\Omega_{gw} = 10^{-5}$, because a background this strong would have changed the balance among light elements (helium and lithium) that were created in the Big Bang (but which did not take form until the Universe had cooled well beyond the phase transitions discussed here).
3. Astrophysical backgrounds. Essentially all objects in the universe are producing gravitational waves at some level. The strongest backgrounds are likely to be between 10^{-6} and 10^{-1} Hz, because most asymmetric relativistic systems that we know about have dynamical timescales between a few seconds and a few days. These are likely to be much stronger than inflation in this frequency band. They are not constrained by the nucleosynthesis bound mentioned in the previous paragraph because they arose after lithium and helium were made.
4. Exotic sources. Many other possibilities have been suggested. Cosmic strings (see next paragraph; they could arise from string theory or from phase transitions) may continuously be producing bursts of radiation that blend into a background. So-called pre-big-bang inflationary models produce any desired level of radiation today by altering the spectrum of fluctuations in the metric that existed before inflation started. Brane-world models for fundamental physics can also include scenarios where branes collide; such models can produce different levels of background radiation, including none!
5. Other compact relativistic objects. We expect to see neutron stars and black holes, but only in certain mass ranges, because of what we

know about the physics of matter that forms them. Neutron stars should exist between about 1 and 2 $M_\odot$; below this range stars are stable to collapse and just form white dwarfs, while above this range neutron stars are unstable and would collapse to black holes. In turn, we do not expect black holes below about $2M_\odot$, again because stars are stable. But in the early universe, the behavior of matter may have been different, and smaller black holes may have formed. Other kinds of matter may exist that forms compact stars: scientists have speculated about massive interacting scalar fields (so-called boson stars) and, less exotically, about quark-matter stars in which a neutron star has undergone a phase transition to matter with non-zero strangeness. Observations of microlensing toward the Large Magellanic Clouds have suggested that the halo of our Galaxy may contain MACHOs, compact stars with masses of about $0.5M_\odot$, which would be puzzling if confirmed.

Sources for LISA

LISA will operate in the low-frequency band between 0.1 mHz and 0.1 Hz. By comparison with sources for ground-based detectors, isolated sources in this frequency range sources either have to have larger masses (the modes of a massive black hole of $10^6 M_\odot$ have a frequency around 1 mHz) or larger separations (compact white dwarf binaries with orbital periods around an hour are known, and pre-coalescence neutron-star binaries will pass through this frequency range thousands of years before they coalesce). LISA's high sensitivity means that some sources will be extremely strong in the LISA data, and more distant sources will still be strong enough to cause a confusion noise background. We discuss some of these issues here.

First, a list of expected sources:

- **Massive black holes.** One of the great surprises of the late 1990s was the discovery that most galaxies contain a massive ($10^6 M_\odot$) or even a supermassive ($10^9 M_\odot$) black hole in their centres. The formation history of these holes is still not clear. They probably formed early in the lifetime of the galaxy and accreted a lot of gas in the early phases, but they may also have grown by merger of smaller black holes. There is growing evidence that even the small proto-galactic clouds whose merger led to the galaxies we see today contained black holes. Most galaxies may have contained thousands of black holes of mass $10^3 M_\odot$ or more, and it is very possible that these have sunk to the center and merged into the large ones. Quasars and active galaxies seem to be powered by supermassive holes, and there is evidence from X-ray studies that gas accretion accounted for most of their growth from 10^6 to $10^9 M_\odot$, but not for all of it.

  LISA has enough sensitivity to see the merger of two black holes in the mass range $10^4 - -10^7 M_\odot$ if it occurs during the mission lifetime anywhere in the Universe. Even very distant events, say at redshift 10, could have very high signal to noise ratios. LISA should therefore be able to answer some of the questions about black hole growth, and about the initial mass spectrum of massive black holes. It should also be able to study in detail the complex merger phase where the two black holes come together. By comparing this radiation with that predicted by supercomputer simulations of black hole mergers, LISA will explore and test general relativity in the strongest possible gravitational fields.

Detecting the radiation from massive black hole binaries will not be a problem. The inspiral phase will be so easy to pick up that LISA will be able to issue a prediction of the merger time and location on the sky at least a month before it happens. This will enable coordinated observing campaigns with X-ray telescopes and ground-based instruments.

- **EMRI: Extreme Mass-Ration Inspiral sources.** Massive black holes in galactic centres do not exist in isolation. They are inside vast clouds of stars that have, from normal stellar evolution, large numbers of neutron stars and stellar-mass black holes (say $10M_{\odot}$). Random encounters among these stars gradually allow the black holes to sink toward the centre, so that near the massive hole there should be an overdensity of stellar holes. Occasionally a random close encounter should put one of these holes on a "plunge" orbit, a highly eccentric orbit aimed nearly directly at the central hole. If the impact parameter is small enough, the first pass near the massive hole will radiate enough energy in gravitational waves to change the orbit from unbound to bound. The smaller hole will then execute a large number of orbits during which its periholon distance (peri*bothron* means closest approach to the hole) remains roughly constant but its apholon distance shrinks. During this time it will be a source of isolated bursts of gravitational waves at periholon. LISA may pick these up from nearby sources but they will not repeat during the LISA mission lifetime. More distant encounters will contribute to an astrophysical background in the LISA band.

  Eventually the smaller hole acquires an orbit where its overall period is in the LISA band. Even then, it may have 10^5 orbits remaining before eventually crossing the horizon. The details of its orbital behaviour are contained in the phase evolution of the emitted gravitational waves. If it is possible to construct accurate matched filters (needed for the data analysis – see Chapter 3) for this radiation then LISA will be able to examine in detail the geometry of the black hole exterior. It will test the uniqueness theorem of general relativity, that all black holes will be members of the Kerr family.

  This exquisite test will, however, not be easy to perform. At present, we do not understand the equation of motion of the small black hole around the large on well enough to construct an accurate filter over so many orbits. The hole follows a geodesic only to lowest order. Corrections to its motion to first order in the mass-ratio of the two holes are required. Some of these are understood, and in principle we have methods to calculate all of them. But we have no efficient means to do so. This is one of the outstanding problems of LISA science that must be solved before LISA launches.

