Coalescing binary black holes in the extreme mass ratio limit

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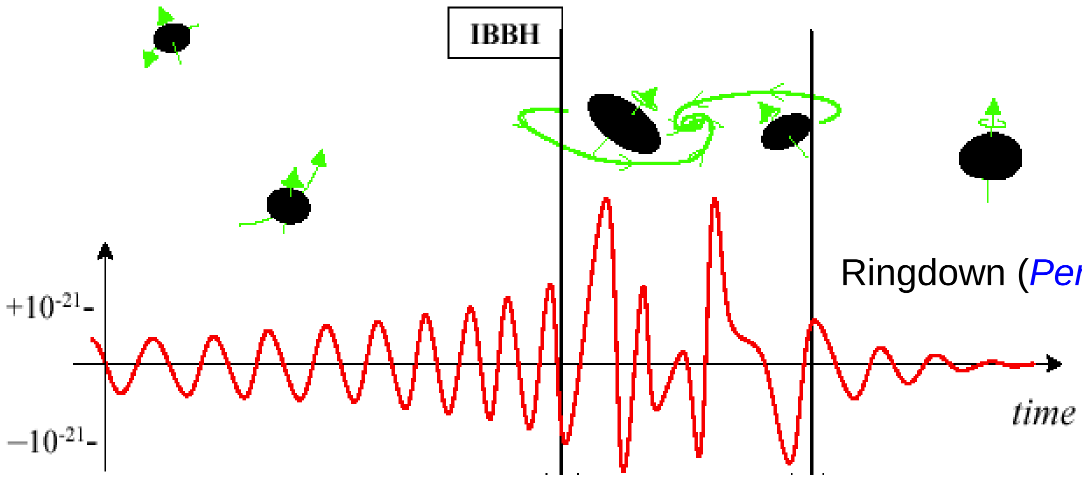
Some years ago they were telling you that…

Among the most promising candidate sources for ground based interferometric GW detectors: coalescing binary systems made of massive (stellar) BHs ($M \sim 30 M_{sun}$).

Most useful part of the waveform is emitted in the last ~5 orbits of the inspiral and during the plunge that takes place after the crossing of the Last Stable Orbit (LSO).

Relativistic speed and highly non-linear gravitational interaction.



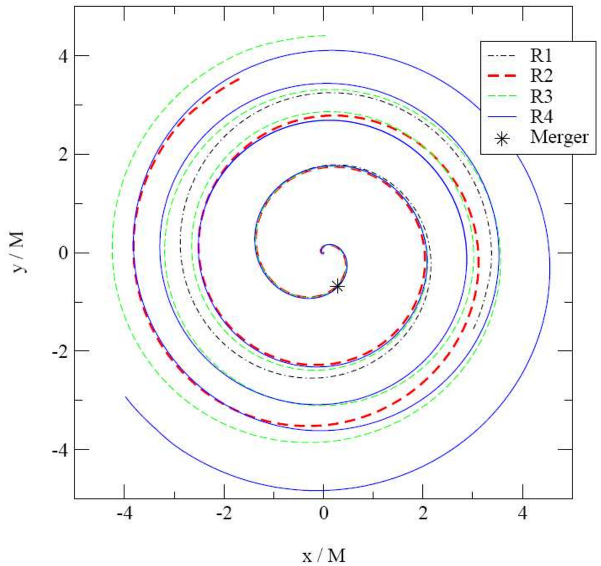
Ringdown (*Perturbation theory*)

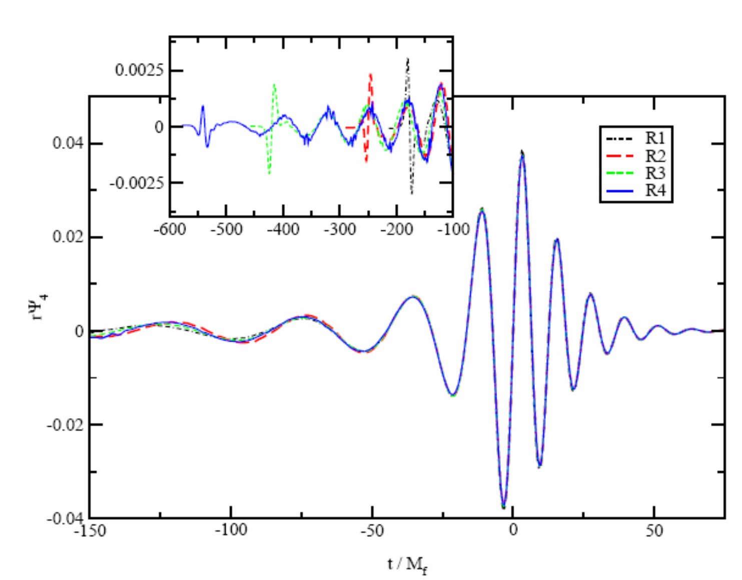
Inspiral (*PN methods*)

Merger: highly nonlinar dynamics.
(*Numerical Relativity*)

…and today you have access to the real thing!

Wonderful success of Numerical Relativity: it is possible to merge black holes and extract Gravitational Waveforms from simulations!





From *Baker et al., 2006*; *4 orbits + merger* of two BHs of equal masses; *l=2, m=2* contribution (99%)

Different groups can collide black holes today (also with spin)

- ✓*F. Pretorius (Alberta, Phys.Rev.Lett.* ***95*** *(2005) 121101)*
- ✓*M. Campanelli, C. Lousto, P. Marronetti, Y. Zlochower (Brownsville-Texas, Phys.Rev.Lett.* ***96*** *(2006) 111101)*
- ✓*F. Herrmann, P. Laguna et al. (PSU, gr-qc/0601026)*
- ✓*P. Diener, D. Pollney, R. Takahashi, et al. (AEI-LSU, Phys. Rev. Lett. 96 (2006) 121101)*
- ✓*J. Gonzalez, U. Sperhake, B. Brugmann, M. Hannam, S. Husa, gr-qc/0610154*

Motivations and overview

However, it is still difficult for NR codes to handle the merger in the *extreme mass ratio limit*

Perturbation theory is still useful today!

Motivations and Overview

Our (*complementary*) problem:

✓BBH merger in the extreme mass ratio, i.e. $m_1=\mu \ll m_2=M$ *(say* $\nu = m_1 m_2/M^2 < 0.1$ *)*

✓Gauge-invariant metric perturbation theory, i.e. solve the linearized Einstein's equation around Schwarzschild background (Zerilli-Moncrief and Regge-Wheeler equations).
Point-particle approximation for the BH of smaller mass.

✓Radiation reaction: 2.5 Post-Newtonian Padé resummed expression of the radiation reaction (damping) force to regularize the badly behaved standard PN expansion
[*Damour, Iyer&Sathyaprakash 1998, Buonanno&Damour 2000*]
Possibility to accurately follow the sequence inspiral-plunge-ringdown.

✓It is an "almost" analytical problem (ODEs and linear PDEs)!

Why should one do this today?

Motivations and Overview

General Motivations

✓In the extreme mass ratio limit (*ν<0.1*), there are *no computations* available to date of the GWs from the plunge (from quasi-circular orbits) coming from the solution of Einstein's equation (in some approximation.)

✓*Gives complementary information to that gained (today) by means of NR simulations.*

Most important motivation: templates and Gravitational Waves detection

✓EOB (Effective-One-Body) framework [*Buonanno-Damour 2000*]

✓*Reproduce (with a certain error) the numerically computed waveform by means of analytical techniques: e.g., quadrupole (improved) formula matched to a superposition of QNMs.*

✓EOB as a *flexible framework* (*i.e., include angular momentum, higher order PN corrections etc.*) to construct reliable banks of templates to be used for detection of GWs from compact binaries.

✓The validation of the EOB approach and philosophy relies on the comparison with (sparse) NR results to tune the parameters of the approximations.
It is an approach complementary to NR simulations, but cannot substitute them.

Effective One Body approach to GR two-bodies dynamics

✓The two body dynamics (at every PN order) is mapped into a representative system composed by an *effective metric* + *a representative point particle.*

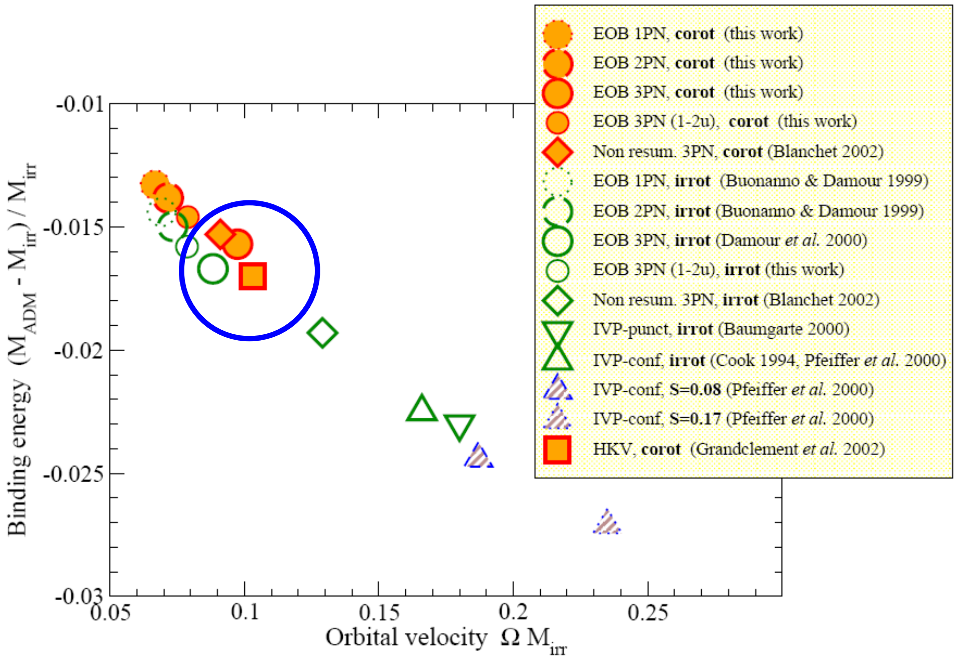
✓The *effective metric* is a deformation of Schwarzschild (or Kerr) at certain PN order.

