Detecting Gravitational Wave from Inspiraling Binaries with a Network of Detectors : Coherent vs. Coincident Strategies

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Outline of Talk

Motivation

Methodology

Key Results

- *Theoretical*
- *Simulations*

Conclusions and Future Directions

Motivation : Why Inspiraling Binaries?

- *Waveform is known with high accuracy*
- *Astrophysically important source*

Motivation : Why Network of Detectors?

- *Increases confidence of detection*
- *Provides directional information*
- *Provides Polarization information*

Degenerate in single detector

Motivation : Comparison of performances of Coherent and Coincident strategies

Coherent Strategy

Combine data from different detectors in a phase coherent manner to yield a single optimal statistics for the network in the maximum likelihood sense.

Improved Sensitivity

Coincident Strategy

Prepare list of candidature for each detector separately and then match them.

Fake Reduction

Methodology : Matched filtering

Waveform well-modeled, hence matched filtering is possible.

$$c(\tau) = \int x(t) q(t+\tau) dt$$

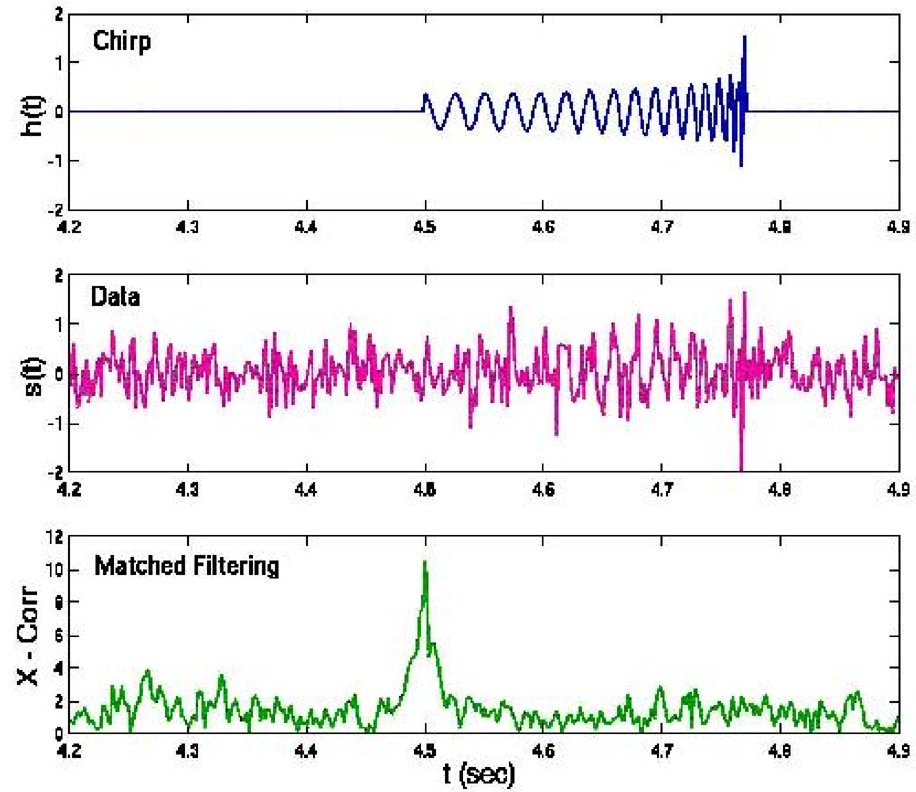
Data

Optimal filter in gaussian noise