Even when good waveform predictions are available, the search algorithm will be very challenging. The parameter space for the filter family will be huge and dense, with small changes in parameters making large changes in the phase evolution by the end of the orbit. Studies indicate that hierarchical methods, analogous to those being developed for ground-based studies of the pulsar problem (mentioned earlier), will be able to solve this problem and detect hundreds of inspiral events per year. But the detailed implementation has not yet been done.

The events detected will come from nearby galaxies, out to redshifts of several tenths. More distant events will create some of the confusion noise LISA must contend with. Exactly how this will affect detection, and just how to pull as many sources as possible out of the confusion background, are problems that have not yet been adequately studied.

- **IMRI: Intermediate Mass-Ratio Inspiral sources.** In the last few years it has been realised that the first generation of stars (Population III) may have produced an abundance of heavy black holes, in the range from a few hundred to a few thousand solar masses. These are called intermediate mass black holes (IMBH). If this happened, then the massive central black holes in galaxies will also capture some of these.

  These sources differ from IMRIs in at least two key details. First, they radiate more strongly, so can be seen further away. Second, it is not clear that their orbits can be calculated accurately enough by the same linear perturbation techniques that are being used to study EMRI orbits, since the mass ratio m/M might be as large as 0.01 in some cases.

- **Compact binary systems.** LISA's data stream will be dominated by radiation from binary systems in the Galaxy. These are not normal binary systems, because they have to have periods of an hour or so to be in the LISA band. They consist of white dwarfs and (very occasionally) relativistic objects like neutron stars and black holes. LISA is expected to be able to resolve thousands of such objects at frequencies above 1 mHz, including perhaps one black-hole/black-hole binary. But there are so many of them at lower frequencies that they will blend together into another confusion noise, and only the nearby systems will be strong enough to resolve. The confusion noise will essentially look like a random, but non-isotropic, background. It falls off rapidly at high frequencies, both because the population of sources thins out and because LISA has better directional sensitivity at higher frequencies (see below), enabling it to separate overlapping sources better.

Most of the systems that LISA can resolve will chirp: their frequency is high enough, hence their orbital separation small enough, that they radiate gravitational waves strongly. This means that LISA will also be able to give distances for these systems, an additional observable that will help identify the systems in other observing instruments.

Extragalactic binaries may also be seen by LISA, including for example a black-hole binary in M31, the Andromeda galaxy. But most extragalactic systems blend into a confusion background at higher frequencies (up to 10 mHz) that is thought to lie just below the LISA shot-noise limit. This is one reason that it does not seem to be worthwhile to raise the power of LISA's lasers, since the gain in sensitivity would be limited by confusion noise.

- **Stochastic background.** LISA can only detect the stochastic background if its noise is higher than instrumental or confusion noise from foreground objects. Radiation with a closure density $\Omega_{\rm gw} \sim 10^{-10}$ would be detectable in LISA's best frequency around 3 mHz. On standard inflation models this seems unlikely, but there are many possibilities for radiation generated either in non-standard cosmological scenarios (string cosmology) or from phase transitions that happened after inflation. Intriguingly, LISA's frequency band contains radiation that would have been emitted by the Universe when its temperature was about 1 TeV, corresponding to the electroweak phase transition. If an excess noise is seen in LISA, the experimeters must have good confidence to ascribe it to gravitational waves and not to instrumental misbehaviour.

  LISA has one in-built way of distinguishing a gravitational wave background from instrumental noise. Since its triangular array in effect gives it three detectors, and since the gravitational wave field has only two degrees of freedom (the two polarizations), LISA can form a linear combination of the outputs of the three detectors that *eliminates* the gravitational wave signal completely. Called the *Sagnac mode*, this is a useful check: if the full LISA output and the Sagnac mode have similar noise, then the noise is probably instrumental; but if the LISA output is higher than the Sagnac noise, then LISA may have detected a background.

- **Unexpected sources.** LISA has such good sensitivity that it may well detect sources that are not know or expected. Cosmic strings are a particularly intriguing candidate. A cosmic string is a one-dimensional "defect", or mass concentration, which would arise naturally in some unified field theories where symmetry breaking occurs as the Universe cools off. The strings essentially trap a small region of space where the symmetry did not break and the vaccum energy is much higher. These strings, which can be megaparsecs or more in length, can have kinks or sharp bends, which would travel along the string at the speed of light and would radiate strongly in certain "beamed" directions, like electrons in a synchrotron. Their characteristic signal would be easy to distinguish from most known systems.

Chapter 2 – Gravitational wave detectors

Timeline in the history of gravitational wave detection

- **1960 – 75** J. Weber develops first bar detector at Univ. of Maryland. Announces 2-bar coincidences at unexpected rate.
- **1970 – 80** Other groups develop similar bar detectors, see no unusual events: Glasgow, Garching, Rome, Bell Labs, Stanford, Rochester, LSU, MIT.
- **1980 – 1994**
  1. Cryogenic bar detectors developed at Rome, Stanford, LSU, Perth (Aus.) Reach below 10^{-19}. A number of joint observations performed, no detections.
  2. Rome and Maryland room-temperature detectors, running at time of SN1987a, report unusualy correlations, including with neutrino observations. If true would represent conversion of hundreds of solar masses into energy. Claimed statistical significance seriously over-estimated.
  3. Ranging measurements to Pioneer spacecraft are used to place limits on gravitational radiation with periods of a few hours.
  4. Prototype interferometers developed at MIT, Garching, Glasgow, Caltech. Typical sensitivity 10^{-18}.
  5. First long coincidence observation with interferometers: the Glasgow/Garching 100-hour experiment.
  6. Major collaborations formed to propose large-scale detectors:
     - LIGO: Caltech & MIT
     - VIRGO: France (CNRS) & Italy (INFN)
     - GEO: Germany & UK
     - AIGO: Australia

