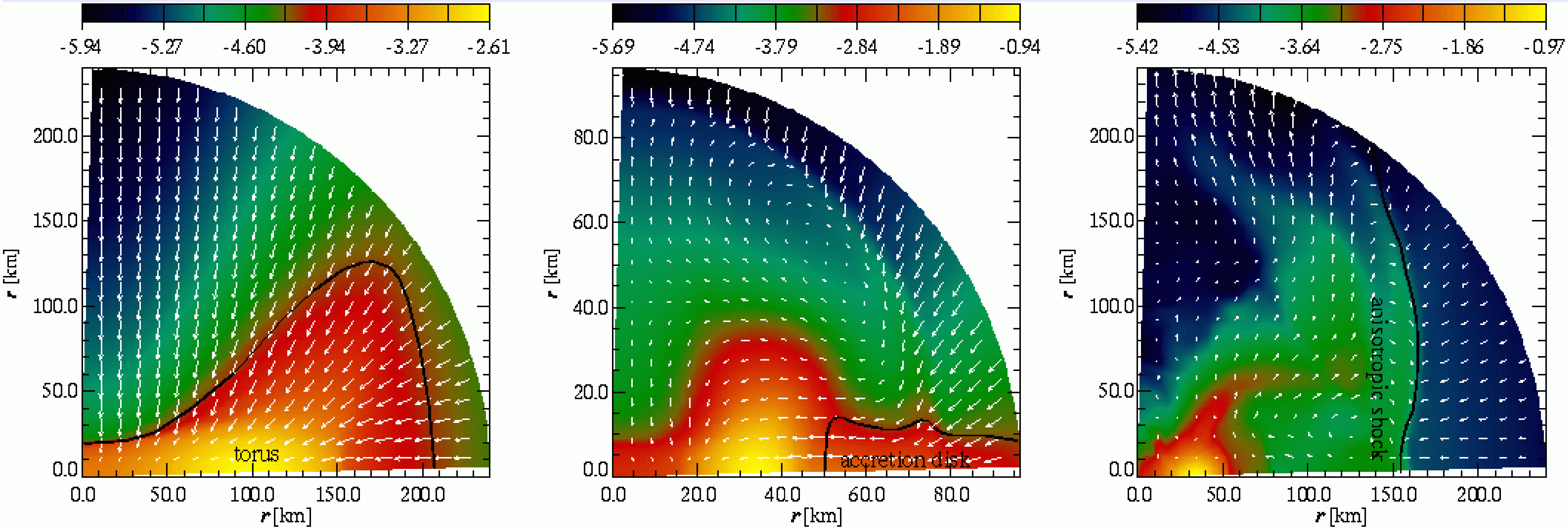
Simulations of general relativistic core collapse

IHP workshop on „Gravitational Wave Data Analysis“, November 2006



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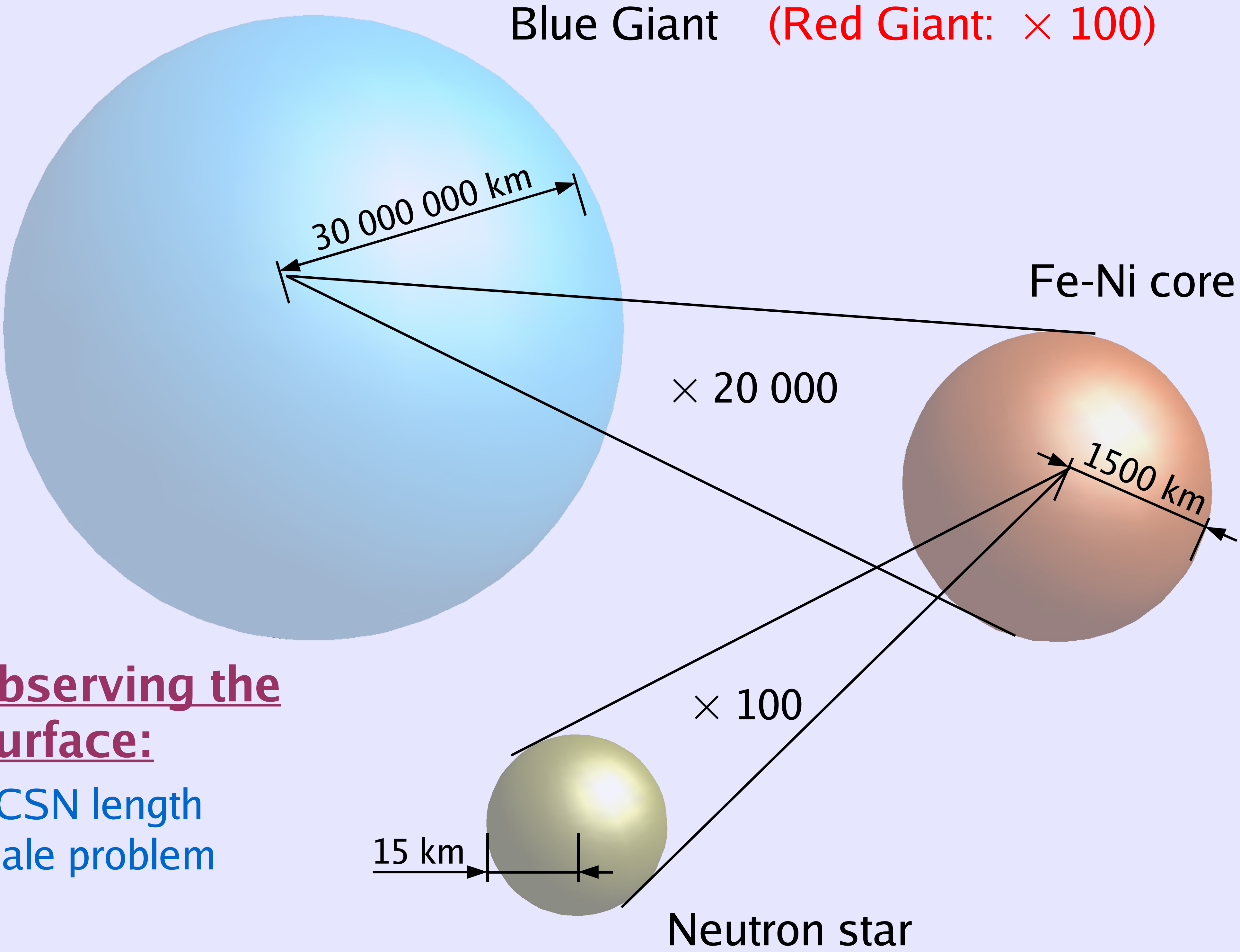
SFB-Transregio-7
Gravitationswellenastronomie

Observing the Surface:

CCSN length scale problem



Blue Giant (Red Giant: × 100)