$$\tilde{q}(f) = \frac{h(f)}{S_h(f)}$$

Known waveform

Noise psd



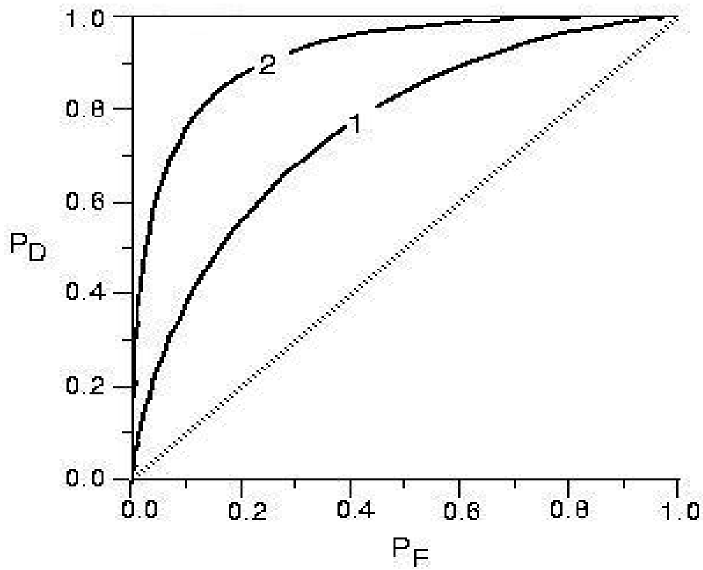
Methodology : Receiver Operating Characteristic (ROC) curves

Noisy data gives rise to two kinds of error

- *False alarm*
- *False dismissal*

ROC curve is a plot of detection efficiency vs. false alarm probability

The upper the curve lies the better the performance of the detector



Methodology : Receiver Operating Characteristic (ROC) curves

Goal : To draw the ROC curve for Coherent and Coincident Strategies and compare the performances

Methodology: assumptions

- *Two detectors have same noise-psd (that of LIGO-I)*
- *The noise is stationary and gaussian*
- *They are at the same place and have same orientation*
- *We consider two cases – uncorrelated noise and correlated noise*

Nearby detectors like two LIGOs at Hanford

Distant detectors - like LIGO and VIRGO

Key Results : Theoretical

Coherent detection (uncorrelated noise)

$$P_{FA} = N_{ind} \exp\left(-\frac{\Lambda^*}{2}\right)$$

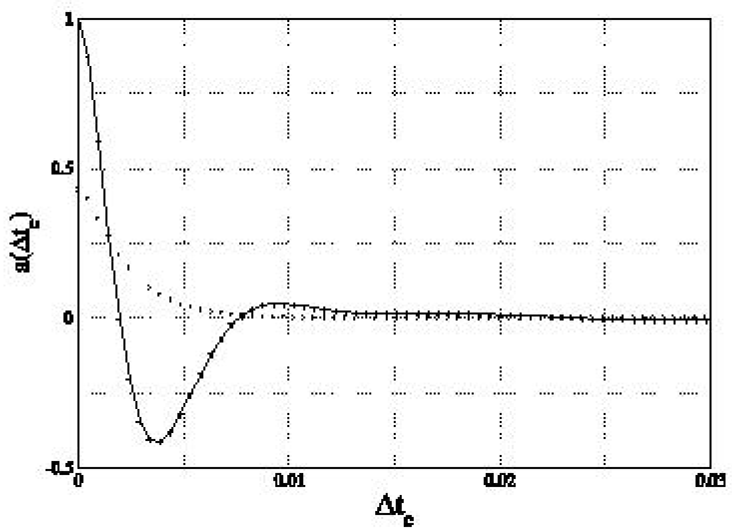
Number of statistically independent templates

↓

Different from total number of templates because of correlation between adjacent templates