- **1990 – 2000**
  1. Ultracryogenic bars constucted in Rome (Frascati) and Legnaro. Expected to reach below 10^{-20}. New generation of spherical or icosahedral solid-mass detectors proposed in USA (LSU), Brasil, Netherlands, Italy. Arrays of smaller bars proposed for the highest frequencies.
  2. Large interferometers funded and constructed:
     - LIGO: Hanford (WA) and Livingstone (LA), 4 km (plus 1 2-km in WA)
     - VIRGO: Pisa, 3 km
     - GEO600: Hannover, 600 m
     - TAMA300: Japan, 300 m, longer one expected
  3. Ranging to interplanetary spacecraft continues to place limits on GWs, but still far below what would be expected from sources.
  4. The space-based low-frequency detector LISA proposed to the European Space Agency and adopted as one of four Corenerstone missions in the Horizons 2000+ forward plan.
- **2000 – present**
  1. Network of 4 bar detectors (Allegro, Nautilus, Auriga, Perth) reports upper limits based on simultaneous observing over a 1-year period.
  2. Rome group reports unexpected correlated events between two antennas, again level of significance not clear, again much stronger than expected GWs.
  3. NASA and ESA agree to develop LISA jointly for launch in 2012 (now 2014). Cooperate on ESA Smart-2 mission (2008) to test key technologies for LISA.
  4. LIGO and GEO begin to take data and analyse results jointly. First upper limits from the large interferometers published.
  5. LIGO reaches within 1.5 of design sensitivity, goes into first long data run (Sept 2005). Early in 2006 LIGO reaches design sensitivity.
  6. Cosmic microwave background groups begin serious plans for detecting “B-mode” signal in CMB polarization, a signature of gravitational waves during the epoch when CMB was emitted. Pulsar astronomers begin first timing experiments (Parkes times 20 pulsars simultaneously) to look for correlated fluctuations that might be caused by gravitational waves passing the Earth with periods of 5 years or so.

Interaction of gravitational waves with beam detectors

In analysing gravitational wave detectors, it is important not to fall into the trap of coordinate confusion. The most convenient gauge for describing a propagating gravitational wave is the TT gauge, which is comoving for freely-falling particles. Convenient though it is, it is **not** the same as the locally Minkowskian coordinate system that would be used by an experimenter to analyze an experiment, particularly in a ground-based detector. Of course, invariant quantities like proper distance can be computed in any coordinate system. It is important to be aware of how coordinates are used in any computation, and very important to ask coordinate-independent questions.

- If a detector is small compared to the wavelength of a gravitational wave, then the geodesic deviation equation can be used to just give a simple extra force on the equipment. Laser interferometers on the Earth can be treated this way. Then the gravitational wave simply produces a force to be measured. The gravitational wave physics is simple. (The physics of the detectors is not!!)
- If a detector is comparable to or larger than a wavelength, then the geodesic deviation equation is not useful: it is only the first term in a Taylor expansion that becomes very clumsy. Space-based interferometers like LISA, accurate ranging to solar-system spacecraft, and pulsar timing are all in this class. They are all **beam detectors**: they use light (or radio waves) to register the waves.
- For large detectors it is easiest to remain in TT gauge and calculate the effect of the waves on the "speed" (coordinate speed) of the light. In the "+" polarisation metric of a plane wave travelling in the positive z-direction, a null geodesic moving in the x-direction satisfies the proper distance relation

  $$ds^2 = -dt^2 + (1 + h_+)dx^2 + (1 - h_+)dy^2 + dz^2 = 0,$$

  which can be solved for its effective speed

  $$\left(\frac{dx}{dt}\right)^2 = \frac{1}{1 + h_+}.$$

  This is a *coordinate speed*, no contradiction to special relativity.

- Suppose one arm of an interferometer is along the x-direction and the wave is moving in the z-direction with a +-polarization of *any* waveform $h_+(t)$ along this axis. (It is a plane wave, so its waveform does not depend on x.) Then from the above proper-distance equation, a photon emitted at time t from the origin reaches the other end, at a fixed coordinate position $x = L$, at the coordinate time
$$t_{far} = t + \int_0^L [1 + h_+(t(x))]^{1/2}\, dx,$$
where the argument $t(x)$ denotes the fact that one must know the time to reach position x in order to calculate the wave field. This implicit equation can be solved in linearized theory by using the fact that h_+ is small. Then we can set $t(x) = t + x$ and expand the square root. The result is
$$t_{far} = t + L + \frac{1}{2}\int_0^L h_+(t + x)dx.$$
In an interferometer, the light is reflected back, so the return trip takes
$$t_{return} = t + 2L + \frac{1}{2}\left[\int_0^L h_+(t + x)dx + \int_0^L h_+(t + L + x)dx\right].$$

- What one monitors is changes in the time for the return trip as a function of time at the origin. This is directly what happens in radar ranging or in transponding to spacecraft, and close to what happens in direct interferometry. So one measures the variation of the return time as a function of the start time t:
$$\frac{dt_{return}}{dt} = 1 + \frac{1}{2}\left[h_+(t + 2L) - h_+(t)\right].$$
This depends only on the wave amplitude when the beam leaves and when it returns. Interestingly it does not involve the wave amplitude at the other end.

- The wave amplitude at the other end does get involved if the wave travels at an angle θ to the z-axis in the $x - z$ plane. If we re-do this calculation, allowing the phase of the wave to depend on x in the appropriate way, and taking into account the fact that h_{xx} is reduced if the wave is not moving in a direction perpendicular to x, we can find
$$\begin{aligned}\frac{dt_{return}}{dt} &= 1 + \frac{1}{2}\{(1 - \sin\theta)h_+(t + 2L) - (1 + \sin\theta)h_+(t) \\ &\quad + 2\sin\theta h_+[t + L(1 - \sin\theta)]\}.\end{aligned}$$
This *three-term relation* is the starting point for analyzing the response of man-made beam detectors.

Analysis of beam detectors

Here we consider the implications of the three-term formula for specific ways in which gravitational waves are being looked for using beam detectors.