✓Conservative dynamics + radiation reaction added.

✓ Padé approximants for radiation-reaction force + EOB resummation of the conservative dynamics.

A. Buonanno and T. D, PRD ***59****, 084006 (1999).*
A. Buonanno and T. D, PRD ***62****, 064015 (2000).*
T. D, PRD ***64****, 124013 (2001).*
T. D, E. Gourgoulhon and P. Grandclément, PRD ***66****, 024007 (2006).*
A. Buonanno, T. D and Y. Chen, arXiv: gr-qc/0508067 (2005).
T. D and A. Gopakumar, PRD ***73****, 124006 (2006).*

Test against NR: circular orbits of corotating BHs
Within the Helical Killing Vector (HKV) approach.
Comparison between the binding energy at LSO.



Why Padé resummation (briefly…)?

Poisson (1995) and Damour-Iyer-Sathyaprakash (1998)…

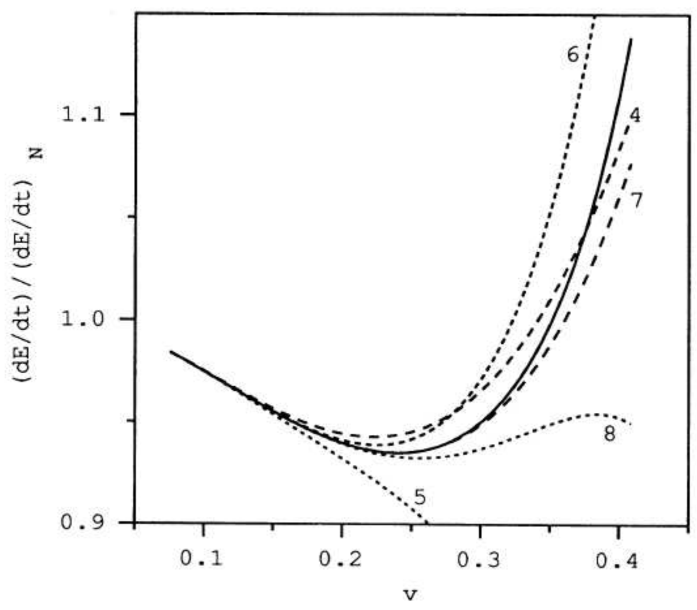


FIG. 1. Various representations of $(dE/dt)/(dE/dt)_N$ as a function of orbital velocity $v = (M/r)^{1/2} = (\pi M f)^{1/3}$. The solid curve represents the exact result $P(v)$, as calculated numerically. The various broken curves represent the post-Newtonian approximations $P_n(v)$, for $n = \{4, 5, 6, 7, 8\}$. The smallest value of v corresponds to an orbital radius r of $175M$; the largest value of v corresponds to $r = 6M$, the innermost stable circular orbit.

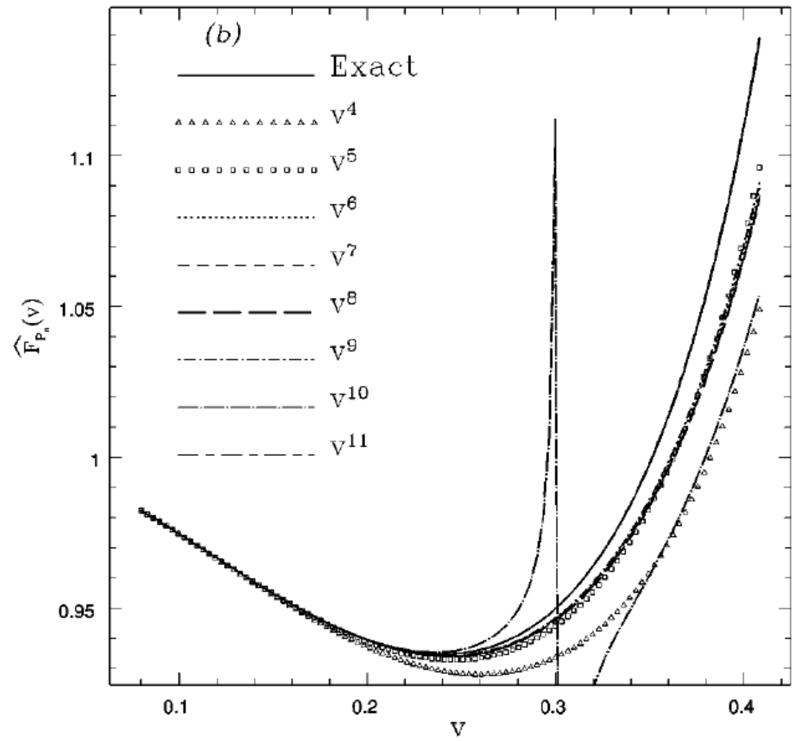
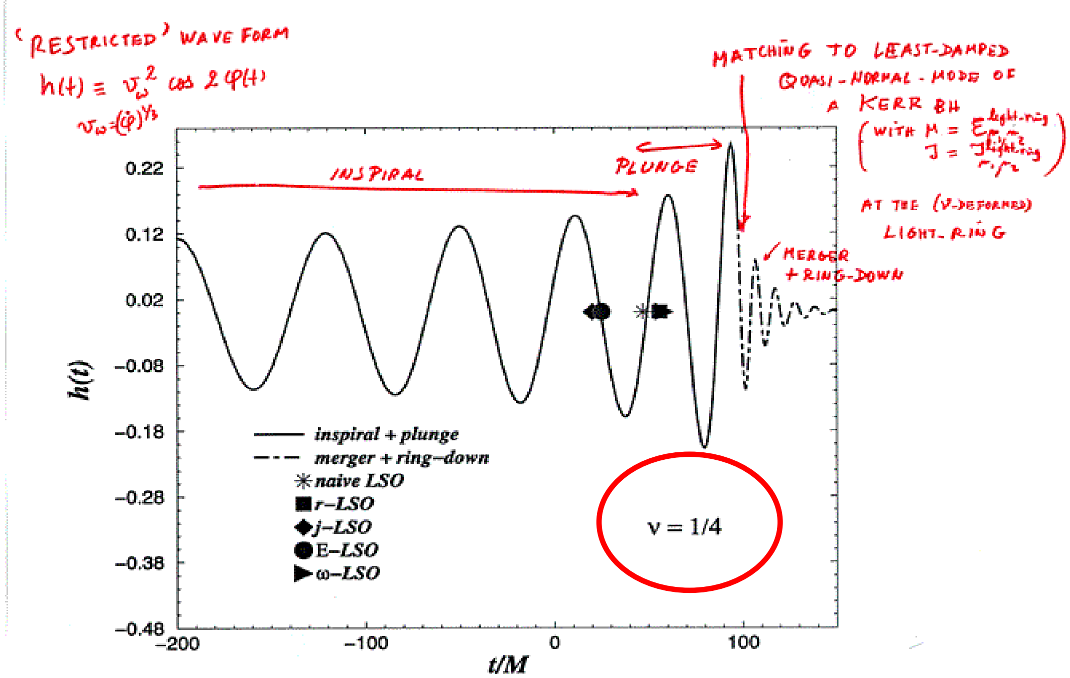


FIG. 3. Newton-normalized gravitational wave luminosity in the test particle limit: (a) T-approximants and (b) P-approximants.

The EOB prediction (2000)

MAIN RESULT: *the plunge is "always" quasi circular (even below the LSO)*



$$\nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

Radial K-energy > azimuthal K-energy during the plunge

Quasi-circular motion

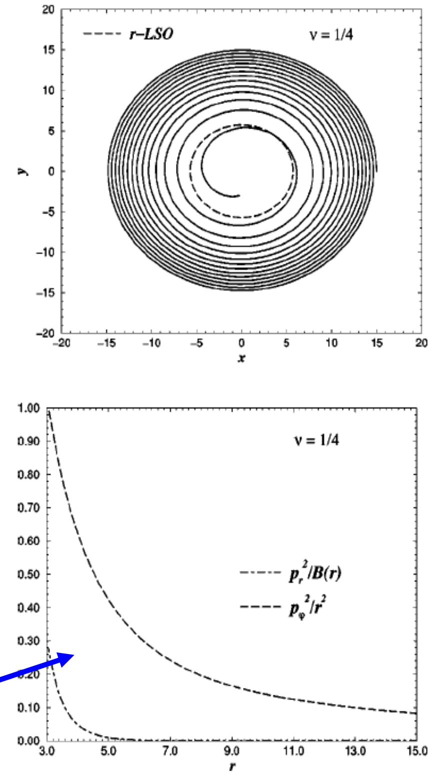


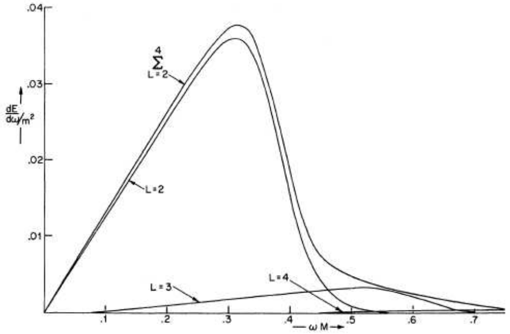
FIG. 1. In the top panel we show the inspiraling circular (relative) orbit for $\nu = 1/4$. The location of the r-LSO, defined by the conservative part of the dynamics, is also indicated. In the bottom panel we compare the two kinetic contributions that enter the Hamiltonian: the "radial" and the "azimuthal" one. The figure shows that the assumption we made of quasi-circularity, i.e. $p_r^2/B(r) \ll p_\varphi^2/r^2$, is well satisfied throughout the transition from the adiabatic phase to the plunge.