↓

Has to be determined from simulations

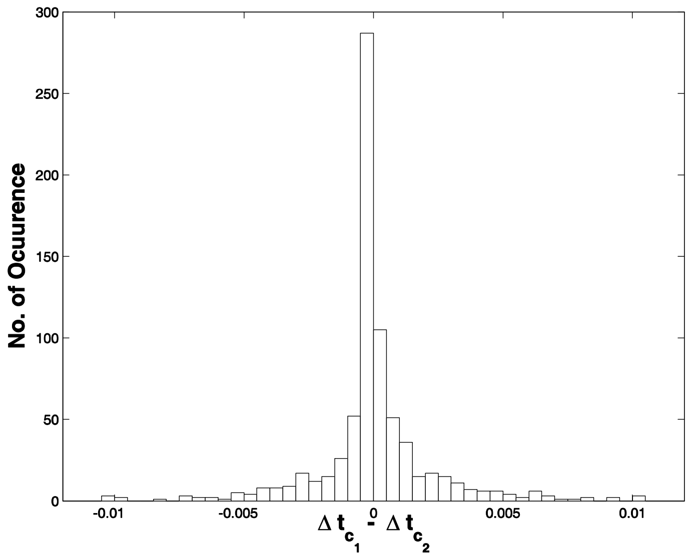


Key Results : Theoretical

Coincident detection (uncorrelated noise)

$$P_{FA} = N_{ind} N_{win} \exp(-\Lambda^*)$$

For the presence of noise, the signal parameters detected by the two-detectors will in general be slightly different. We must allow a certain range in the parameter space while matching the two lists



To be determined from simulation

Key Results : Theoretical

Coherent detection (uncorrelated noise)

Amplitude of incident wave

$$P_{DE} = 1 - \int_0^{\Lambda^*} \frac{d\Lambda}{2} \exp\left[-\frac{\Lambda + 2A^2}{2}\right] I_0\left(A\sqrt{2\Lambda}\right)$$

Coincident detection (uncorrelated noise)

$$P_{DE} = \left(1 - \int_0^{\Lambda^*} \frac{d\Lambda}{2} \exp\left[-\frac{\Lambda + A^2}{2}\right] I_0\left(A\sqrt{2\Lambda}\right)\right)^2$$

Correlated detectors

Noise correlation : $\langle n_1(f) n_2^*(f') \rangle = \frac{1}{2} \epsilon(f) S_h(f) \delta(f - f')$

Stationary noise

Effective Noise correlation coefficient: $\epsilon_0 = 4 \int_{f_l}^{f_u} df \, \frac{\epsilon(f) |\tilde{s}(f)|^2}{S_h(f)}$

Upper cut-off Lower cut-off

Search template

Key Results : Theoretical

Coherent detection (correlated noise)

$$P_{FA} = N_{ind} \exp\left(-\frac{\Lambda^*}{2(1+\epsilon_0)}\right)$$

Coincident detection (correlated noise)

$$P_{FA} = N_{ind} N_{win} \int_{\rho^*}^{\infty} d\rho_2 \int_{\rho^*}^{\infty} d\rho_1 \frac{\rho_1 \rho_2}{1-\epsilon_0^2} \exp\left[-\frac{\rho_1^2+\rho_2^2}{2(1-\epsilon_0^2)}\right] I_0\left(\frac{\epsilon_0 \rho_1 \rho_2}{1-\epsilon_0^2}\right)$$

Key Results : Theoretical

Coherent detection (correlated noise)

$$P_{DE} = 1 - \int_0^{\Lambda^*} \frac{d\Lambda}{2(1+\epsilon_0)} \exp\left[-\frac{\Lambda + A^2}{2(1+\epsilon_0)}\right] I_0\left(A\frac{\sqrt{2\Lambda}}{1+\epsilon_0}\right)$$