- **Ranging to spacecraft.** Both NASA and ESA perform experiments in which they monitor the return time of communication signals with interplanetary spacecraft for the characteristic effect of gravitational waves. For missions to Jupiter and Saturn, for example, the return times are of order $2 - 4 \times 10^3$ s. Any gravitational wave event shorter than this will appear 3 times in the time-delay: once when the wave passes the Earth-based transmitter, once when it passes the spacecraft, and once when it passes the Earth-based receiver. Searches use a form of data analysis using pattern matching. Using two transmission frequencies and very stable atomic clocks, it is possible to achieve sensitivities for h of order 10^{-13}, and even 10^{-15} may soon be reached.
- **Pulsar timing.** Many pulsars, particularly the old millisecond pulsars, are extraordinarily regular clocks, with random timing irregularities too small for the best atomic clocks to measure. If one assumes that they emit pulses perfectly regularly, then one can use observations of timing irregularities of single pulsars to set upper limits on the background gravitational wave field. Here the 3-term formula is replaced by a simpler two-term expression, because we only have a one-way transmission. Moreover, the transit time of a signal to the Earth from the pulsar may be thousands of years, so we cannot look for correlations between the two terms in a given signal. Instead, the delay is a combination of the effects of waves at the pulsar when the signal was emitted and waves at the Earth when it is received.

  If one simultaneously observes two or more pulsars, the Earth-based part of the delay is correlated, and this offers a means of actually detecting long-period gravitational waves. Observations require timescales of several years in order to achieve the long-period stability of pulse arrival times, so this method is suited to looking for strong gravitational waves with periods of several years. Observations are currently underway at a number of observatories, most notably at the Parkes Observatory, which is monitoring 20 pulsars and may increase the number to 40. In the future, vast coherent arrays of radio antennas (the Square Kilometer Array) will provide excellent data for potentially hundreds of sources simultaneously.

- **Interferometry.** An interferometer essentially measures the difference in the return times along two different arms. The response of each arm will follow the three-term formula, but with different values of θ, depending in a complicated way on the orientation of the arms relative to the direction of travel and polarization of the wave. Ground-based interferometers are small enough to use the small-L formulas we derived earlier. But LISA, the space-based interferometer, is larger than a wavelength of gravitational waves for frequencies above 10 mHz, so a detailed analysis based on the 3-term formula is needed.

  Notice that the correct way to describe the interaction of an interferometer with a gravitational wave is in terms of time-delays: each arm is a kind of "light-clock" and we compare the time-keeping of each with the other as they are stretched by the gravitational wave. Non-experts frequently assume that the principle of operation of an interferometer is different, that (for example) it measures the number of wavelengths of light that "fit" along an arm and then looks for changes in that number as the mirrors swing. This is not the case. If it were, then the frequency of the light would have to be stable to parts in 10^{21}, the same accuracy as that of the "length" measurement. No laser could be that stable. Instead, an interferometer measures proper time delays, and to first order this is independent of the wavelength of the light being used.

An issue that frequently arises in discussions of gravitational wave detection is concern that not all effects of general relativity have been taken into account. Thus, some people coming to the subject for the first time worry that the 'stretching of space' will stretch out the wavelength of the light in the interferometer and thus negate the chance of measuring it. This question arises particularly if one thinks of an interferometer as measuring the number of wavelengths along an arm; as pointed out above, this is an incorrect description of how an interferometer works. But even when one discusses detection in terms of time-delays, the question arises of whether the gravitational wave will affect the speed of light and thus corrupt or negate the measured time-delay. The answer to this worry is that such questions are coordinate-dependent. We have done our analysis above in a TT coordinate system, where the free particles remain at fixed locations and the coordinate speed of light changes. On the other hand, one could do the same computation, at least for beams shorter than the wavelength of the gravitational wave, in a freely-falling frame centered on one end of the beam. In such a frame, the other end of the beam moves its coordinate position (because coordinates measure proper distance in such a frame) but the speed of light is invariant (again because a freely-falling frame duplicates special relativity accurately). So in such a coordinate system the effect comes from proper distance changes. This point of view is in fact the most convenient one for laboratory experiments, but we have used the TT gauge above to derive the beam formula because it is robust and works for long beams that extend well outside the domain of validity of any local inertial frame. The lesson, however, is that one can have a picture in which the proper distance changes, or one in which the speed of light changes, or some mixture, but in any coordinate system the answer for the proper-time of the returning beam will come out the same, since that is a coordinate-invariant measure.

LISA: a gravitational wave detector in space

Ground-based interferometers are limited to frequencies above a few Hz, mainly because local disturbances in the Newtonian gravitational field are larger than expected gravitational waves at frequencies lower than this. To observe below 1 Hz one must go into space.

LISA consists of 3 independent spacecraft arrayed as an equilateral triangle, with laser beams along each of the arms. The spacecraft can be taken to be point particles following geodesics, so the analysis is similar to other beam detectors. However, LISA's arms are 5×10^6 km, comparable to the wavelength of a gravitational wave of frequency 25 mHz, so for frequencies higher than this one cannot use the small-arm approximation.

Given its 3 arms, LISA can be regarded as having three independent interferometers, made from the arms that join at any two of the three vertices. One of the interferometers responses can be found from the other two at low frequencies where the short-arm approximation holds, but not at high frequencies. The interferometers are independent in terms of their gravitational wave response: they sense two different polarisations of the gravitational wave. But since the two detectors share a common arm, their instrumental noise in the two detectors is not independent. Thus, LISA can determine the polarisation of an incoming wave without needing another detector, but it cannot do a cross-correlation experiment between its two detectors in order to improve its sensitivity to a stochastic background of gravitational waves.

LISA's spacecraft will be placed in a special set of orbits about the Sun, in such a way that as they orbit they remain in a roughly equilateral triangle in a plane tilted at 60^{O} to the ecliptic. The triangle rotates backwards relative to the orbital motion. In this way, LISA's antenna pattern sweeps across the sky, so that a long-lived source has roughly the same chance of being seen in most locations. The modulation of the signal by the motion around the Sun allows LISA to find source positions; LISA essentially synthesizes a gravitational wave telescope with a diameter of 2 AU. This only is effective, however, at frequencies above about 1 mHz, because below this frequency the wavelength of the gravitational wave is larger than the size of the orbit, and the modulation effects become less detectable.