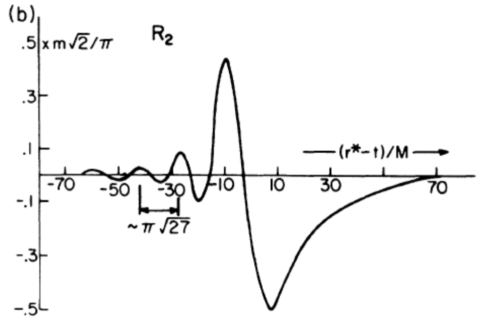
The extreme mass ratio: long (*perturbative*) history

Early GWs calculations: Radial plunge of a particle from infinity into a nonspinning BH

DRPP, *Phys. Rev. Lett.* ***27****, 1466 (1971).*
DRT, *Phys. Rev. D* ***5****, 2932 (1972).*

Precursor-Burst-Ringdown
$ME/\mu^2 = 0.0104$.

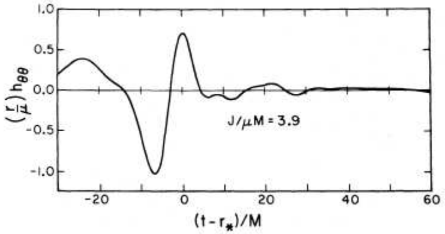


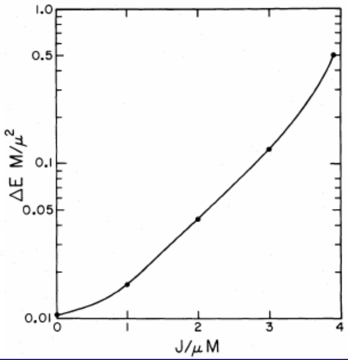


Particle plunging from infinity *with angular momentum.*

S. Detweiler and E. Szedenits, Astrophys. J. ***231****, 211 (1979).*
K.I. Oohara and T Nakamura, Prog. Theor. Phys. ***70****, 757 (1983)*

ME/μ^2 enhanced as much as a factor of 50.

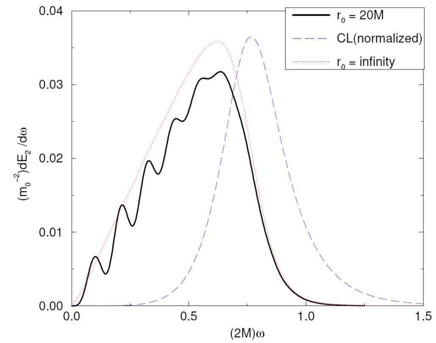




Most recent refinements: radial plunge from *finite distance.*

C.O. Lousto and R.H. Price, Phys. Rev. D ***56****, 6439 (1997).*
K. Martel and E. Poisson, Phys. Rev. D ***66****, 084001 (2002).*

Effect of initial data: interference bumps.



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Metric perturbations of a Schwarzschild spacetime

Remark: Regge-Wheeler and Zerilli-Moncrief equations from the 10 Einstein equations.
Gauge-invariant and ***coordinate-independent*** (*in t,r*) formalism.

[*Regge&Wheeler1957, Zerilli1970, Moncrief1974, Gerlach&Sengupta1978, Gundlach&Martin-Garcia2000, Sarbach&Tiglio2001, Martel&Poisson2005, Nagar&Rezzolla 2005]*

In Schwarzschild coordinates:

$$\partial_t^2 \Psi^{(\mathrm{o})}_{\ell m} - \partial^2_{r_*}\Psi^{(\mathrm{o})}_{\ell m} + V^{(\mathrm{o})}_{\ell}\Psi^{(\mathrm{o})}_{\ell m} = S^{(\mathrm{o})}_{\ell m}$$

odd-parity (Regge-Wheeler)

$$r_* = r + 2M\ln[r/(2M) - 1]$$

$$\partial_t^2 \Psi^{(\mathrm{e})}_{\ell m} - \partial^2_{r_*}\Psi^{(\mathrm{e})}_{\ell m} + V^{(\mathrm{e})}_{\ell}\Psi^{(\mathrm{e})}_{\ell m} = S^{(\mathrm{e})}_{\ell m}$$

even-parity (Zerilli-Moncrief)

$$\lambda = \ell(\ell+1)$$

In the *wave zone*: GW amplitude, emitted power and angular momentum flux

$$h_+ - \mathrm{i}h_\times = \frac{1}{r}\sum_{\ell,m}\sqrt{\frac{(\ell+2)!}{(\ell-2)!}}\left(\Psi^{(\mathrm{e})}_{\ell m} + \mathrm{i}\Psi^{(\mathrm{o})}_{\ell m}\right)\ {}_{-2}Y^{\ell m}(\theta,\phi) + \mathcal{O}\left(\frac{1}{r^2}\right)$$

$$\frac{dE}{dt} = \frac{1}{16\pi}\sum_{\ell,m}\frac{(\ell+2)!}{(\ell-2)!}\left(\left|\frac{d\Psi^{(\mathrm{e})}_{\ell m}}{dt}\right|^2 + \left|\frac{d\Psi^{(\mathrm{o})}_{\ell m}}{dt}\right|^2\right)$$

$$\frac{dJ}{dt} = \frac{1}{32\pi}\sum_{\ell,\,m}\left\{\mathrm{i}m\frac{(\ell+2)!}{(\ell-2)!}\left[\dot{\Psi}^{(\mathrm{e})}_{\ell m}\bar{\Psi}^{(\mathrm{e})}_{\ell m} + \dot{\Psi}^{(\mathrm{o})}_{\ell m}\bar{\Psi}^{(\mathrm{o})}_{\ell m}\right] + c.c.\right\}$$

The particle dynamics

Hamiltonian formalism (conservative part of the dynamics)

$$\hat{H}_{\rm eff} = \sqrt{A\left(1+\frac{p_\varphi^2}{\hat{r}^2}\right)+p_{r_*}^2}$$

$$A(\hat{r}) = B(\hat{r})^{-1} = 1-2/\hat{r}$$
$$p_r = \hat{P}_r = P_R/\mu$$
$$p_\varphi = \hat{P}_\varphi/M = P_\varphi/(\mu M)$$
$$p_r = p_{r_*}A^{-1}$$

$$\dot{\hat{r}} = \frac{A}{\hat{H}_{\rm eff}}p_{r_*}\ ,$$

$$\dot{\varphi} = \frac{A}{\hat{H}_{\rm eff}}\frac{p_\varphi}{\hat{r}^2}\ ,$$

$$\dot{p}_\varphi = \hat{\mathcal{F}}_\varphi\ ,$$

$$\dot{p}_{r_*} = -\frac{\hat{r}-2}{\hat{r}^3\hat{H}_{\rm eff}}\left[p_\varphi^2\left(\frac{3}{\hat{r}^2}-\frac{1}{\hat{r}}\right)+1\right]$$

$$\dot{\hat{r}}_* = \frac{p_{r_*}}{\hat{H}_{\rm eff}}\ .$$

Non conservative part of the dynamics

$$\hat{\mathcal{F}}_\varphi = -\frac{32}{5}\mu\omega^5\hat{r}^4\frac{\hat{f}_{\rm DIS}}{1-\sqrt{3}\omega\hat{r}}$$

Padé resummed estimate at *2.5 PN* of the angular momentum flux [***TD, BI & BS, PRD 57, 885 (1998) , Buonanno-Damour, PRD 62, 064015 (2000)***]
Consistent below LSO [***TD & AG, PRD 73, 124006 (2006)***]

Explicit evolution of R_* of the particle

The source terms

Even-parity

$$S^{(\mathrm{e})}_{\ell m} = -\frac{16\pi\mu Y^*_{\ell m}}{r\hat{H}\lambda[(\lambda-2)r+6M]}\left\{\left(1-\frac{2M}{r}\right)\left(\hat{P}^2_\varphi+r^2\right)\partial_{r_*}\delta(r_*-R_*(t))\right.$$
$$+\left\{-2\mathrm{i}m\left(1-\frac{2M}{r}\right)\hat{P}_{R_*}\hat{P}_\varphi+\left(1-\frac{2M}{r}\right)\left[3M\left(1+\frac{4\hat{H}^2 r}{(\lambda-2)r+6M}\right)\right.\right.$$
$$-\frac{r\lambda}{2}+\frac{\hat{P}^2_\varphi}{r^2(\lambda-2)}\left[r(\lambda-2)(m^2-\lambda-1)+2M(3m^2-\lambda-5)\right]$$
$$\left.\left.\left.+\left(\hat{P}^2_\varphi+r^2\right)\frac{2M}{r^2}\right]\right\}\delta(r_*-R_*(t))\right\}$$

Odd-parity

$$S^{(\mathrm{o})}_{\ell m} = \frac{16\pi\mu\partial_\theta Y^*_{\ell m}}{r\lambda(\lambda-2)}\left\{\left[\left(\frac{\hat{P}_{R_*}\hat{P}_\varphi}{\hat{H}}\right)_{,t}-2\hat{P}_\varphi\frac{r-2M}{r^2}-\mathrm{i}m\frac{r-2M}{r^3}\frac{\hat{P}_{R_*}\hat{P}^2_\varphi}{\hat{H}^2}\right]\delta(r_*-R_*(t))\right.$$
$$\left.+\left(1-\frac{P^2_{R_*}}{\hat{H}}\right)\hat{P}_\varphi\partial_{r_*}\delta(r_*-R_*(t))\right\}, \qquad (22)$$

$$\lambda = \ell(\ell+1)$$

Numerics and tests with geodesic motion

Numerics

✓Couple of wave-equations: standard numerical techniques (*Lax-Wendroff*)

✓Smoothing the delta-function ($\sigma \ll M$). Extensive testing ($\sigma \approx \Delta r_*$ *is ok*)
In practice, the *finite-size* effects are irrelevant (we shall see tests of this in next slides)

$$\delta(r_* - R_*(t)) \equiv \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(r_* - R_*(t))^2}{2\sigma^2}}$$

Tests: Geodesic motion

✓ Circular and radial orbits: comparison with literature [*waveforms and energy*]
[*KM, PRD **69**, 044025 (2004)*, *KM & EP, PRD **66**, 084001 (2002)*, *COL & RHP, PRD **55**, 2124 (1997)*]

✓*Circular orbits:* good agreement for energy and angular momentum fluxes.