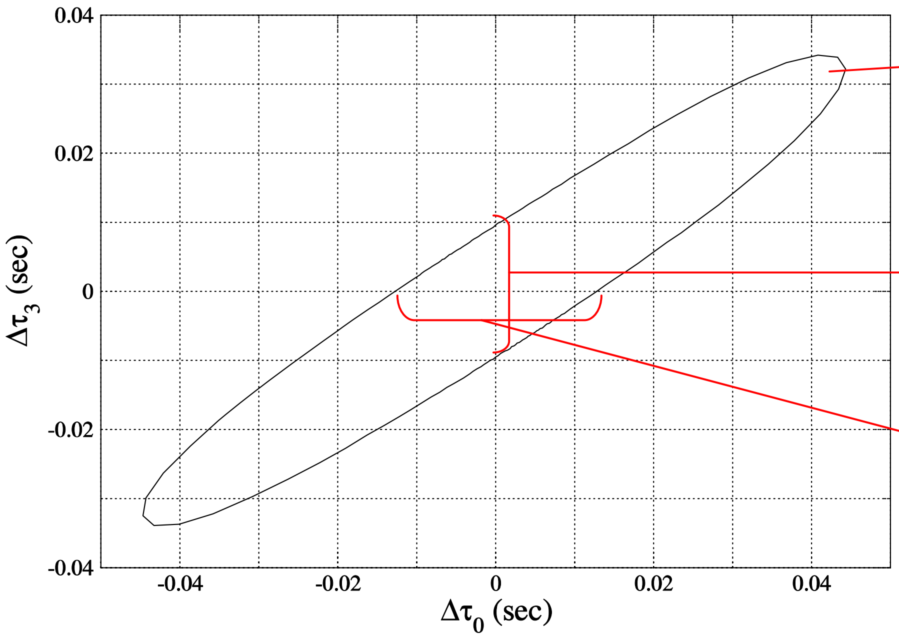
Coincident detection (correlated noise)

$$P_{DE} = \int_{\rho^*}^{\infty} d\rho_1 \int_{\rho^*}^{\infty} d\rho_2 \frac{\sqrt{\rho_1\rho_2}}{2\pi(1-\epsilon_0)A}\left[1 + \frac{\epsilon_0(\rho_1+\rho_2)}{(1-\epsilon_0)A}\right]^{-1/2} \exp\left[-\frac{(\rho_1 - A)^2 + (\rho_2 - A)^2 - 2\epsilon_0(\rho_1 - A)(\rho_2 - A)}{2(1-\epsilon_0^2)}\right]$$