30 000 000 km

× 20 000

Fe-Ni core

1500 km

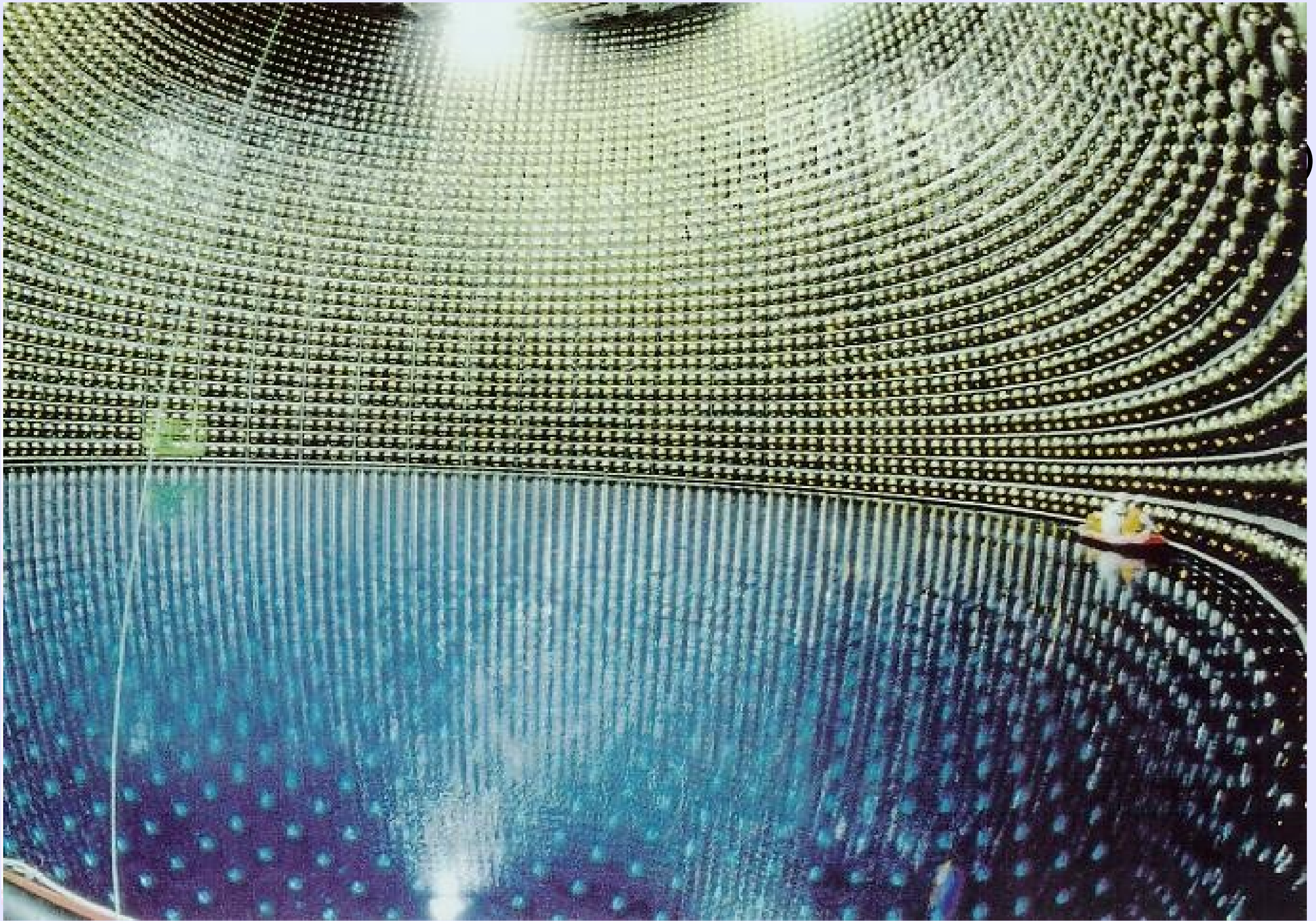
× 100

15 km

Neutron star

Looking into the heart of a core collapse supernova

- through observations of neutrinos
  (up to now only SN1987A)



Looking into the heart of a core collapse supernova

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  (not yet occured! Would provide kind of Rosetta stone!)

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Looking into the heart of a core collapse supernova

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- through simulations

  (already a 40 year effort ; extremely complex &
  expensive 6D radiation-hydrodynamics problem
  requiring $\sim 10^{21}$ operations / simulation
  or ~1CPU-yr @ 30 Teraflop / simulation)

Core collapse supernovae: neutrino-driven delayed explosion

(Wilson ´82, Bethe & Wilson '85)

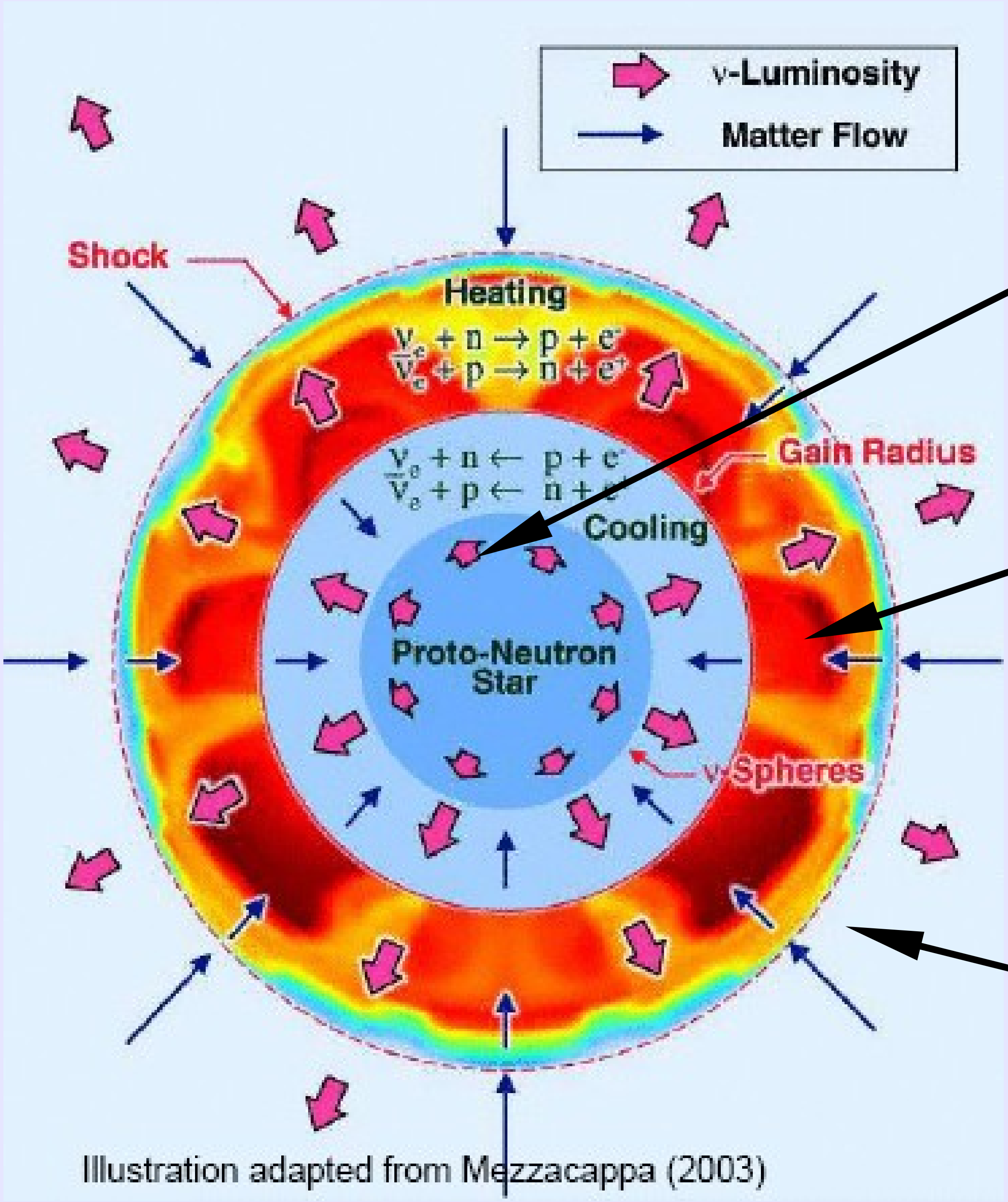


Illustration adapted from Mezzacappa (2003)

neutrinos diffuse out of opaque proto-neutron star ($\tau_\nu \sim 1$)