Numerics and tests with geodesic motion

Circular orbits (comparison with Martel 2004)

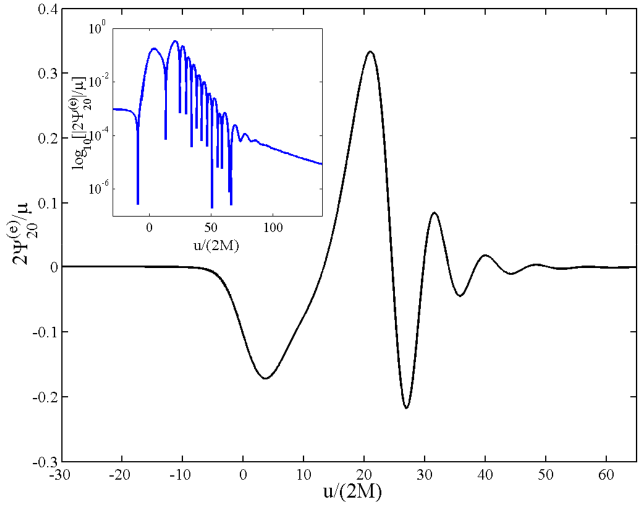
Table 1. Energy and angular momentum fluxes extracted at $r_{\rm obs} = 1000M$ for a particle orbiting the black hole on a circular orbit of radius $r = 7.9456$. Comparison with the results of Martel [29].

($\Delta r_* = 0.02M$)

ℓ	m	$(\dot{E}/\mu^2)_{\rm here}$	$(\dot{E}/\mu^2)_{\rm Martel}$	rel. diff.	$(\dot{J}/\mu^2)_{\rm here}$	$(\dot{J}/\mu^2)_{\rm Martel}$	rel. diff.
2	1	8.1998×10^{-7}	8.1623×10^{-7}	0.4%	1.8365×10^{-5}	1.8270×10^{-5}	0.5%
	2	1.7177×10^{-4}	1.7051×10^{-4}	0.7%	3.8471×10^{-3}	3.8164×10^{-3}	0.5%
3	1	2.1880×10^{-9}	2.1741×10^{-9}	0.6%	4.9022×10^{-4}	4.8684×10^{-8}	0.7%
	2	2.5439×10^{-7}	2.5164×10^{-7}	1.1%	5.6977×10^{-6}	5.6262×10^{-6}	1.2%
	3	2.5827×10^{-5}	2.5432×10^{-5}	1.5%	5.7846×10^{-4}	5.6878×10^{-4}	1.7%
4	1	8.4830×10^{-13}	8.3507×10^{-13}	1.6%	1.8999×10^{-11}	1.8692×10^{-11}	1.6%
	2	2.5405×10^{-9}	2.4986×10^{-9}	1.7%	5.6901×10^{-8}	5.5926×10^{-8}	1.7%
	3	5.8786×10^{-8}	5.7464×10^{-8}	2.3%	1.3166×10^{-6}	1.2933×10^{-6}	1.8%
	4	4.8394×10^{-6}	4.7080×10^{-6}	2.7%	1.0838×10^{-4}	1.0518×10^{-4}	3.0%

Radial plunge (Comparison with Lousto-Price 1997)

- ✓Conformally flat initial data
- ✓Radial plunge along z-axis

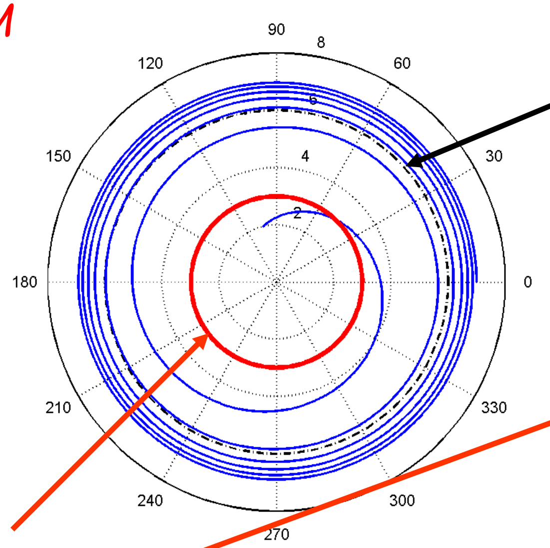


The orbit: transition from inspiral to plunge

Setting up initial data for particle dynamics

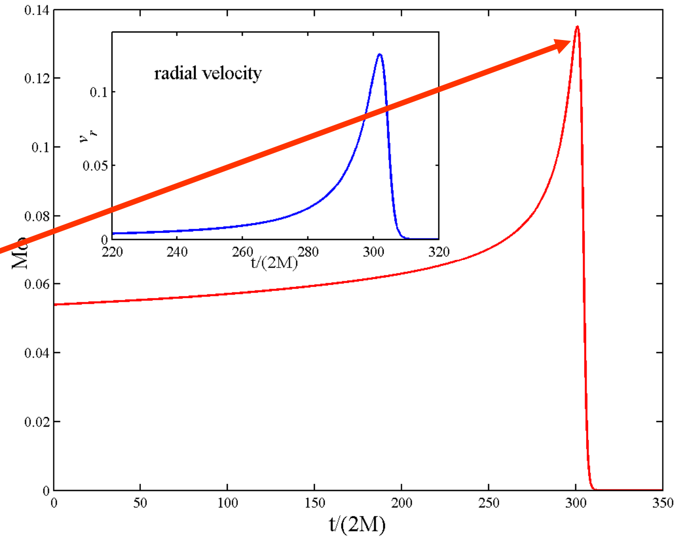
Solve the EoM in the adiabatic approximation to first order beyond the adiabatic approximation, i.e. $p_r \neq 0$

$\mu = 0.01M$
$r_0 = 7M$



Last Stable Orbit(LSO) r = 6M

($\approx$1.5 orbits more before the merger)

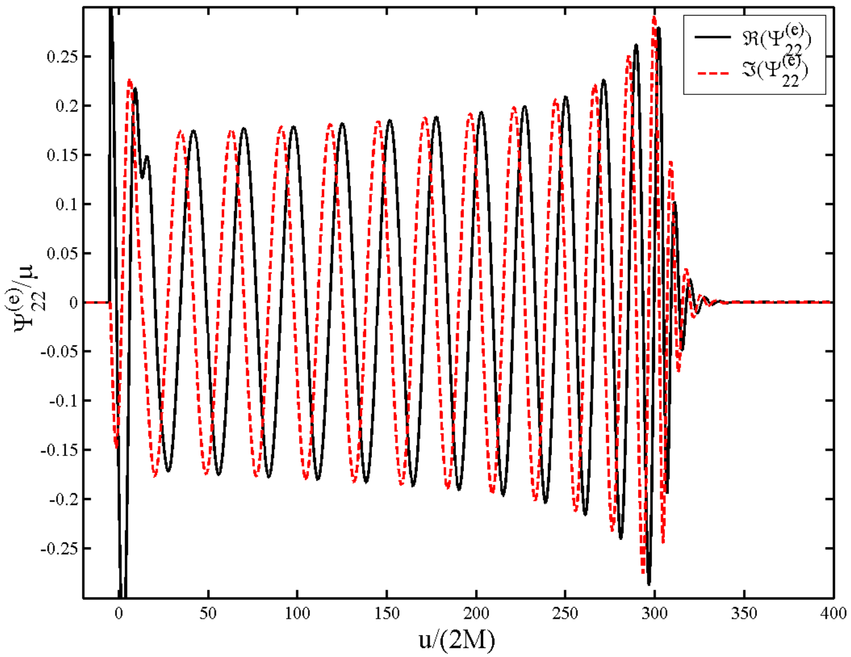


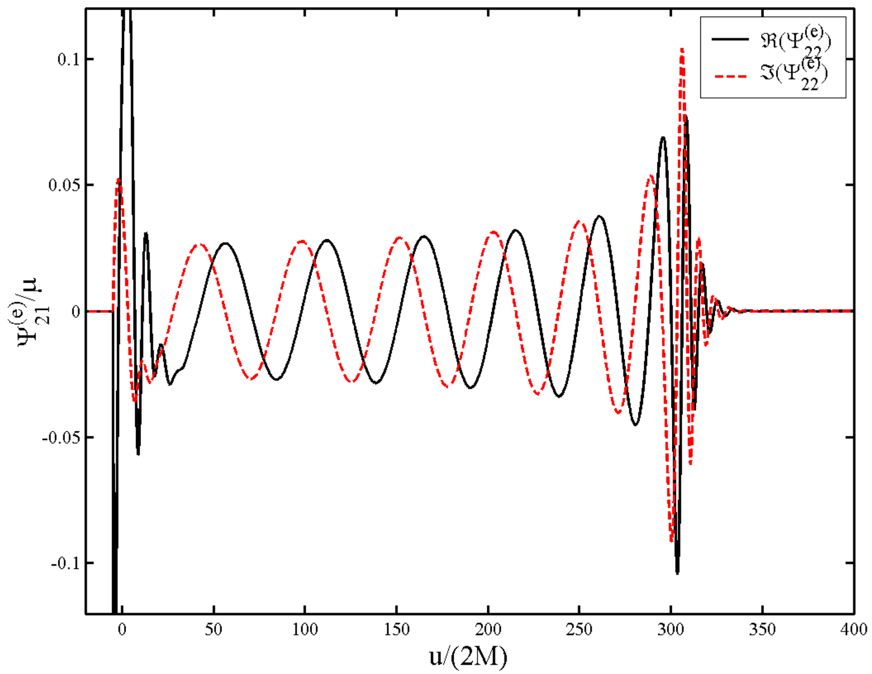
Light Ring r = 3M

Setting up initial data for gravitational perturbation

Initially, no GW perturbation. Initial burst of unphysical radiation radiated away and causally disconnected from the rest of the dynamics (the system has the time to adjust itself to the correct configuration).