Key Results : Simulation

$$\langle s(f;\lambda) | s(f;\lambda+\Delta\lambda) \rangle = 1 - MM$$

Minimal mismatch



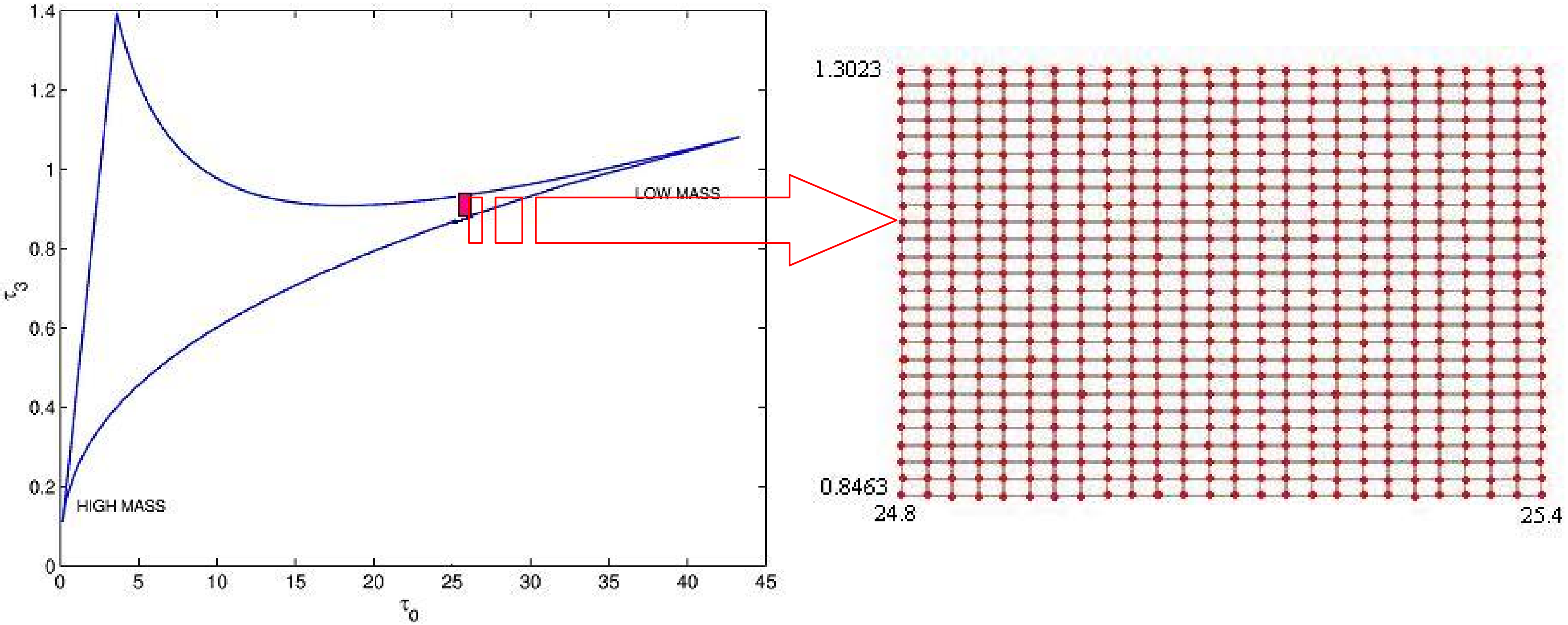
contour of 0.03 minimal mismatch

19ms

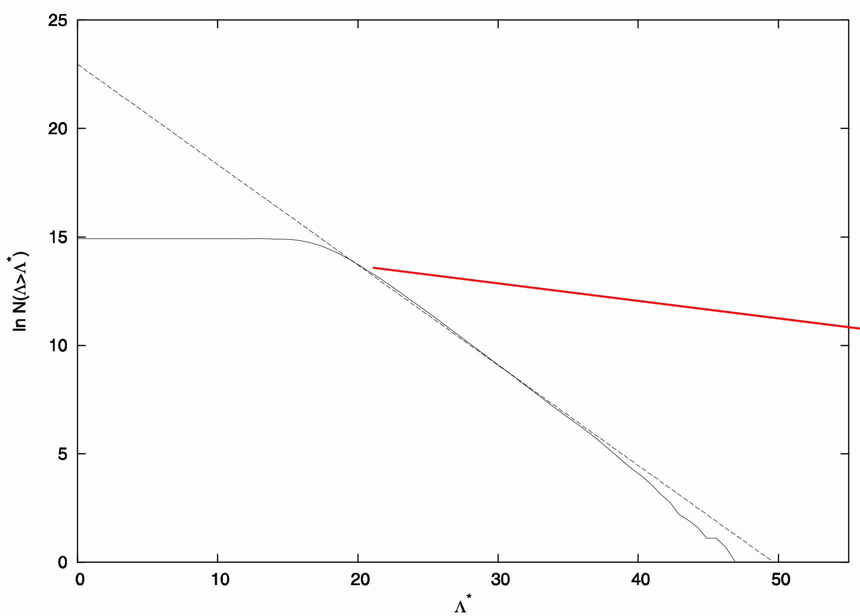
25 ms

Approximately constant over the entire range of parameter space

Key Results : Simulation



Key Results : Simulation



$$N_{FA} = N_{sim} N_{ind} \exp\left(-\frac{\Lambda}{2}\right)$$