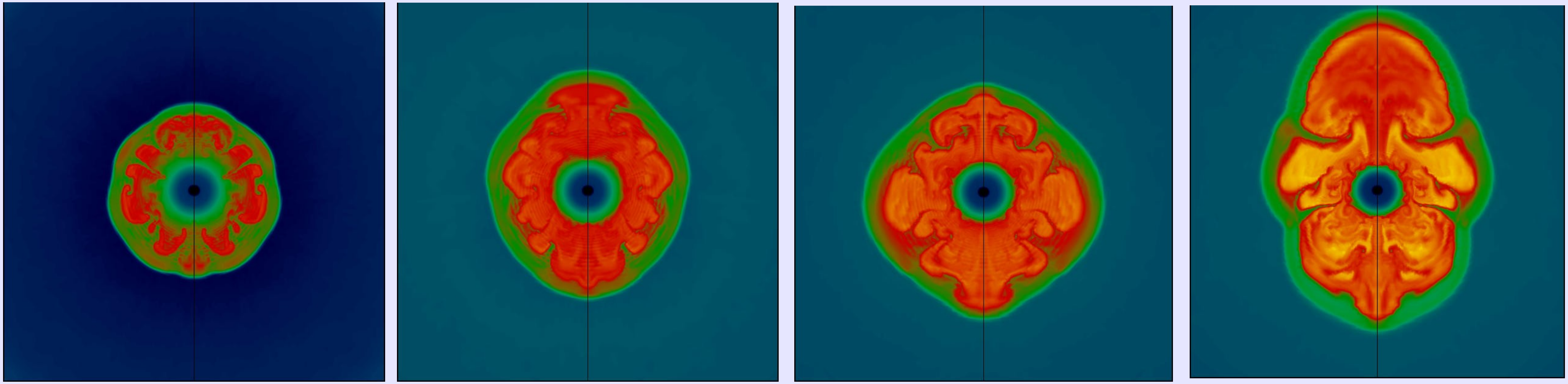
neutrinos heat matter in semi-transparent ($\tau_\nu \sim 1$) post-shock region ---> convection with coexisting downflows and rising hot bubbles sets in

neutrinos stream freely through stellar envelope ($\tau_\nu << 1$)

State-of-the-art 2D hydrodynamic simulations with

Boltzmann ν-transport, microphysical EOS, relativistic gravity, and progenitors from (1d) stellar evolutionary calculations

2D HD + (1.5D + 2D) NuTrans: $3\ 10^{17}$ ops/simu, i.e. 10^{7}s @ 30Gflops, or
10^{5}s @ 3Tflops



Snapshots from an axisymmetric 180^{o} simulation of a non-rotating 11.2 M$_{sun}$ progenitor
(Buras, Rampp & Janka 2003)

--> weak explosion (0.3 foe)!

Gravitational waves

(Einstein quadrupole formula)

$$h_{jk} = \frac{2G}{c^4}\frac{1}{R}\frac{d^2Q_{jk}}{dt^2} \sim \frac{R_s}{R}\frac{v^2}{c^2}$$

R_s=1 km , v/c=0.1 , R=10kpc ---> **$h \sim 10^{-20}$** *

time-dependent mass-energy quadrupole moment in core collapse supernovae due to

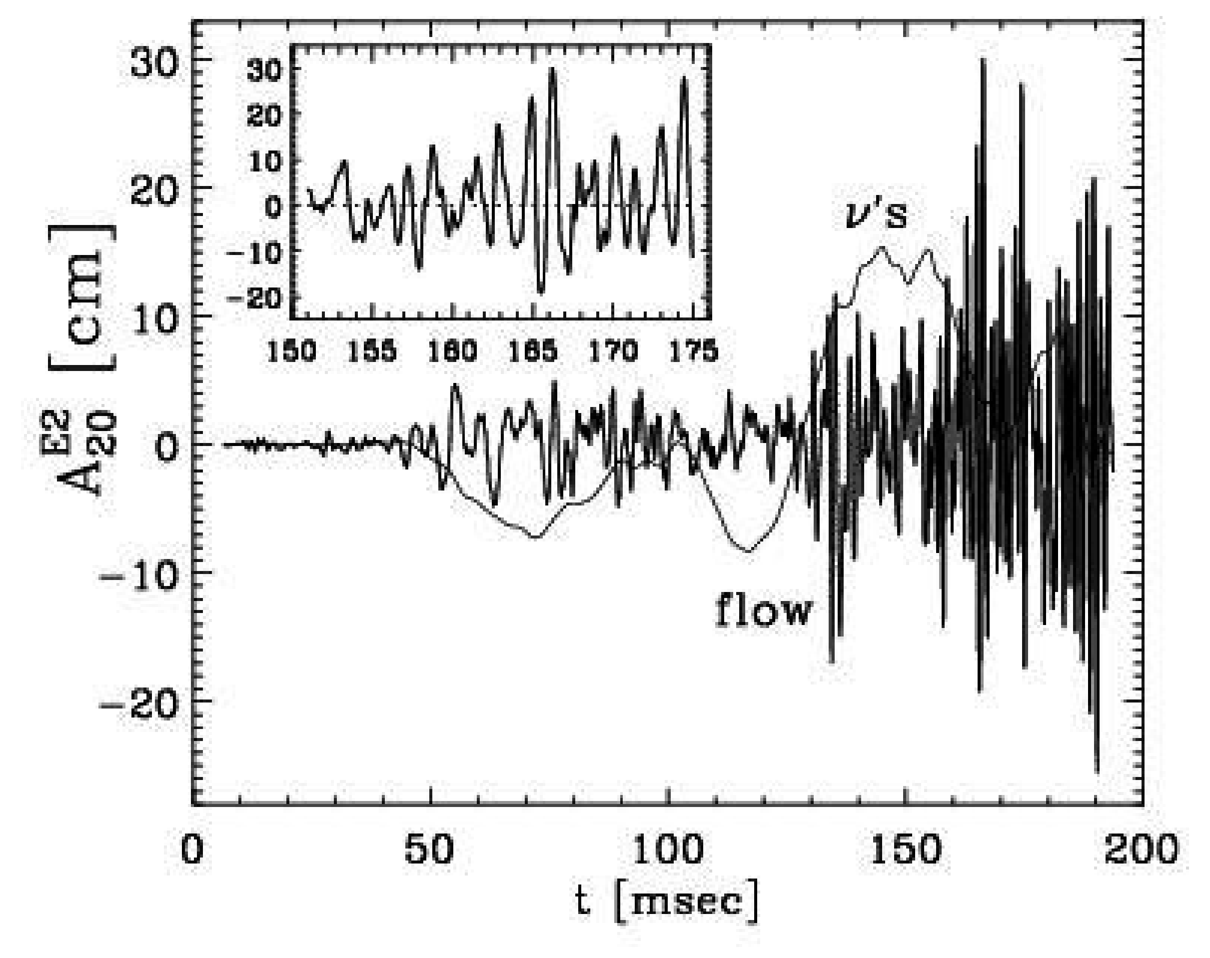
- convection in proto-neutron star
  - convection in neutrino heated hot bubble
    - anisotropic neutrino emission
      - any other non-radial instability (e.g. SASI, AAC)