LISA deals with similar limitations as ground-based interferometers, but in very different ways:

1. **Shot noise.** This is photon counting noise, coming from the quantisation

of the light energy on the photodetector. The more energy in the light, the lower the noise relative to the signal. LISA collimates the beam from a laser and directs it to the spacecraft at the other end of the arm using a 30-cm mirror. The diffraction limit of this for light with a wavelenght of 1 micron is of order one microradian. Over a distance of 5×10^6 km, therefore, the center of the beam spreads out to several kilometers. The 30-cm mirror on the other spacecraft intercepts only a tiny fraction of the transmitted light, and if LISA relied on this light being reflected, again with a large diffraction pattern, then the intensity of the returned light would be negligibly small. Instead, LISA will have active optical transponders: the incoming light will be amplified by an on-board laser, which returns a much stronger beam. With these "active mirrors", LISA needs only 1 W lasers in order for its shot noise limit to provide excellent sensitivity. This is a very comfortable requirement. Shot noise is the limiting noise at frequencies above about 1 mHz.

2. **Vibration.** Space is not totally empty. The radiation pressure from the Sun is a constant force that would push the spacecraft out of their positions, and on top of that the Sun is not a constant radiator. Fluctuations in its intensity at the level of one part in 10^3 are normal. Solar storms produce intense fluxes of particles, which can exert forces on the spacecraft. LISA copes with these disturbances by using a freely-flying "proof mass" inside the spacecraft at the end of each arm as the reference point for the interferometry. The spacecraft itself is an active shield, sensing the position of the proof mass and firing very weak (micro-Newton) thrusters to compensate the external forces and prevent the proof mass from being disturbed. Since each spacecraft carries two proof masses, one for each arm terminating at it, the control algorithm for the thrusters is not trivial, but it can be described. This form of disturbance isolation is called "drag-free technology", and will become more and more useful in experiments in space in the future. The accuracy with which the sensing of the proof mass can be done decreases with decreasing frequency, so vibration noise dominates shot noise below about 1 mHz.

3. **Laser frequency noise.** In LISA, there are 6 active lasers, each with independent frequency noise. LISA copes with this by a very clever algorithm for combining the signals from the several beams in linear combinations with various time-shifts that cancel out the frequency noise. The algorithm for doing this is called "time-delay interferometry", or TDI.

> The result is that the system can produce three TDI combinations that essentially represent the signals from the three interferometers associated with each of the vertices of the triangle. The first version of TDI imagined that the arm-lengths in LISA were constant in time, but a more recent version allows them to vary in realistic ways. Understanding TDI is essential for anyone who intends to work on data analysis for LISA.

In addition, LISA has to contend with one further “noise”: **confusion noise**. We will see later that LISA has enough sensitivity to see a huge number of sources. For some of them the nearest ones will stand out individually but the more distant ones will blend together into a random confusion background of gravitational waves. Separating the resolvable sources from the more distant ones is a challenge to LISA’s data analysis that the ground-based detectors do not have to deal with, and we will discuss this later. Over much of LISA’s frequency range, the confusion limit is not far below the shot noise limit, which explains why it is not worth putting more powerful lasers on board. At low frequencies, below 1 mHz, the confusion noise actually dominates other noise sources, in particular sensor noise in the drag-free system.

Interestingly, at low frequencies the fact that the signals from the three interferometers are not independent leads to a noise veto: there is a linear combination of the three TDI output signals that cancels out all gravitational wave signals at low frequencies, and therefore can be used to measure the instrumental noise. This so-called “Sagnac mode” (referred to earlier) can be used therefore to monitor the detector. This does not work quite this way at higher frequencies (above about 10 mHz), because when we lose the short-arm approximation then the three interferometers have independent responses whose relative phase (time-of-arrival) depends on the direction the wave is coming from. It is possible to null out waves from a particular direction by taking a particular phased combination of three TDI signals, but it is not possible to null out all gravitational waves at once at high frequencies.

LISA’s frequency “window” extends from about 0.1 mHz (below which confusion noise and sensor noise make observing difficult) up to about 0.1 Hz. Its best range is between 1 mHz and 10 mHz. Above this, the cancellations that happen because the arms are a good fraction of a wavelength degrades the sensitivity of LISA.

LISA Pathfinder

Because no mission has ever demonstrated drag-free technology at anything like the degree of precision that LISA requires, the space agencies (ESA and NASA) are cooperating on a small predecessor mission to test the technology. Called LISA Pathfinder, it will consist of just one spacecraft containing two proof masses and some laser optics needed to replicate the behaviour of one LISA arm. Because the "arm" will be short (all within one satellite), there will not be any significant gravitational wave sensitivity, but the drag-free system and the sensing system can be made to operate near the LISA requirement. This mission, due to fly in 2009 at the Earth-Sun Lagrange point L1, will provide the reassurance that the delicate measurements needed for LISA can be performed automatically in space.

The development of LISA Pathfinder has allowed the LISA instrumentation groups an opportunity to solve some of the principal design and engineering problems that LISA will encounter. It therefore will make it possible to develop the full LISA mission with more confidence.

Chapter 3 – Data Analysis

Since ground-based detectors are of relatively low sensitivity, sophisticated data analysis strategies are necessary even to recognise gravitational wave events. LISA's confusion-limited sensitivity presents equally demanding (but different) challenges. The art and science of data analysis are key components of all the searches for gravitational waves. In order to understand LISA data analysis one must begin with a foundation in ground-based data analysis.

Matched filtering.

The assumption of (ideal) ground-based signal analysis is that one is fighting against an instrumental or environmental noise that is broadband, spread over spectrum, stationary, and Gaussian or nearly Gaussian in its statistics. Now, if you can look for a signal in one small part of spectrum, you fight against only the noise in that band, so this improves signal-to-noise performance. The classic technique that takes advantage of this is the Fourier transform, where the amplitude sensitivity (signal-to-noise ratio, SNR) is proportional to $T^{1/2}$, where T is the observation time. Generally, if one has an accurate waveform prediction, it is possible to achieve the same kind of gain in amplitude sensitivity, a gain that is proportional to the square-root of the number of cycles in the waveform. This generalisation is called matched filtering.

In the simplest implementation of matched filtering, one just takes a correlation of the noisy data, x_j, with a predicted template signal h_j. The resulting statistic is

$$c = \Sigma_j x_j h_j.$$

If x is uncorrelated with h then the expectation value of this is zero. If x additionally contains a signal proportional to h then the expectation value is non-zero and is proportional to $||H||^2$. If the value of c is large enough, then one should be ready to declare a detection. The "noise" in the correlation c is the variance of the correlation, which is non-zero even though the expectation value vanishes.