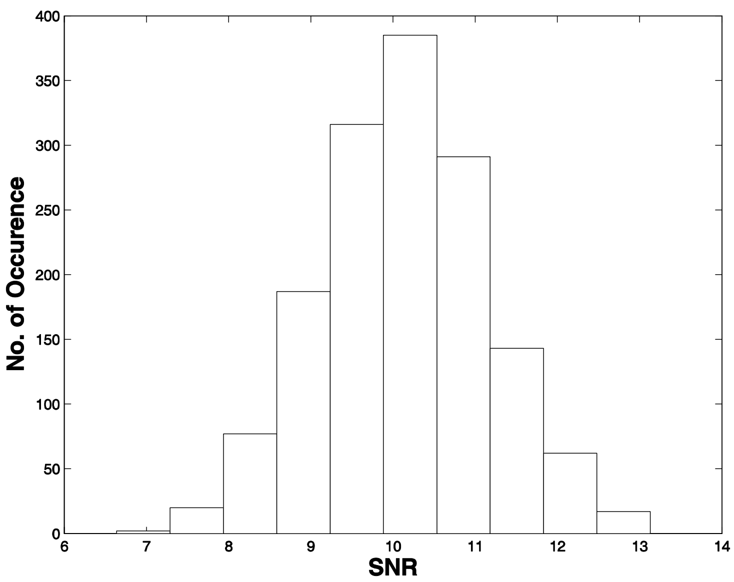
Number of simulations

19% of the total templates are statistically independent

Key Results : Simulation

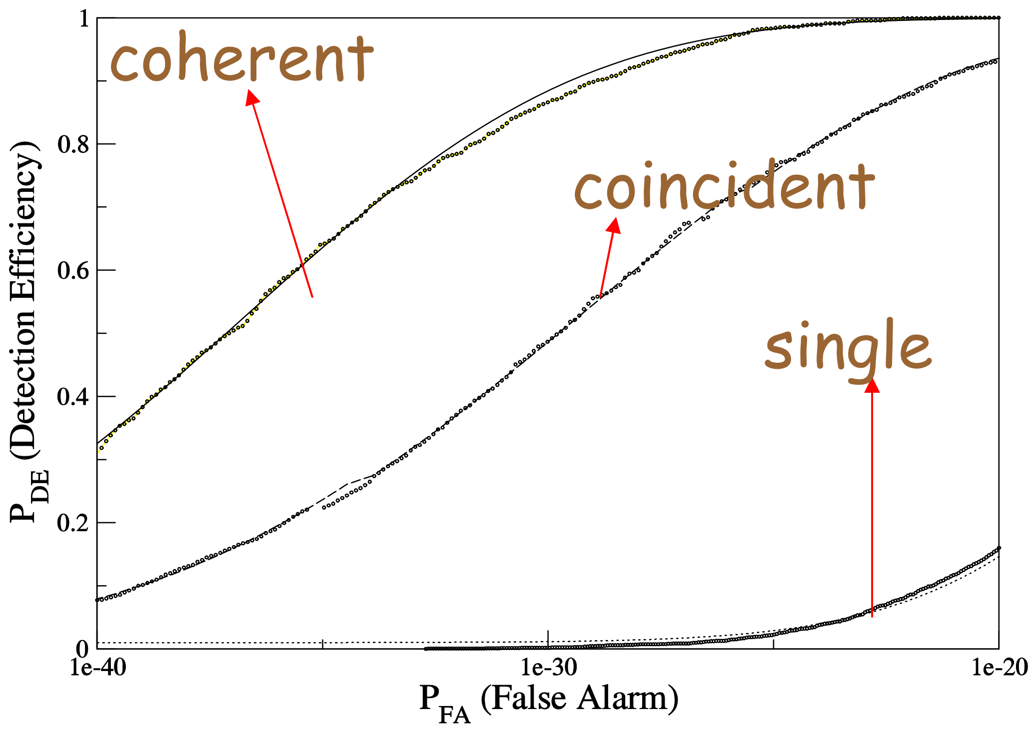
The size of window depends upon the threshold. We have ascertained by simulation a window size for each of the parameters such that 99% of the observed values fall within the window. So we detect signal for 97% of the time.