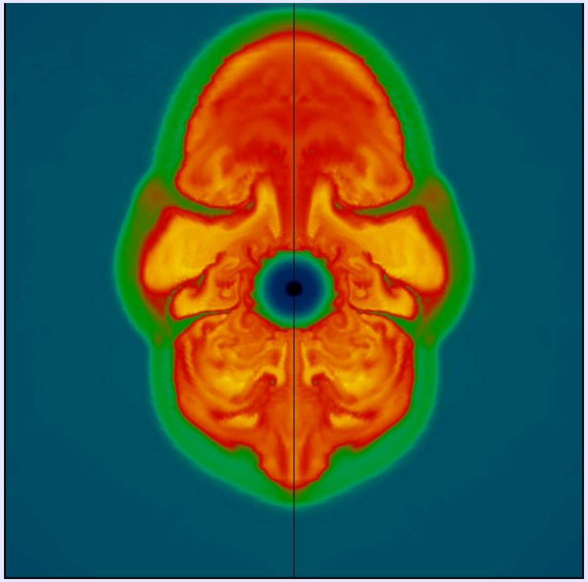
generically produced by any CCSN

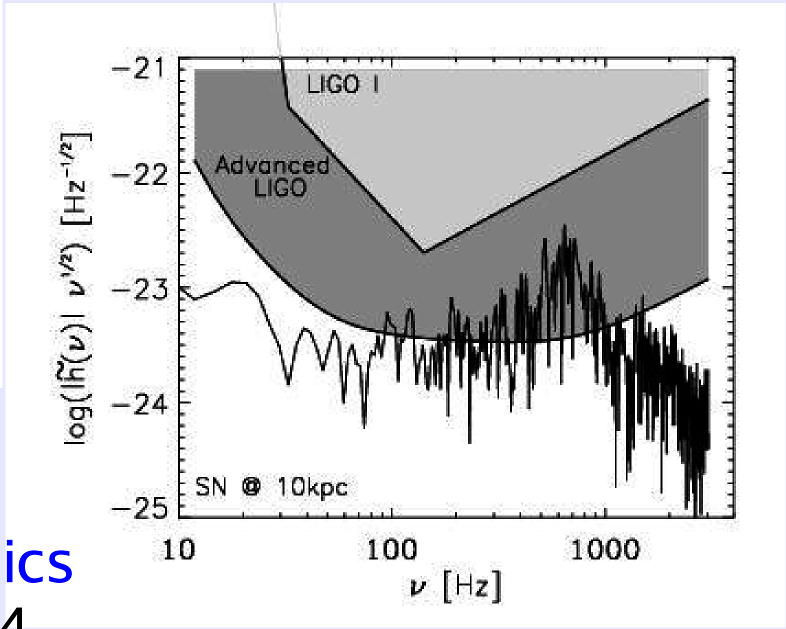
and due to rotation and magnetic fields

* [measuring the distance earth-sun with an accuracy of 1 nm]