Notice that if the signal is a sinusiod of frequency f, $h = A\sin(2\pi ft)$, then the correlation is just the sine-transform of the data at that frequency,

$$c = A\Sigma_{j=0}^{N-1} x_j \sin 2\pi f t_j.$$

Matched filtering is a generalisation of the signal-recognition ability of the Fourier transform.

In practice one does the correlation in the Fourier domain. By Parseval's theorem on Fourier transforms, c can also be found from the Fourier transforms $\tilde{x}$ and $\tilde{h}$:

$$c = \frac{1}{N}\Sigma_{k=0}^{N-1}\tilde{x}_k\tilde{h}_k{}^*,$$

where (for N samples of data in the time domain)

$$\tilde{x}_k = \Sigma_{j=0}^{N-1}x_j e^{-2\pi ijk/N},$$

and similarly for $\tilde{h}_k$. If data are sampled with intervals Δt then the index j represents the sample at time $j\Delta t$, and the index k represents a frequency whose value is $f_k = k/(N\Delta t) = k/T$, where T is the total observation time. Note, therefore, that the frequency resolution in the discrete case is $\Delta f = 1/T$.

There are two reasons for preferring the Fourier domain.

1. The first reason is that one usually does not know when to expect the signal, so that there is an arbitrary time-of-arrival parameter τ in the waveform, $h(t-\tau)$. The (continuous) Fourier transform of $h(t-\tau)$ is $\tilde{h}(f)\exp(2\pi if\tau)$. When this is translated into formulas for discretely-sampled data, the previous equation becomes

   $$c_J = \frac{1}{N}\Sigma_{k=0}^{N-1}\tilde{x}_k\tilde{h}_k{}^*e^{2\pi iJk/N},$$

   where J is the integer indexing the discrete sampling time corresponding to τ, i.e. such that

   $$jk/N = f_k\tau.$$

   The expression for c_J is itself just an inverse Fourier transform back to the time domain (as represented by the time-shift index J), and it can be done with the fast FFT algorithm. *Thus, in the Fourier domain one can perform the time-sliding of a template much more efficiently and rapidly than in the time domain.*

2. The second – and more fundamental – reason for working in the Fourier domain is that the time-domain correlation is only an optimum detection statistic if the noise is white, independent of frequency. This is almost never the case in real experiments. If there are frequencies where the noise is louder, then these should have less weight. Let us assume that the data x_j consists of noise n_j plus an expected signal h_j:

   $$x_j = n_j + h_j, \qquad < n_j >= 0,$$

where $<>$ denotes the expectation value of the random variable. If the noise is *stationary*, then the noise at different *frequencies* will be uncorrelated. (The different samples of the noise in the time domain will generally still be correlated with one another, but the assumption of stationarity implies that their correlation depends only on their separation in time, not on the absolute time when they are sampled. When one goes to the Fourier domain, this independence of an overall time-shift implies that the Fourier components of the noise defined above are uncorrelated with one another.) In terms of continuous Fourier transforms, this means that there is a (real) function $S(f)$ such that

$$< \tilde{n}(f)\tilde{n}^*(f') >= S(f)\delta(f - f').$$

We call $S(f)$ the *power spectral density* (p.s.d.) of the noise. It has dimensions of squared noise per unit frequency. In discretely sampled data, we have

$$< \tilde{n}_k\tilde{n}^*{}_{k'} >= S_k\delta_{kk'},$$

where the (discrete) FFT power spectrum $S_k =< |\tilde{n}_k|^2 >$ is related to the p.s.d. at the associated frequency f_k by

$$S(f_k) = \frac{\Delta t}{N} S_k.$$

Given all this machinery, we are now ready to write down the optimum detection statistic in the frequency domain – the *matched filter*:

$$C = \Sigma_{k=0}^{N-1} \frac{\tilde{x}_k\tilde{h}_k{}^*}{S_k}.$$

All the detector groups plot $S(f)^{1/2}$ vs f to show their sensitivity. They take the square root so it can be compared directly to the amplitude of any expected signal. And they calibrate S to the detector's strain sensitivity (response to a gravitational wave) by making sure that the noise n in the last equation is measured in strain (not in voltage or some other measure more directly connected to what is sampled from the detector output). This calibrated strain spectral density is called $S_h(f)$.

Notice that matched filtering is better for signals that have longer wave-trains. In the time-domain, this is clear: the correlation builds up signal power faster than noise power. In the frequency domain, what happens is that a longer signal has a narrower spectrum and therefore fights against less noise bandwidth. The two complementary pictures lead to the same conclusion, that (roughly speaking) the amplitude signal-to-noise ratio (SNR) of matched filtering improves as the square root of the number of cycles of the waveform, just as happens with the simple Fourier transform. This means that we can dig long-lived signals out of the noise when they are intrinsically weaker than shorter signals. (Roughly speaking again, the total signal power recieved by the detector is about the same for the same SNR, provided one does optimal matched filtering and the signals have the same frequency.) The penalty one pays for being able to see weaker signals is that one has to construct a template that matches their waveforms cycle-by-cycle over the whole duration of the filter. This can be a challenge to our theoretical understanding of the signals. It usually also presents a challenge to our computational resources. The reason is that, when signals depend on parameters (masses of stars, position on the sky – see next section), a longer-lived signal is usually more sensitive to getting the matching value of the parameter exactly right than a shorter signal would be. So the number of templates one has to use to get a good match over all cycles typically goes up very rapidly as the number of wave cycles in the filter gets longer.

Matched filtering is the *optimal* linear detection statistic if the noise is Gaussian. No linear transform of the data can do better. But if the noise is more complicated then one must be more careful. In the ground-based projects, detector groups spend large efforts on understanding the statistics of their noise. For LISA the confusion background of weak gravitational wave signals will be Gaussian at small enough amplitudes (where the central limit theorem holds for the dozens or more sources that are superimposed) but there is a non-Gaussian transition between this level and the signals that are strong enough to be resolved. This is an issue that needs research.

Parameter searches.

The template h is generally only known to be a member of a family with several parameters, such as the masses of the objects. The search must be done individually for each member of this family. This raises problems of

- computer power: can we search over the whole family?
- resolution: are we taking templates closely spaced enough?
- accuracy: how well can we measure the parameters?
- significance: what is the chance of a random correlation?