Key Results : Simulation



SNR	Window in time of coalescence	Window in τ_0	Window in τ_3
14	3.4 ms	75 ms	38 ms
13	3.4 ms	75 ms	57 ms
12	3.9 ms	125 ms	76 ms
11	3.9 ms	175 ms	95 ms
10	4.9 ms	200 ms	95 ms
9	5.9 ms	250 ms	133 ms
8	6.8 ms	300 ms	152 ms
7	8.3 ms	375 ms	190 ms

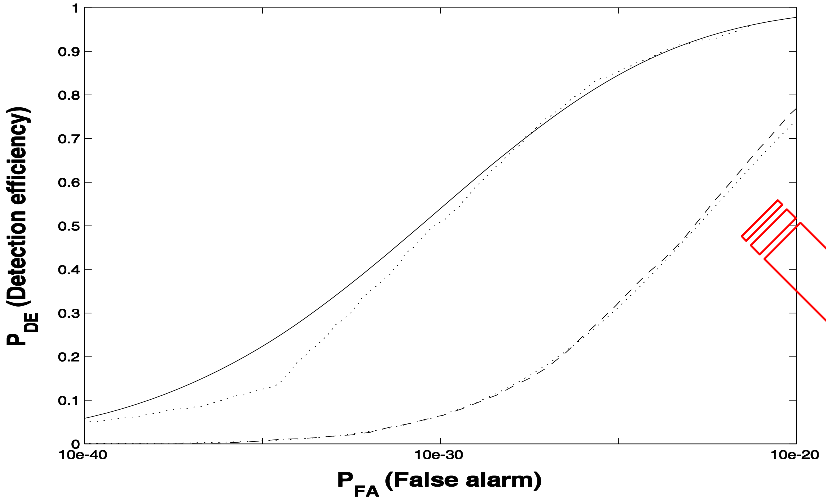
Key Results : Simulation



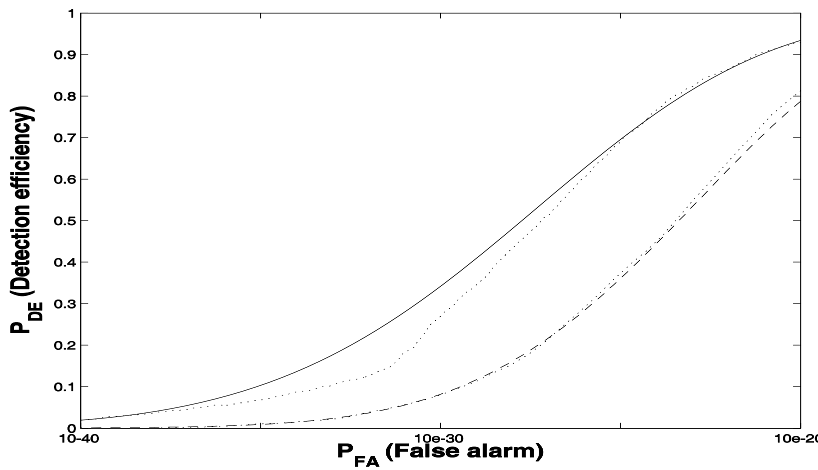
SNR = 10
uncorrelated
noise

Key Results : Simulation

- *Performance decreases*
- *Difference between coherent and coincident methods lessens*