GWs from a non-rotating 11.2 M_{sol} star



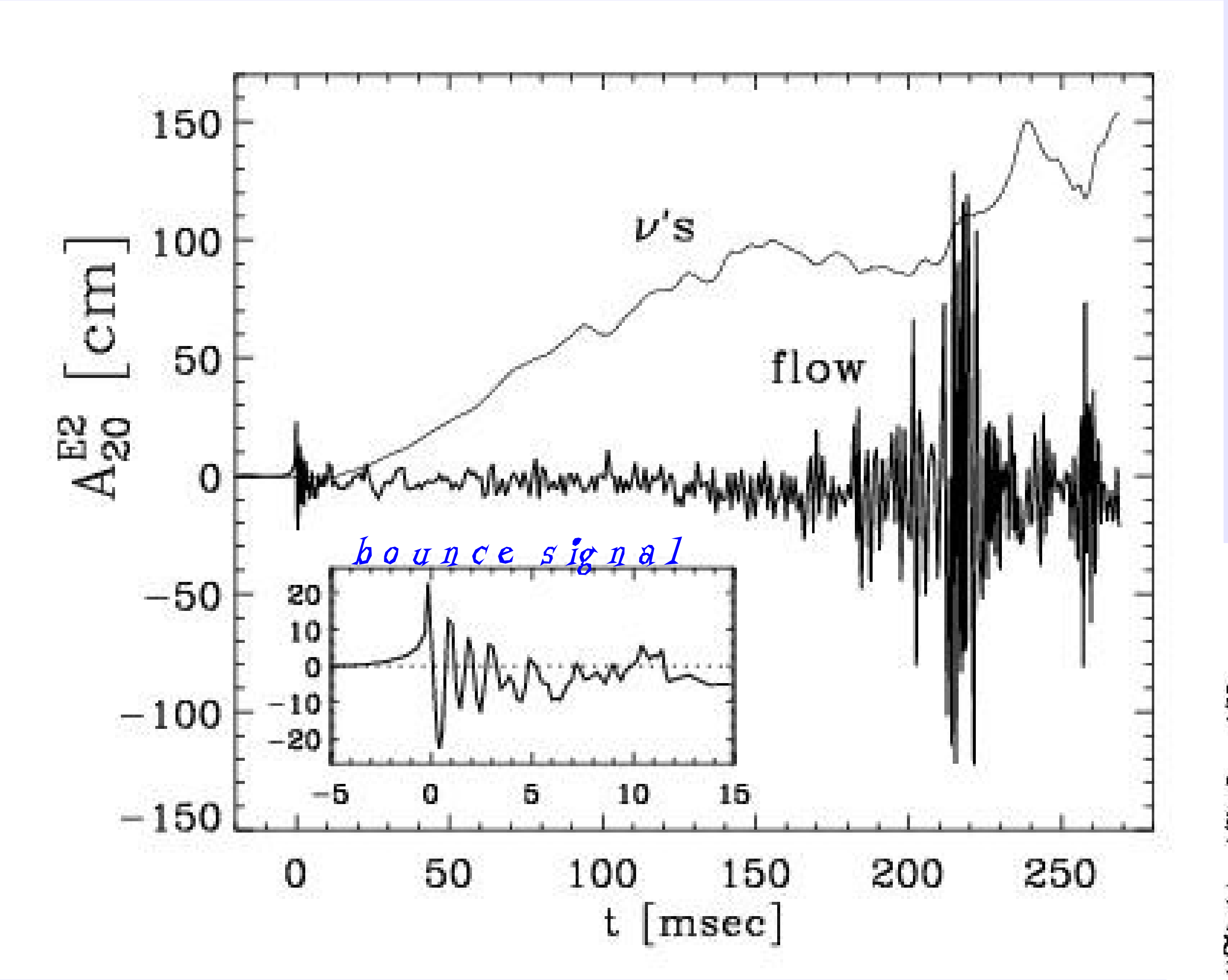


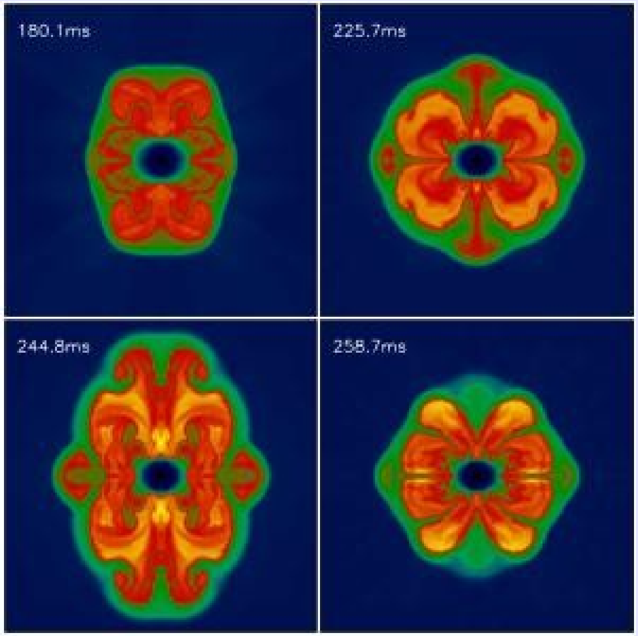


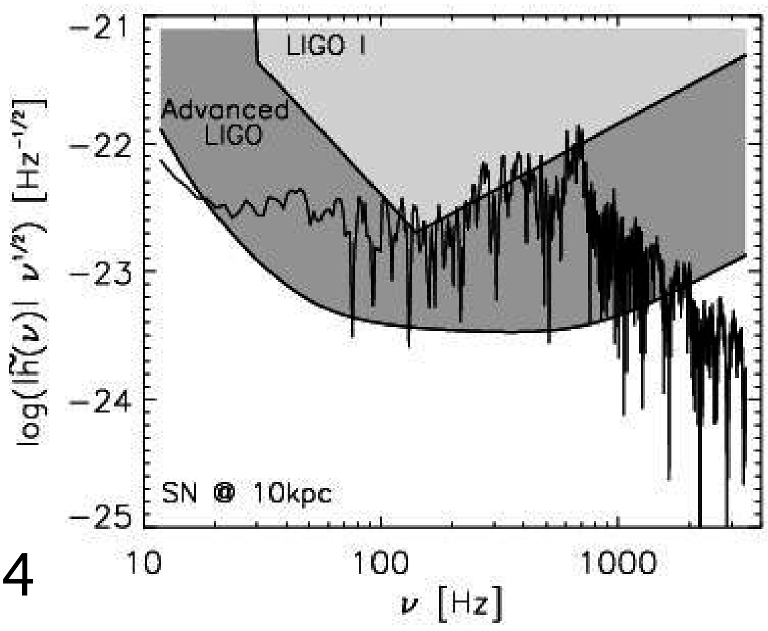
Models with detailed micro and transport physics

Müller, Rampp, Buras, Janka & Shoemaker '04

GWs from a rotating 15 M_{sol} star







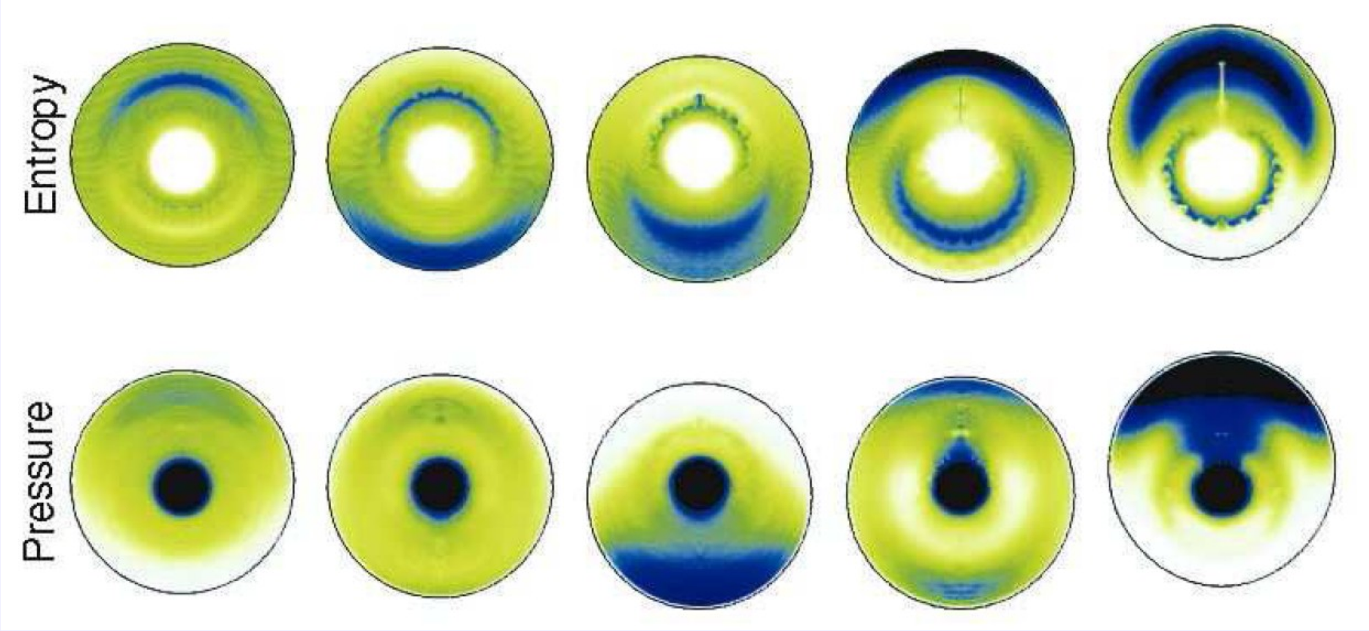
Müller, Rampp, Buras, Janka & Shoemaker '04

Additional instabilities:

(i) Standing Accretion Shock Instability (SASI)

Blondin et al. '03

low-mode oscillatory instability of flow behind standing accretion shock (“sloshing”)