Gravitational Waveforms: l=2



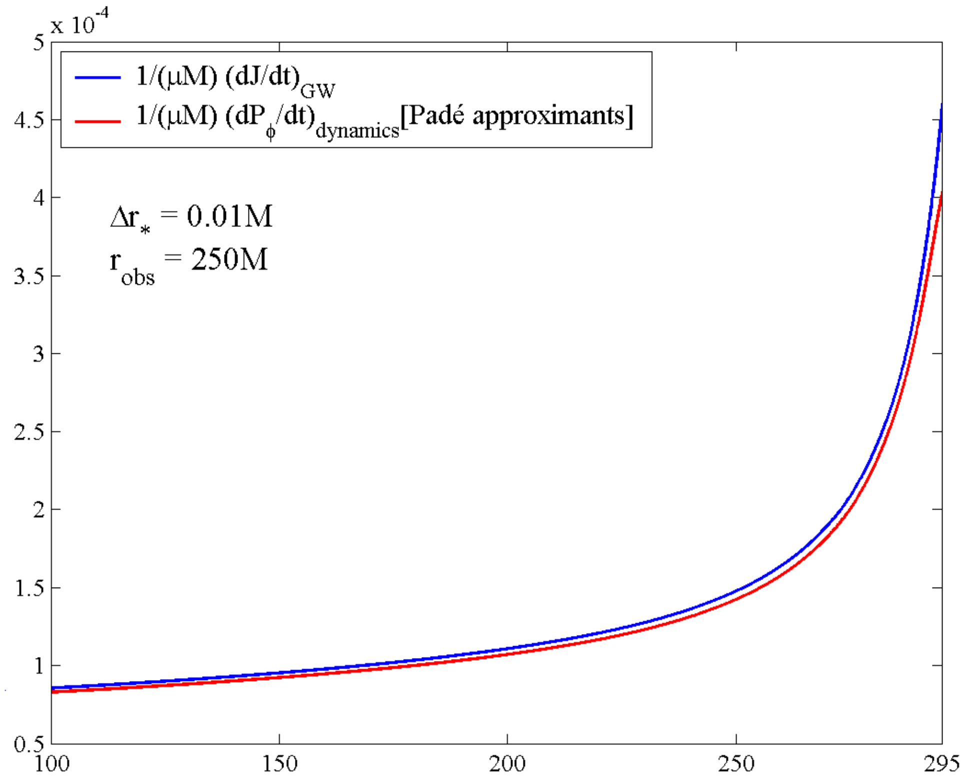


Smooth transition:
inspiral-plunge in the waveforms

Crossing the LSO: *u/2M≈ 240*

Crossing the Light-Ring: *u/2M ≈301*

Consistency check: angular momentum flux



Consistency between GWs radiated angular momentum and orbital decay

Difference less 5% until (roughly) the light ring

Energy and angular momentum released in GWs

Radiation during the plunge (high multipoles)

Table I: Energy and angular momentum emitted at infinity (observer at $r_{\rm obs} = 250M$ by a particle with $\mu = 0.01M$ during the plunge phase only; the integrals are done from $r \simeq 5.9865M$, corresponding to retarded time $u/(2M) = 240$.

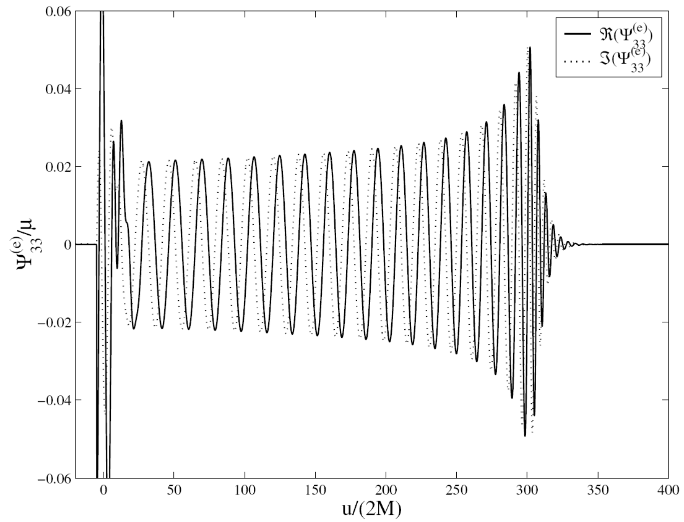
ℓ	m	$(M/\mu^2)E^\infty$	J^∞/μ^2
2	0	9.8×10^{-4}	0
	1	2.06×10^{-2}	0.084
	2	3.3×10^{-1}	2.994
3	0	3.4×10^{-5}	0
	1	5.6×10^{-4}	1.2×10^{-3}
	2	8.1×10^{-3}	3.9×10^{-2}
	3	1.05×10^{-1}	8.5×10^{-1}
4	0	1.7×10^{-6}	0
	1	2.4×10^{-5}	3.6×10^{-5}
	2	3.3×10^{-4}	1.1×10^{-3}
	3	3.5×10^{-3}	1.8×10^{-2}
	4	4.2×10^{-2}	3.2×10^{-1}

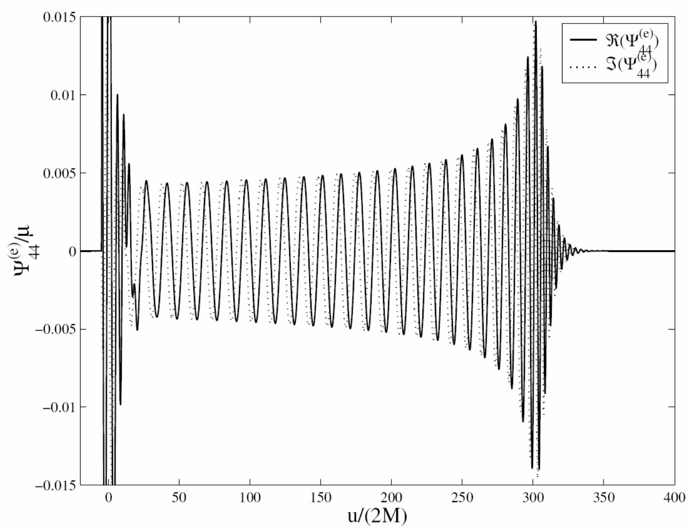
Total Emission

$ME/\mu^2 \approx 0.5$

$J/(\mu M) \approx 0.04$

[at ≈6M, $J/(\mu M) \approx 3.45$]



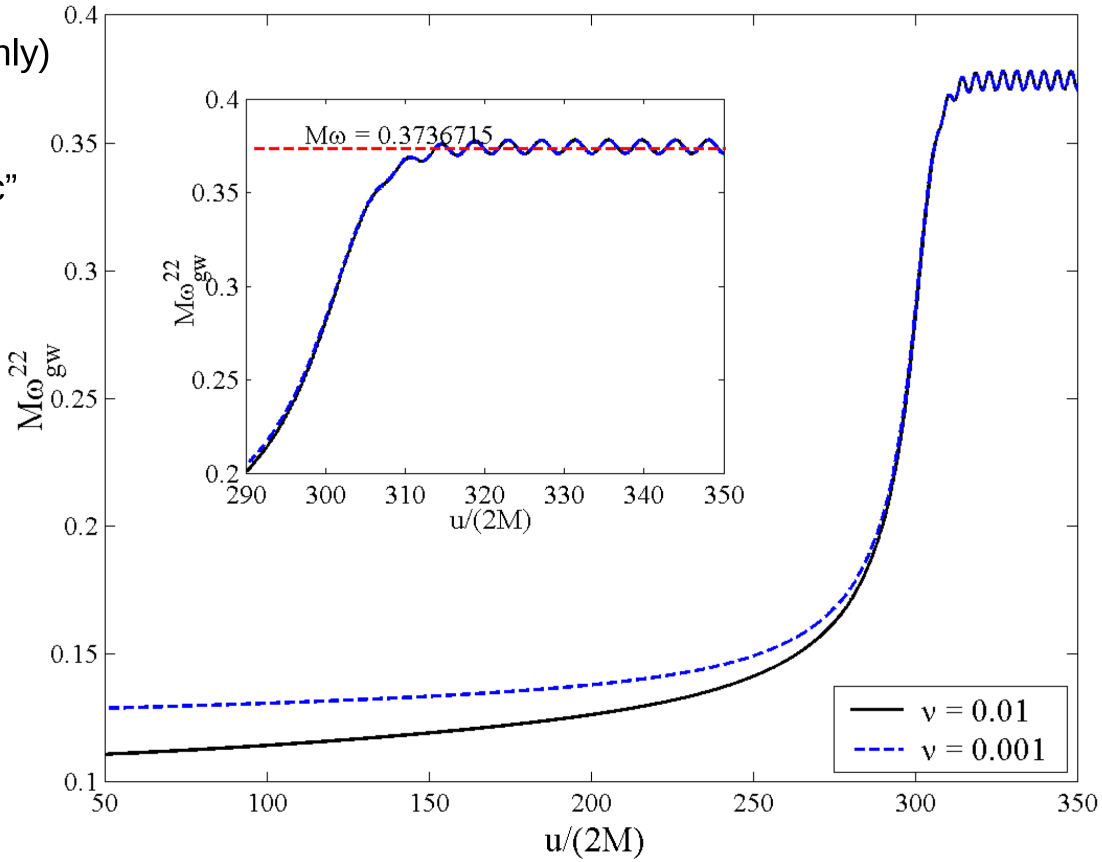


Universality (*dependence on* ν) of the numbers?

- ✓Universal behaviour only after (roughly) u/2M = 280.
- ✓Smaller ν to reach a "quasi-geodesic" plunge starting from the LSO.