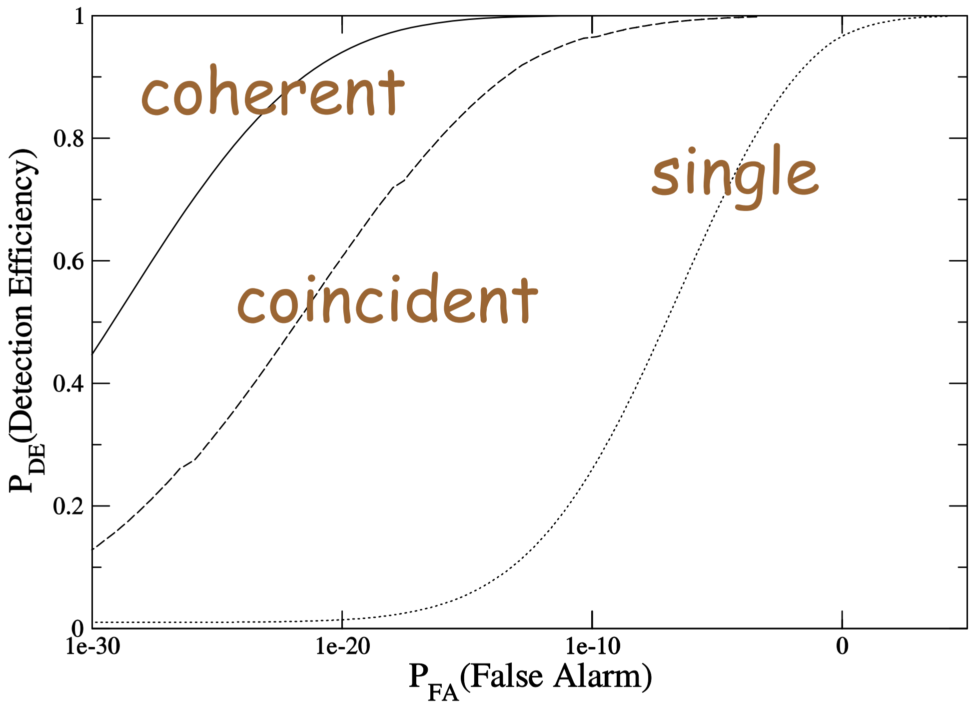
SNR = 10, ε_0 = 0.2



SNR = 10, ε_0 = 0.3

Key Results : Simulation

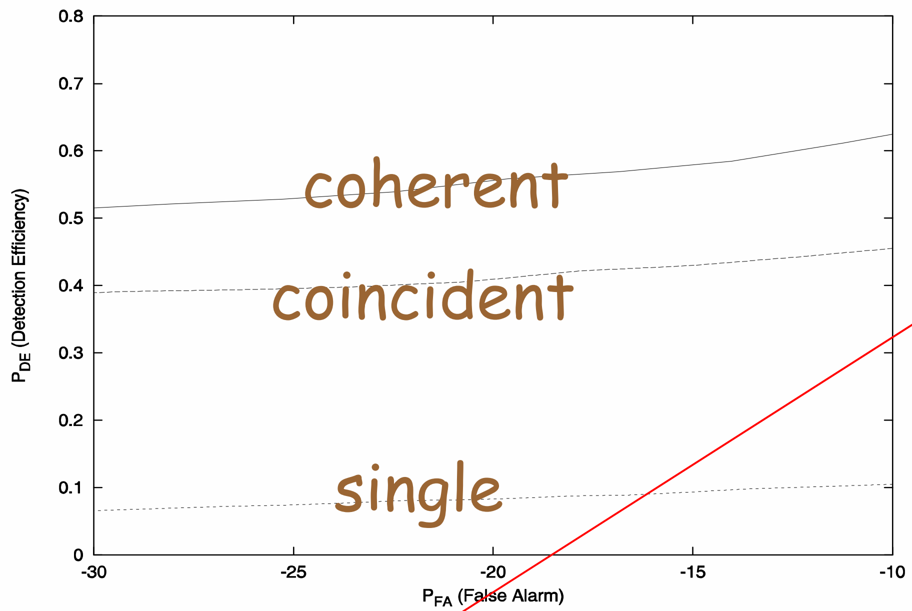
ROC curve for one year data



SNR = 10 uncorrelated noise

Key Results : Simulation

Exclude galactic sources



As far as LIGO I can see for SNR = 7

ROC curves for sources uniformly distributed in the sky over a distance 0.4 Mpc – 26 Mpc with uniformly distributed polarization angle for 1 year data

Conclusion

Coherent strategy outperforms coincident strategy by an amount 35% - 45%.

Future Directions

- *It is possible to generalize this for arbitrarily oriented detectors*
- *It may be possible to generalize this for burst sources*

Thank You