All searches are compute-intensive in one way or another. The most demanding search problem in the ground-based domain is the search for unknown *gravitational wave pulsars* over the whole sky. Since many nearby pulsars may not be known from radio observations, a search in gravitational waves is desirable. To get sufficient sensitivity it is felt that these searches may last several weeks or months, but during that time the signal is modulated (Doppler-shifted) by the motion of the Earth. The modulation pattern depends on location, and after a 1-year observation there are 10^{13} different resolvable locations that have to be searched independently! The AEI's Merlin computer, a teraflop-class cluster, works full time on this, but even this computer power this will only allow us to survey a small fraction of the sky. Other collaboration clusters will help but not suffice.

To address this problem, the Milwaukee and AEI groups have recently released Einstein@Home, a screen-saver that may provide significantly more power than in-house clusters. It has become the most powerful "computer" in the LIGO Scientific Collaboration (LSC), delivering about 70 Tflops continuously.

Burst searches.

The concept of matched filtering is useful only if you have an expected waveform. For "burst" sources, such as expected from gravitational collapse, our waveform knowledge is poor. Such searches, as conducted by the LSC, look for unusual amounts of signal power in the output of detectors, usually within narrow wavebands, and always in coincidence with other detectors. The coincidence demand reduces the background from instrumental artifacts. With three or more detectors it is possible to triangulate the location of a source from the time-delays between the arrival of the signal at the different detectors. The LSC is developing network analysis methods to ensure that, if events are in coincidence, the responses of the detectors are consistent with one another for a source in the deduced location on the sky and with the inferred polarization.

Data analysis for LISA

LISA's data analysis is more complicated than that for ground-based detectors. As described earlier, LISA has actually three (TDI) output data streams, each of which has a different antenna pattern. These can therefore be combined optimally to increase the sensitivity to particular directions. They can also be combined in the Sagnac mode to cancel gravitational waves and characterize instrumental noise.

At a basic level, LISA sources will be found by matched filtering, as for ground-based signals. The signal-to-noise ratios will typically be larger and so the sources will be easier to identify. Searching for galactic binary systems, provided they are resolvable, will not pose a serious challenge. Modulation by LISA's motion needs to be removed, but this is much less of a problem than at the higher ground-based frequencies. The signals from black hole coalescences will typically be very stong, and can be computed using the same post-Newtonian and numerical techniques as are used for ground-based searches for black-hole binaries. But EMRI and IMRI signals pose a more serious challenge due to the size of the parameter space, i.e. the number of filters that must be searched; in addition to this, we need to do more work to develop perturbation methods to predict waveforms accurately enough. The hierarchical methods developed for ground-based pulsar searches will give a good starting point for handling the complexity of these searches. Estimates suggest that the problem, while complex, can be done.

A potentially more serious complication than signal complexity is the fact that LISA sources overlap and and so the analysis must fight confusion as well. It will not be possible to filter accurately for an EMRI source, for example, using normal matched filtering, if the data stream also contains a strong signal from a massive black hole coalescence. The coalescence signal needs first to be removed in order to make the "noise" against which the EMRI signal is filtered as close to Gaussian as possible. LISA data analysis will therefore consist of an iterative set of steps in which strong sources are identified and removed, followed by weaker ones, until most sources are identified. Then this must be repeated, since once the weaker ones are known the stronger ones can be removed more accurately. Removal of signals accurately requires, of course, the measurement of all their parameters, which is the astrophysical information that scientists want from LISA.

Signal removal is not a trivial problem; it requires some intelligence in the software. It is possible to characterise a given strong signal as a "LISA source" in a number of ways. For example, the waveform from an inspiralling pair of massive black holes could be identified as coming from such a system, but it could also be Fourier analysed and represented as the superposition of thousands of single-frequency binaries radiating coherently together to produce an apparent coalescence signal. Now, a human data analyst would have no problem choosing the simpler characterisation of the signal as a coalescence over its representation as a host of binaries, but we must build this level of sophistication into the automatic computer search software to prevent it from going down the wrong route. While this example is rather trivial, one can imagine that when there are three or four superposed signals it becomes harder, and when the incoming signal is one that we did not anticipate, i.e. is not in LISA's catalogue of expected sources, then we might have great difficulty identifying it.

A final issue is that many of LISA's sources will be radiating for years; not only binaries but also certain EMRIs might last a long time. After, say, five years of observing the parameters of the signals (their frequency, polarisation, etc) will be better known than in the first year, and this will allow them to be removed even more accurately from data from, say, the first year which has already been processed. In turn, this will improve the identification and measurement of the radiation from coalescences and other transients that occurred in the first year.

Most of the problems I have described here are open research problems. We do not yet have optimum solutions to any of them, much less a data analysis system that can cope with all of them. Nevertheless, there is no lack of ideas for attacking these problems, nor are the problems unique to LISA: signal detection against confusion is a major research field. It seems reasonable therefore to be confident that the situation will be well in hand in another five years or so.

Chapter 4 – Future prospects

LISA is a cooperation between ESA and NASA, which both agencies regard as a high-priority mission. However, both agencies are beset with funding problems, so at the time of this writing (2006) the timetable for LISA is not secure. LISA Pathfinder is well into industrial development and so is expected to stick to its launch date in 2009. In ESA's current timetable, LISA itself will fly in 2017. This is much later than necessary on technical grounds and is determined solely by the expected availability of funding. NASA has an even more difficult problem, which is that it needs to choose among several missions that it was hoping to develop in parallel: LISA, an X-ray mission called Constellation X, and a mission to explore the dark energy and the accelerating expansion of the universe (JDEM). In 2007 NASA will decide which of these goes first; if LISA is not chosen then its launch may well be delayed even beyond 2017.

This uncertainty is particularly at odds with the interest of the scientific community. Work on LISA's science — its sources and their astrophysics — is very active, and the number of papers being published on LISA-related science far exceeds that which any other proposed space mission has ever stimulated. At this time, there is a concerted effort to develop the techniques of data analysis, addressing the challenges referred to earlier.