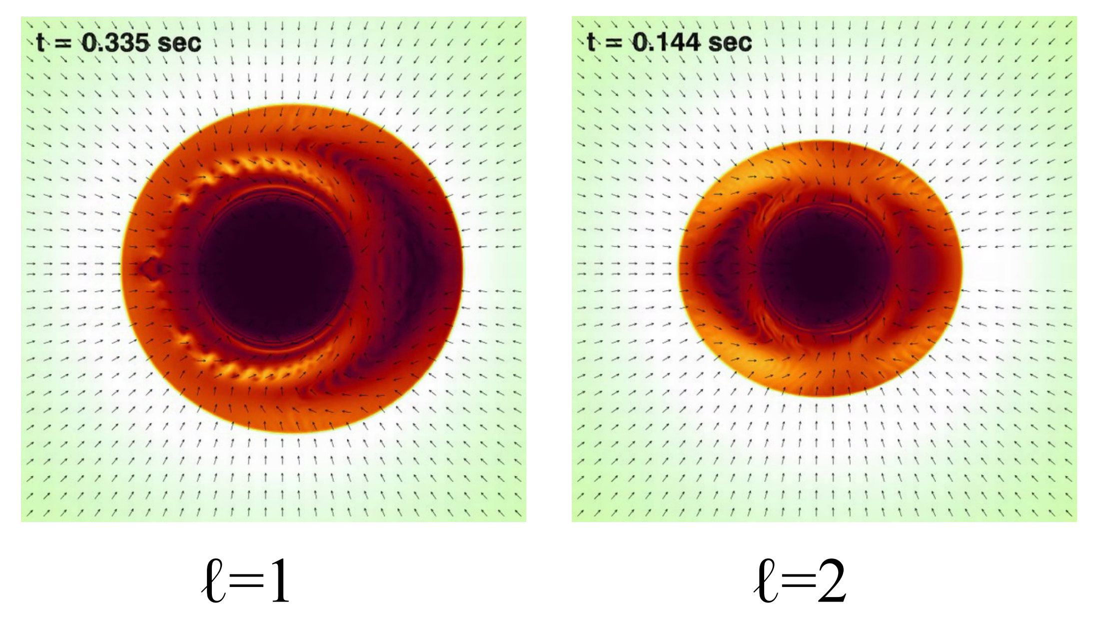
same behaviour is found if neutrino cooling and a microphysical EOS are included

(Blondin et al. '05, Ohnishi et al. '05)

(ii) Advective-Acoustic Cycle (AAC)

Foglizzo '02 (accretion disks),
Scheck et al., '06

- AAC is a non-radial, low-mode oscillatory instability that can grow (and trigger explosions) under conditions which do not allow for the growth of convection (i.e., short advection time scale, small entropy gradient, small initial perturbations)
- neutrino heating is boosted (by a factor 2) by AAC and convection

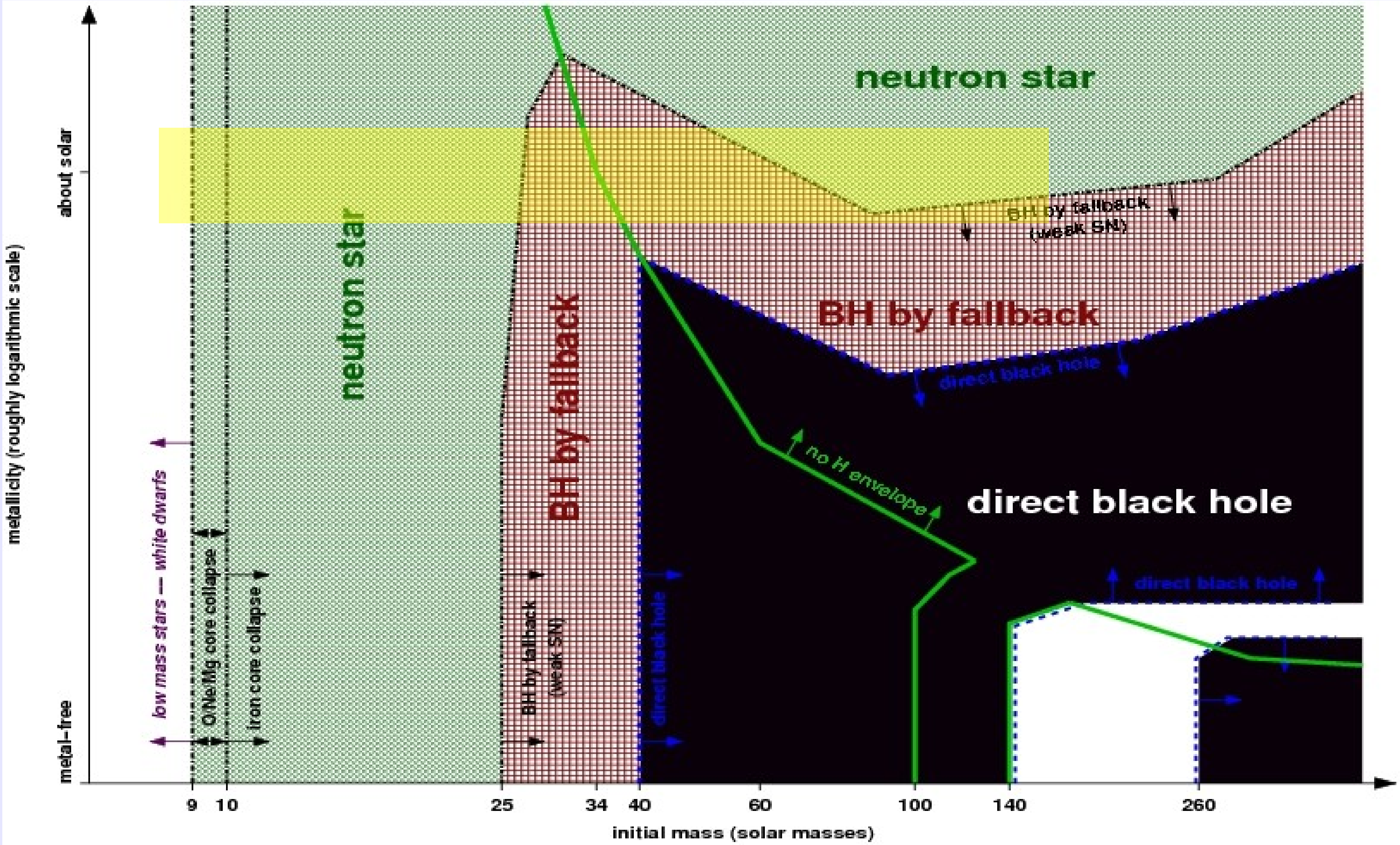


ℓ=1 ℓ=2

AAC is likely responsible for the excitation of low-ℓ modes, which cause large neutron star kicks