Instantaneous GW frequency:

$$\omega_{\rm gw}^{\ell m} = -\Im\left(\frac{\dot{\Psi}_{\ell m}^{(e/o)}}{\Psi_{\ell m}^{(e/o)}}\right)$$



Finite-size (the Gaussian) effects on the source?

The δ-*function* is approximated by a finite-(tiny)size Gaussian. Is this allowed?

There are two (analytically equivalent) ways of writing the sources:

$$S^{(e/o)}_{\ell m} = G^{(e/o)}_{\ell m}(r,t)\delta(r_* - R_*(t)) + F^{(e/o)}_{\ell m}(r,t)\partial_{r_*}\delta(r_* - R_*(t))$$

standard

and (using integration by parts)

$$S^{(e/o)}_{\ell m} = \tilde{G}^{(e/o)}_{\ell m}(R_*(t))\delta(r_* - R_*(t)) + F^{(e/o)}_{\ell m}(R_*(t))\partial_{r_*}\delta(r_* - R_*(t))$$

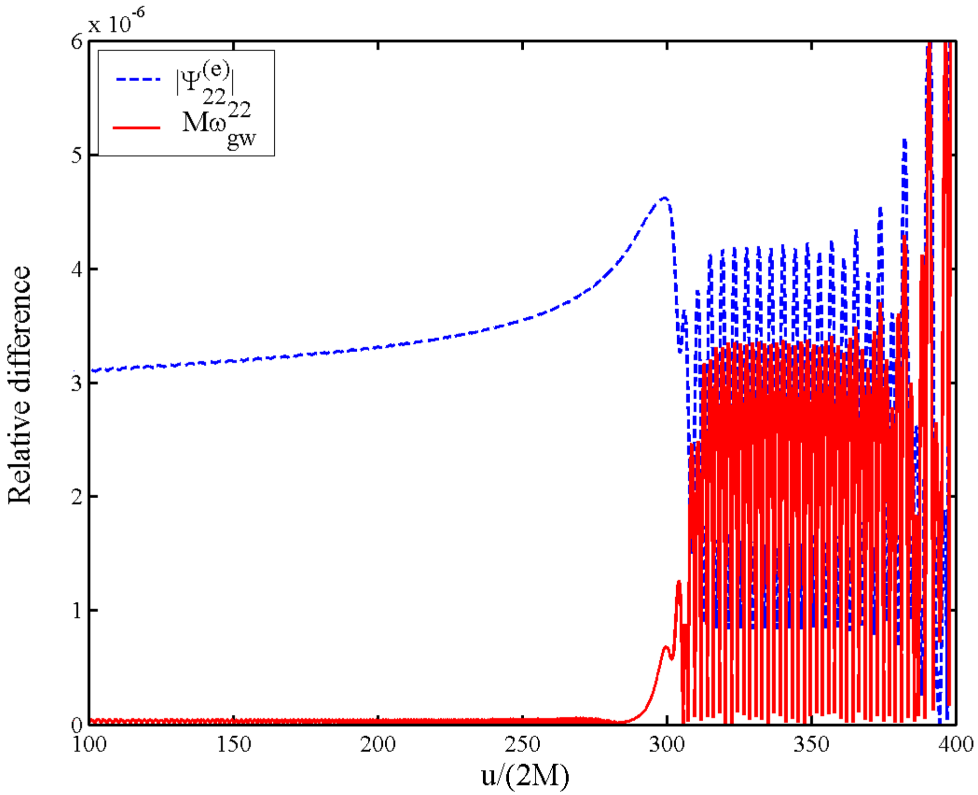
where

$$\tilde{G}^{(e/o)}_{\ell m}(R_*) = G^{(e/o)}_{\ell m}(R_*) - \left.\frac{dF^{(e/o)}_{\ell m}}{dr_*}\right|_{r_*=R_*}$$

One may be worried that, when going on a discrete grid, these two "numerically unequivalent" surces can give relevant differences

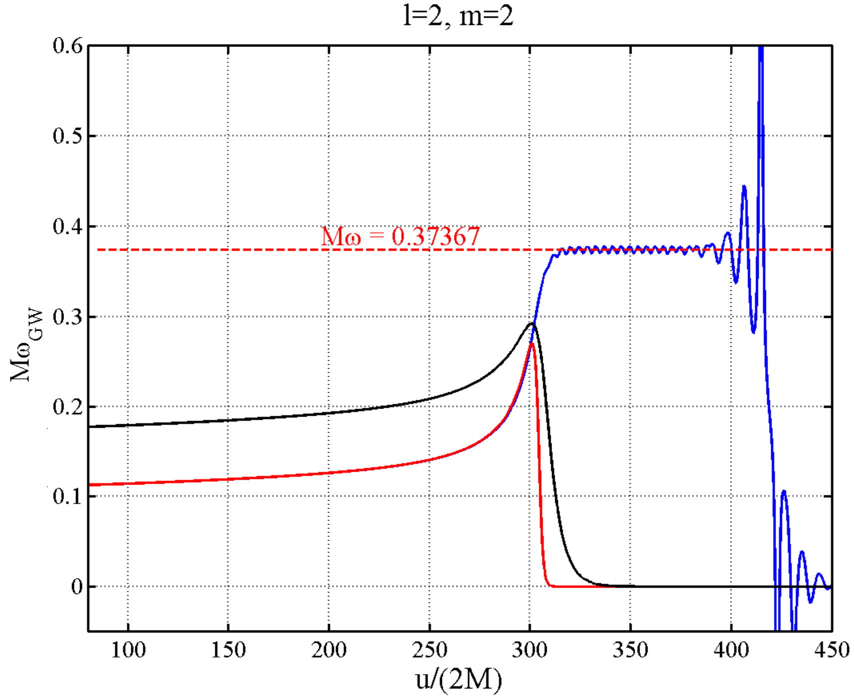
Finite-size (the Gaussian) effects on the source?

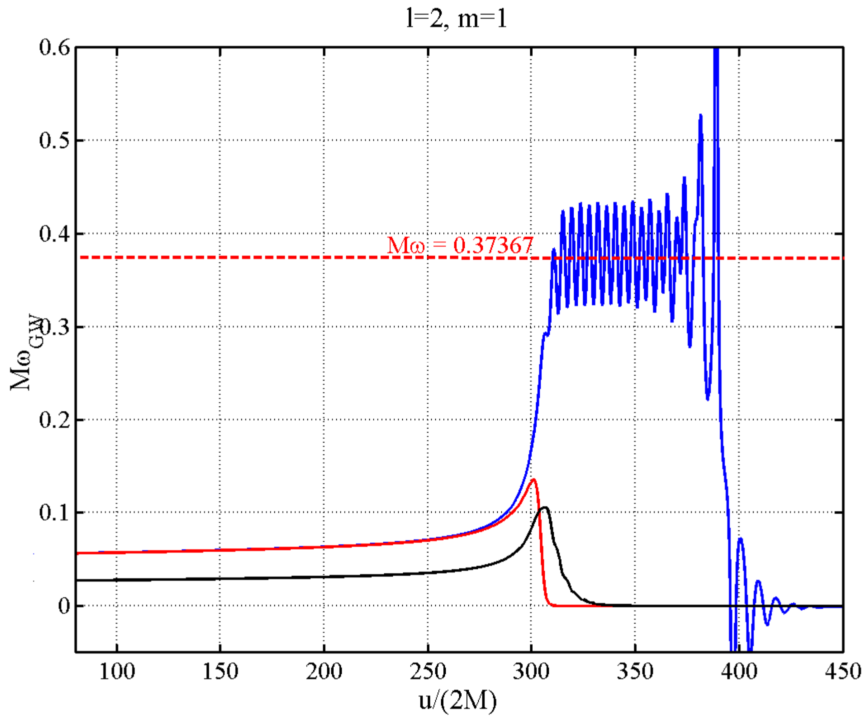
In practice, they are equivalent!



In the simulation we use $\sigma \approx \Delta r^* = 0.01M$. Convergence as soon as $\sigma \ll M$

GW modulus and instantaneous frequency: l=2





- Red line: *m x (orbital frequency).*
- Blue line: *instantaneous GW frequency.*
- Black line: *modulus of the master functions.*

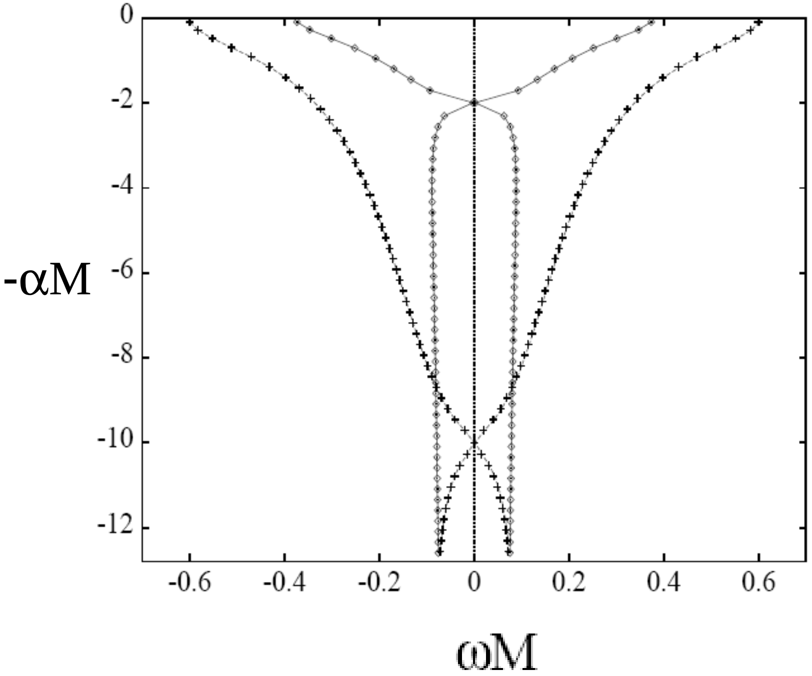
$$\omega_{GW}^{(e/o)} = -\Im\left(\frac{\dot{\Psi}^{(e/o)}}{\Psi^{(e/o)}}\right)$$

QUESTION: can we (approximately) reproduce this behaviour by means of analytical formulae?