The data analysis development is being coordinated by the agencies and by the LISA International Science Team (LIST), which is a group of European and American scientists chosen by the agencies to lead the development of LISA. A centrepiece of the data analysis work is a series of Mock Data Challenges, in which simulated LISA data, containing hidden artificial signals, is released to the community, who are challenged to find the signals. As the series of challenges goes on, they will become more and more realistic, hence more and more challenging. By the end, scientists who respond to this challenge will need to be able to demonstrate that they can resolve the kind of confusion expected for real sources.

The expected science return of LISA is something that we are still learning about, as more and more astrophysicists begin to explore the implications of expected LISA detections. LISA on its own or working with other missions and observing programs (such as Gaia and Constellation-X or its European counterpart XEUS) will address a wide range of scientific and astrophysical issues. What follows is only a partial list.

- Testing strong-field general relativity. LISA will observe black-hole coalescence with very high signal-to-noise ratios, allowing detailed comparison with numerical simulations. It will also observe hundreds of EMRI signals, each of which contains a wealth of information about the detailed geometry of the metric in which the infalling object is moving. It should be possible to determine whether or not this metric is the Kerr metric, to accuracies of a percent or better. This will test general relativity, its black-hole uniqueness theorem, and even cosmic censorship (the conjecture that naked singularities, of the kind associated with black holes spinning with $a/M > 1$).

- Testing the speed of gravitational waves. If gravitational waves do not move at the speed of light then they presumably undergo dispersion, with different frequencies moving at different speeds. Signals from very distant sources, such as black hole binaries at $z = 1$, would be distorted as they rise in frequency from what they would be in standard general relativity. This could provide a very sensitive test of deviations from general relativity.

- Observing black holes directly. With high signal-to-noise ratio, LISA will verify that black holes exist, will measure the masses and spins of two holes in a binary system before they coalesce, and will measure the final mass and spin from the ringdown radiation emitted after coalescence. This will give us an unprecedented picture of black-hole dynamics.

- Take a census of supermassive black holes, intermediate-mass black holes, and stellar-mass black holes. By observing supermassive coalescences, IMRI signals, and EMRI signals, LISA will probe the distributions of these objects in mass and in redshift. We have few, if any, other ways to obtain this information. The information will illuminate many issues, such as the formation rates of IMBs, the evolution of central star clusters around black holes, and the role of supermassive black holes in galaxy formation itself.

- Resolve issues about the growth of supermassive black holes. Black holes in galactic centres certainly grow by accreting gas, as we see in quasars and AGNs. But is different for smaller black holes like the one in our own Galaxy, which does not have an accretion disc? Do such holes grow by merger with other black holes? The answer bears on models for structure formation in the early universe.

- Measure the evolution of the dark energy with redshift. Because LISA can measure the luminosity distance to the supermassive black hole coalescences that it sees, there is a chance to do cosmogony with LISA. If the galaxy in which a coalescence happens can be identified (for example, it is thought that X-ray emission might turn on after a coalescence, as disturbed gas begins to accrete on the quiescent final black hole) then a comparison of the redshift with the distance can sensitively measure the temporal variation of the acceleration of the universe. The formal accuracy of the distance, with errors less than 0.1%, makes this attractive. The eventual accuracy may be limited by gravitational microlensing to a few percent, however, because of the small random magnifications and hence errors in the distance.

- Make a census of close white-dwarf binaries, black-hole binaries, binaries of neutron stars with black holes, and other compact objects in the Galaxy. These poorly understood endpoints of stellar and binary evolution are difficult to observe, but LISA would see them everywhere in the Galaxy. When observing Gaia sources, this information would be complementary and would reveal much about the physics of the late stages of stellar evolution.

- Observe exotic and unexpected systems. LISA has much more sensitivity than ground-based detectors, so it is better placed to do “discovery science”, to uncover things that nobody had thought of as gravitational wave sources or which we cannot predict with any assurance. Cosmic strings are one possibility, brane-world exotic effects are another.

- Observe a cosmological background of gravitational waves. LISA does not have the sensitivity to see the radiation predicted by inflation, but it might see radiation from the electroweak phase transition, or from rather exotic string cosmology scenarios for generating backgrounds. While a long shot, this is arguably the most fundamental and important observation LISA could make.

Reference List

To learn more about the subjects covered here, you can consult a number of references. The articles and books below are standard references. You should also consult the website of Living Reviews in Relativity (http://relativity.livingreviews.org/) for an increasing number of up-to-date reviews of a number of these topics, some of which are listed below.

For LISA the best references are the articles in the proceedings of the LISA Symposium, which is held once every two years. The proceedings always contain up-to-date reviews as well as progress on specific issues that we have treated here. The LISA Science Team has set up the LISA Scientific Community (LISC) website at http://www.lisa-science.org/. It is a good source of general information about LISA and will become a place where the most up to date information on the project and its astrophysics will be available. You can sign up for email notices and use it to get in touch with experts.

- **Gravitational wave detectors.** Conference volumes on detector progress appear more than once per year these days. You can find progress reports on detectors on the web sites of the different groups. The references I give here are more tutorial, aimed at introducing the subject.
  - Saulson, P. R., *Fundamentals of Interferometric Gravitational Wave Detectors*, (World Scientific, Singapore, 1994).
  - Blair, D.G., ed., *The Detection of Gravitational Waves*, (Cambridge University Press, Cambridge England, 1991).
  - Jim Hough and Sheila Rowan, "Gravitational Wave Detection by Interferometry (Ground and Space)", Living Rev. Relativity 3, (2000), 3.: http://www.livingreviews.org/lrr-2000-3
  - Sanjeev V. Dhurandhar and Massimo Tinto, "Time-Delay Interferometry", Living Rev. Relativity 8, (2005), 4.: http://www.livingreviews.org/lrr-2005-4 [This is about TDI, the information streams that contain signal information in LISA.]
- **Sources of gravitational waves and data analysis.** Again, there have been a number of conference publications on this subject. The first two references are reviews of sources. The third surveys the problem of data analysis, which I only touched on in my lectures here. The fourth is devoted to data analysis. The fifth is highly technical.
  - Thorne, K. S., "Gravitational Radiation", in Hawking, S.W., Israel, W., eds., *300 Years of Gravitation*, (Cambridge University Press, Cambridge, 1987), p. 330–458.
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