Fate of stellar core depends on progenitor mass & metallicity

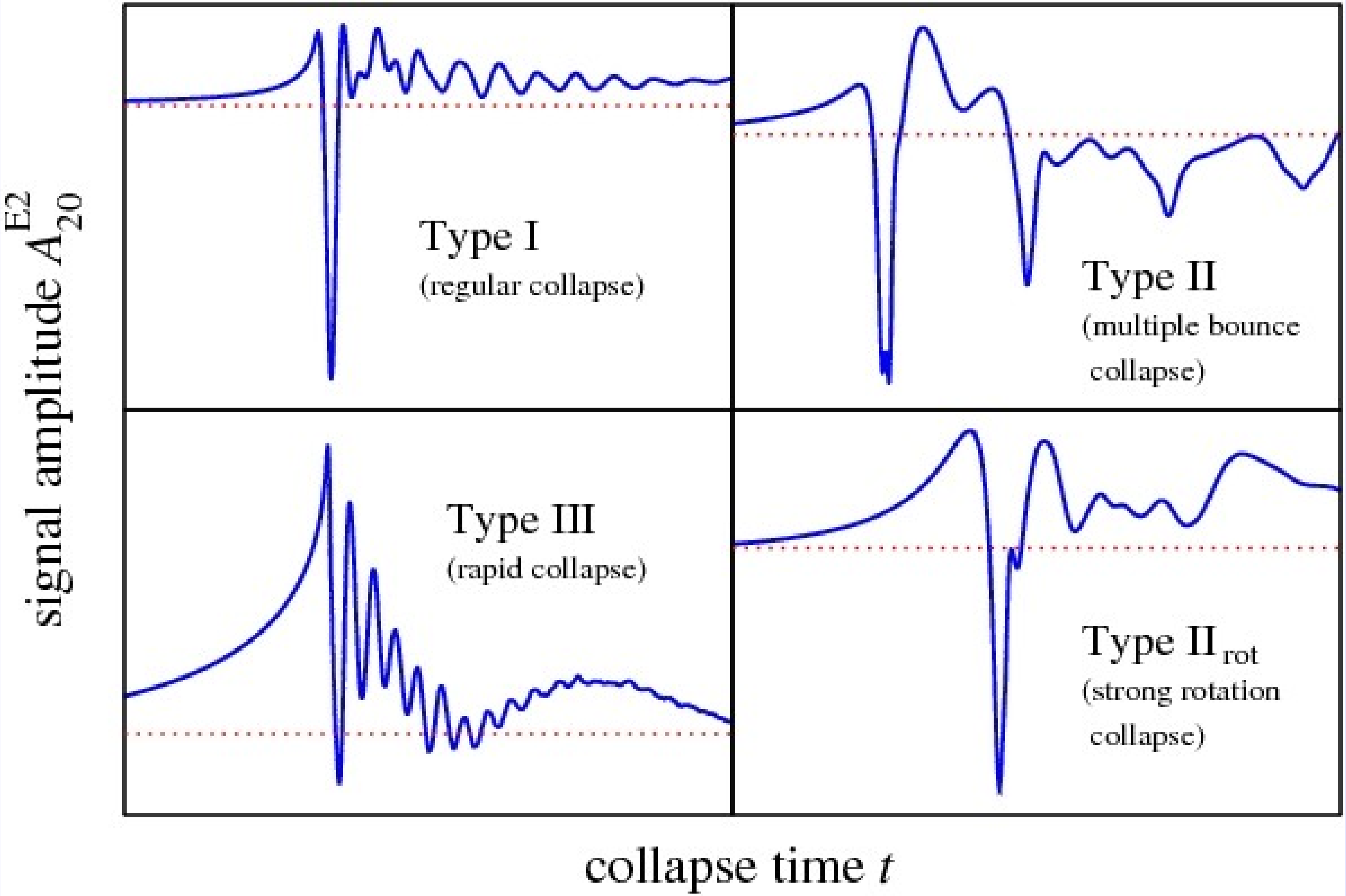
(Heger et al., '03)



Collapse dynamics & waveform types in rotational core collapse

(Newtonian gravity, axisymmetric models)

Mönchmeyer et al., '91: 4 models with tabulated EOS



typical signal strength & frequency:

$h \sim 10^{-20}$ at 10 kpc

$f \sim$ 500Hz - 1 kHz

Zwerger & Müller '97: 78 models with simple analytic EOS

Method R: The Effective Relativistic TOV Potential for a Self-Gravitating Fluid

Idea for simple approximation to general relativistic gravity:

- Keep Newtonian kinematics of a Newtonian hydrodynamics code.
  $\Longrightarrow$ Advantages: Simple structure of equations, no gauge effects, no stability issues, . . .
- Replace Newtonian potential by stronger effective “relativistic” potential
  (similar to strategy in vacuum: approximate black hole gravity by Paczyńsky–Wiita potential).
  [Paczyńsky and Wiita, A&A, 1980]

First attempt:
Use gravitational potential from TOV structure equations (equilibrium state after core bounce).
[Rampp and Janka, A&A, 2002]

$$\Phi(r) = -4\pi \int_0^\infty dr'\, r'^2 \frac{\rho}{|r - r'|}$$

$$\downarrow$$

$$\Phi_{\mathrm{TOV}}(r) = -4\pi \int_r^\infty \frac{dr'}{r'^2} \left(\frac{m_{\mathrm{TOV}}}{4\pi} + r'^3 (P + P_\nu) \right) \frac{1}{\Gamma^2} \left(\frac{\rho(1+\epsilon) + P}{\rho} \right)$$

with TOV mass $\quad m_{\mathrm{TOV}}(r) = 4\pi \int_0^r dr'\, r'^2 \left(\rho(1+\epsilon) + E + \frac{vF}{\Gamma} \right).$

Terms in blue are neutrino contributions.

Method A: Spherical Improvements of the Effective Relativistic TOV Potential

Results with Method R are close to relativistic simulations.

But: Central density is always overestimated!

$\Longrightarrow$ Incorrect location of supernova shock front, accretion rates, and neutrino emission.
[Liebendörfer et al., APJ, 2005]

Obvious reason for mismatch with consistently relativistic results:

Combination of "relativistic" gravity and Newtonian kinematics
(unlimited velocities, no nonlinear effects, underestimated inertia).

New approach: Modify TOV potential such that it is effectively weakened again!

We have successfully tested four modifications of TOV potential. [Marek et al., A&A, 2005]

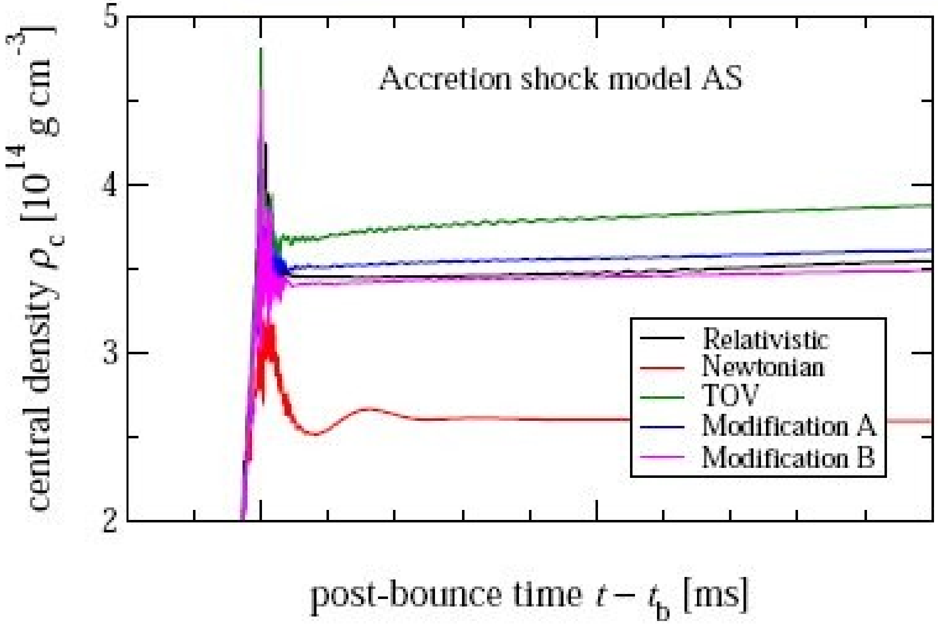
Best result: Add factor Γ to TOV mass: $\boxed{m_{\mathrm{TOV}}(r) = 4\pi \int_0^r dr'\, r'^2\, \Gamma\left(\rho(1+\epsilon) + E + \frac{vF}{\Gamma}\right).}$