ANSWER (for l=2, m=2 for now): YES! (in a few slides)…

Analysis of QNMs signature: oscillations in ω_{gw}

Why oscillations in the GW frequency?

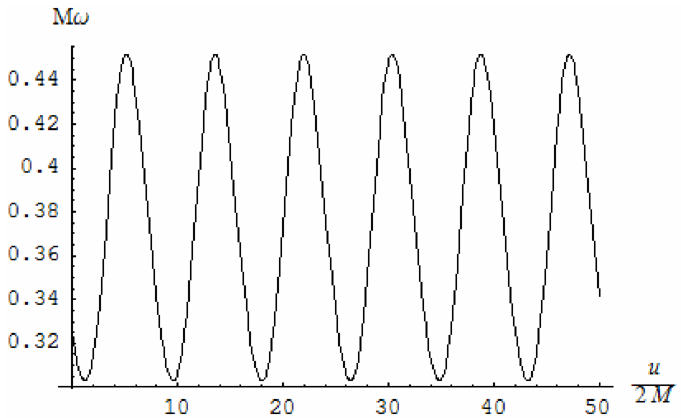


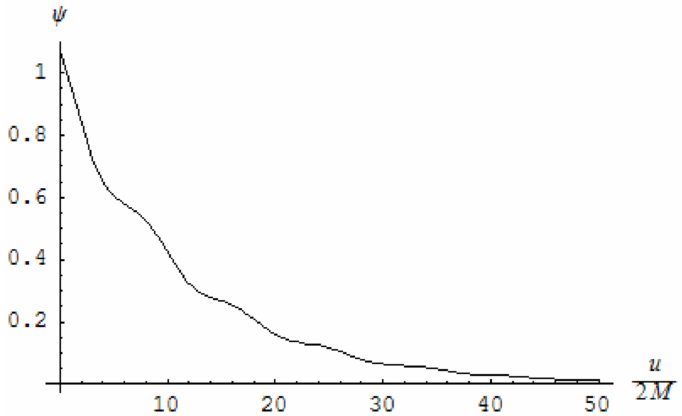
$$\Psi = e^{-\sigma_0^+ t} + \epsilon e^{-\sigma_0^- t}$$

$$\sigma_n^{\pm} = \alpha_n \pm \mathrm{i}\omega_n$$

$$\omega_{GW} = \frac{1 - 2\epsilon \sin(2\omega_0 t) - \epsilon^2}{1 + 2\epsilon \cos(2\omega_0 t) + \epsilon^2}\ \omega_0$$

$$|\Psi| = \sqrt{1 + 2\epsilon \cos(2\omega_0 t) + \epsilon^2}\ e^{-\alpha_0 t}$$





Analytic matching to a superposition of QNMs

EOB Philosophy: match (at ≈ the light ring) QNMs to (some) analytical quadrupole formula

How can one (operatively) do this matching?

What about the accuracy of this procedure (needs comparison with numerical results)?

Newtonian quadrupole

$$\Psi_{22}^{N} \equiv \nu\sqrt{\frac{\pi}{30}}\frac{d^2}{dt^2}\left\{R^2 e^{-2\mathrm{i}\varphi}\right\}$$

$$\Psi_{22}^{NQC} = -4\dot{\varphi}^2\nu\sqrt{\frac{\pi}{30}}R^2 e^{-2\mathrm{i}\varphi}$$

quasi-circular approximation

Improved quadrupole with PN (*resummed*) corrections

$$\Psi_{22}^{F} = \nu\sqrt{\frac{\pi}{30}}\frac{d^2}{dt^2}\left[F_{22} r^2 e^{-2\mathrm{i}\varphi}\right]$$

$$\Psi_{22}^{FQC} = -4\dot{\varphi}^2\nu\sqrt{\frac{\pi}{30}}F_{22} r^2 e^{-2\mathrm{i}\varphi}$$

quasi-circular approximation

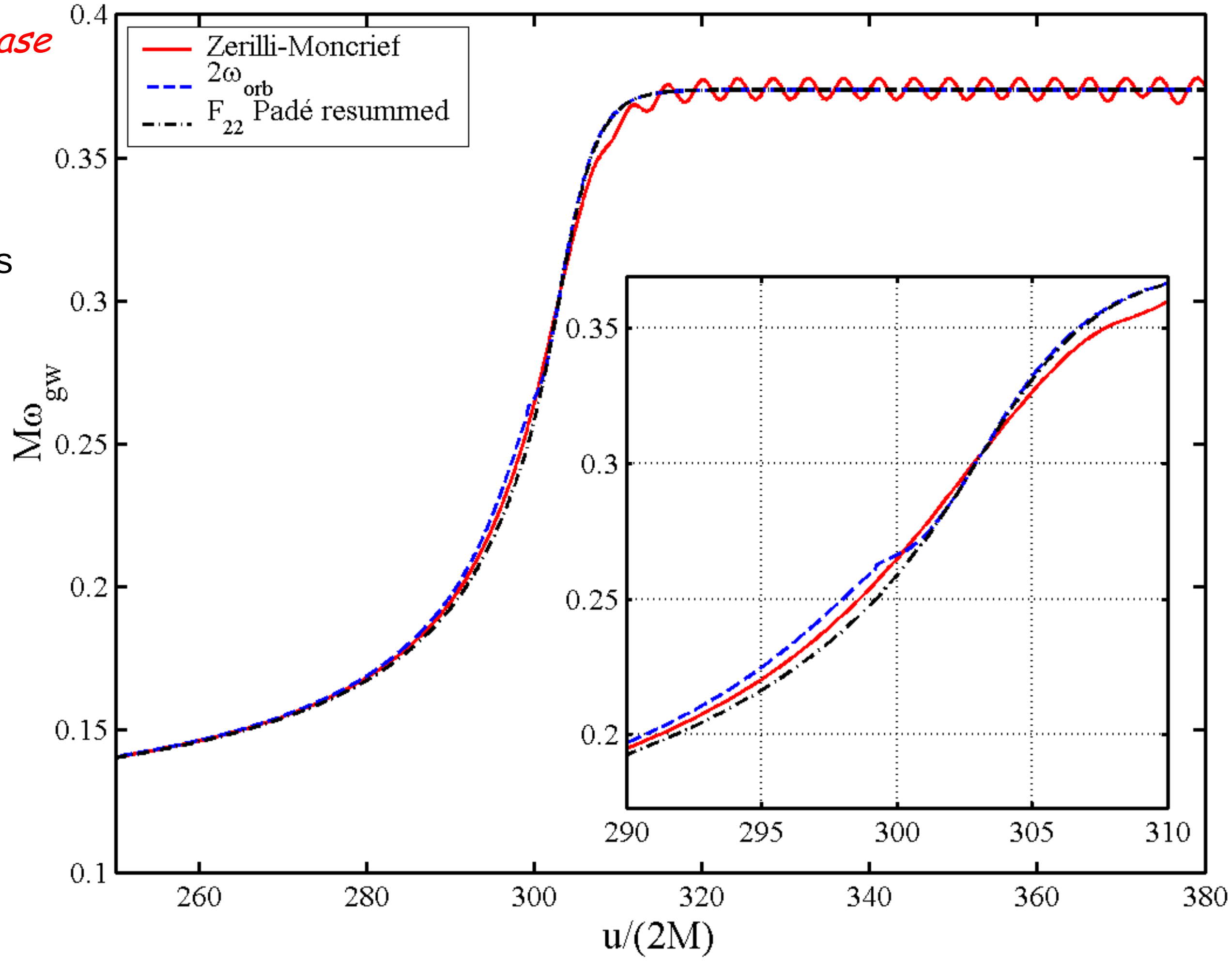
Analytic matching to QNMs ringing

EXAMPLE: the l=2, m=2 case

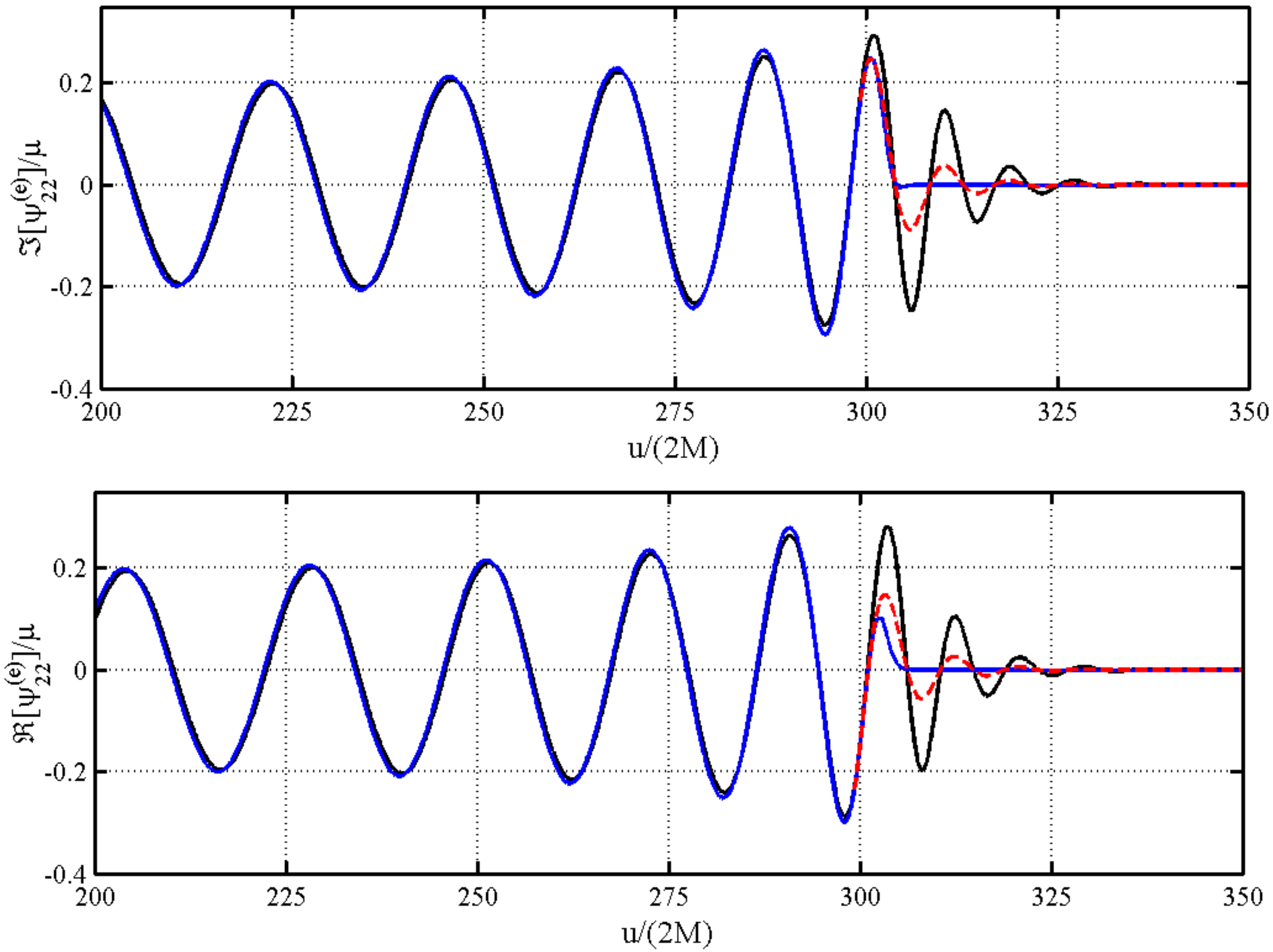
Match to 5 (positive ω) QNMs

$$\Psi_{22}^{QNM} = \sum_{n=0}^{N} C_n e^{-\sigma_n^+ t}$$

$$\sigma_n^{\pm} = \alpha_n \pm \mathrm{i}\omega_n$$

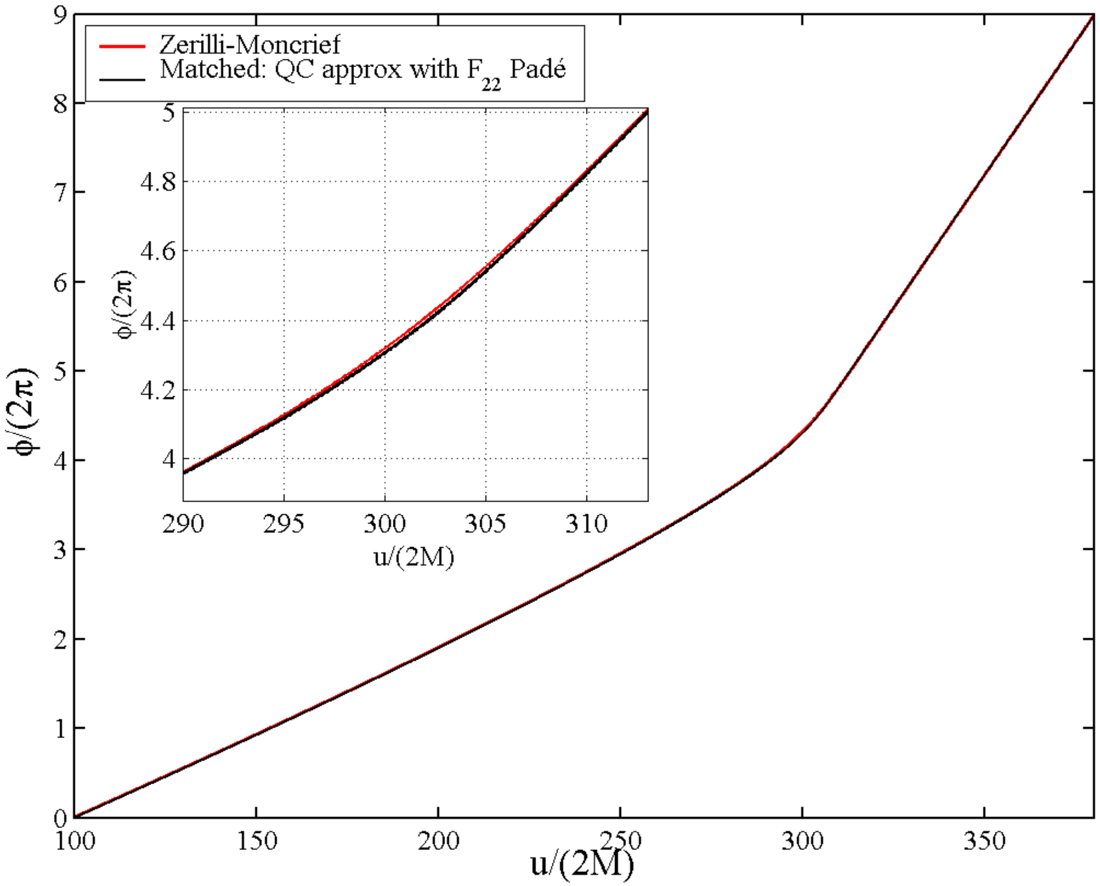


Analytic matching to QNMs ringing

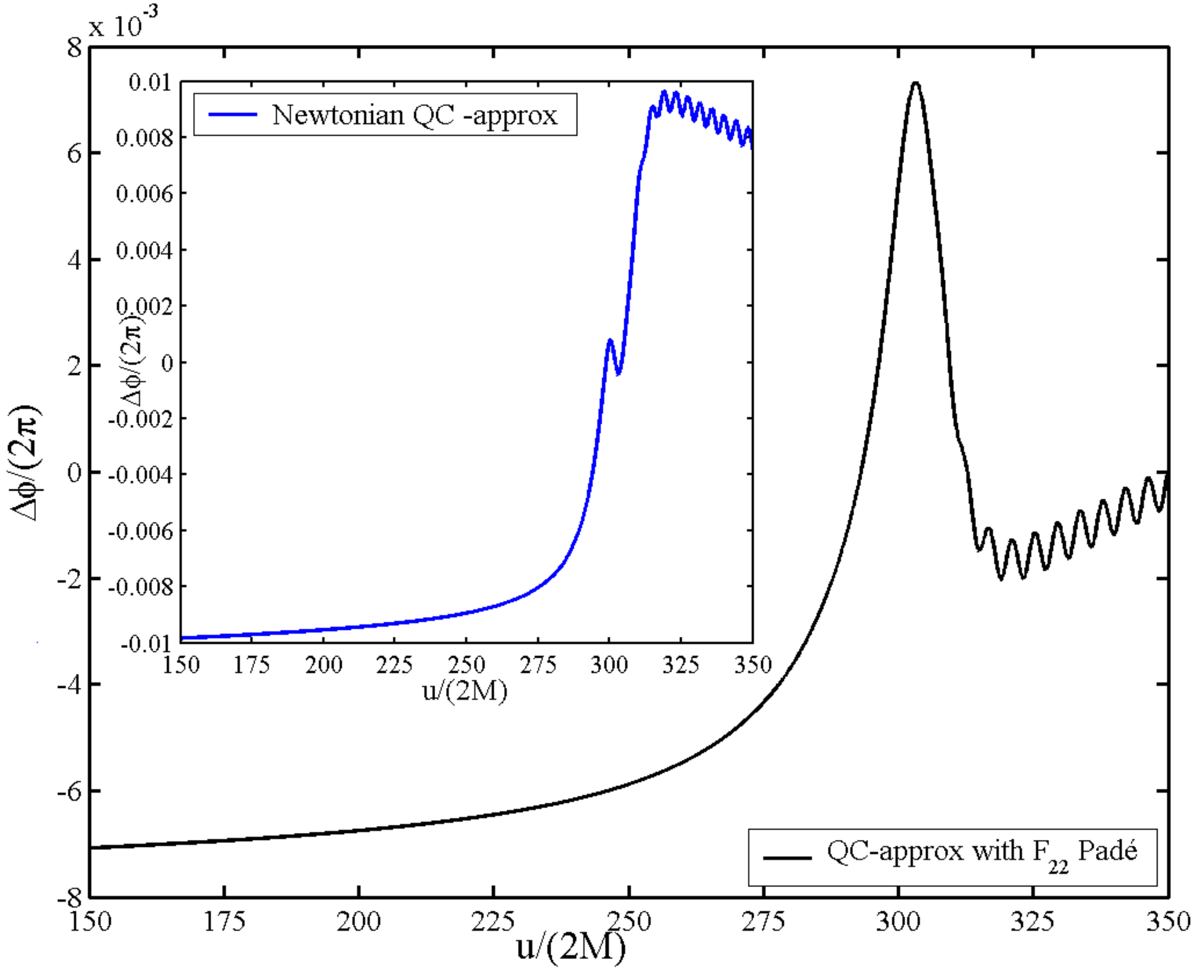


Phase

Most important result: numerical and matched phase almost coincide!



Difference in phase



Less then 0.01 of a cycle of difference between the numerical and the matched phase!

Conclusions

✓ We solved (within certain approximations) the problem of BBH merger in the extreme mass ratio limit.

Waveforms…(and whatever…)

✓Analytical formulae based on EOB philosophy can *well* reproduce the behaviour of the phase during the transition inspiral-plunge (*maximum error of 1% of a cycle*) .

✓First steps towards the possibility of building accurate banks of templates for GWs detection using the EOB framework.

Further work

- ✓ Study the complete (*3PN*) EOB dynamics in the comparable mass case and implement the same "matching" tools developed here in that situation.

✓Compare with data coming out from Numerical Relativity simulations