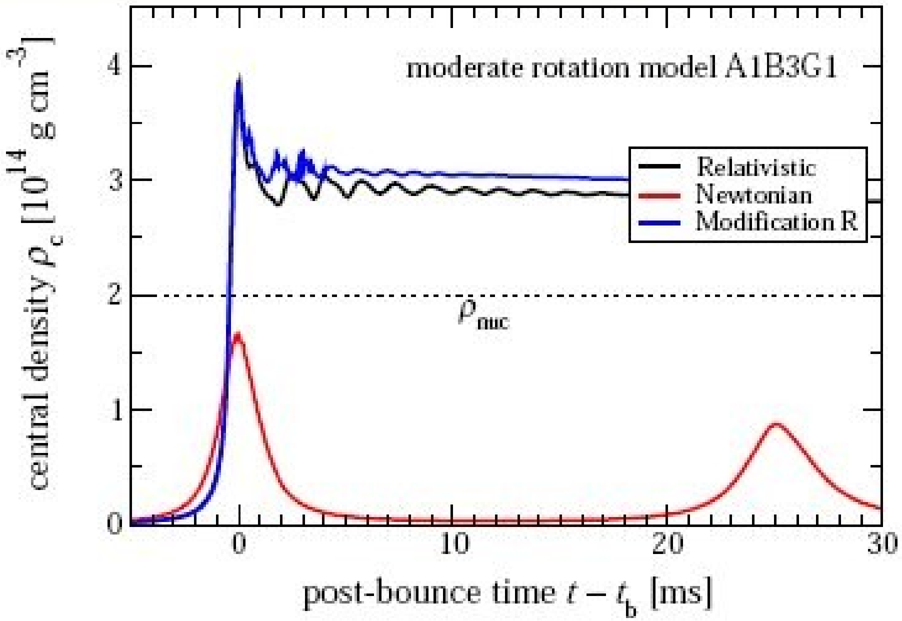
This can be interpreted as integration over coordinate volume instead of proper volume.

The Quality of the Effective Relativistic Potentials

Our tests with simple EOS and microphysical EOS with full Boltzmann neutrino transport show:

Method A yields very good agreement with general relativistic results!





Restrictions of this approximation:

- At most moderate gravity (only regular neutron star, no collapse to black hole).
- At most moderate rotation (otherwise approximation gives too low central density).

Method A is also used in simulations of rotating core collapse in MHD. [Obergaulinger et al., A&A, 2006]

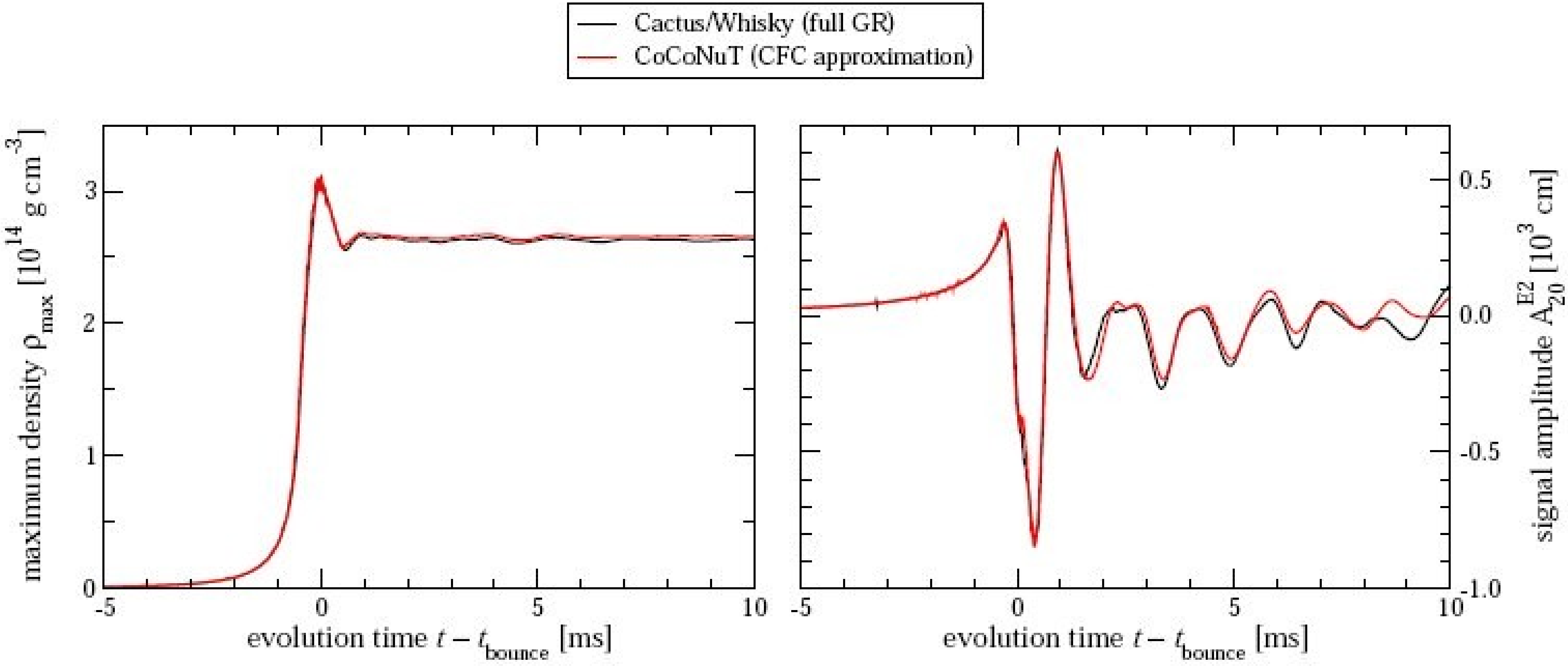
Improved version:
Method A is generalized to strong rotation. [Müller, Müller, and Dimmelmeier, in preparation, 2006]

The Quality of the Conformal Flatness Approximation

Comparison of results obtained using Method CFC and full general relativity proves:

[Shibata and Sekiguchi, PRD, 2005; Dimmelmeier et al., Proc. Albert Einstein's Century Conference, Paris, 2006;
Ott et al., PRL, submitted]

CFC is excellent approximation of full general relativity for supernova core collapse!



Compared to differences between numerical codes and coordinate choices:

Differences between full general relativity and Method CFC are typically smaller!

Method CFC+: The 2PN Extension of Method CFC

To estimate quality of Method CFC and extend its range of applicability:
Include deviation from conformal flatness up to second post-Newtonian order.
[Cerdá-Durán et al., A&A, 2005]

	Fully relativistic:	CFC:	CFC+:
Condition for 3-metric:	γ_{ij}	$\gamma_{ij}^{\mathrm{CFC}} = \phi^4 \hat{\gamma}_{ij}$	$\gamma_{ij}^{\mathrm{CFC+}} = \gamma_{ij}^{\mathrm{CFC}} + h_{ij}^{\mathrm{TT\,2PN}}$

As in Method CFC: Einstein equations reduce to elliptic equations.

$$\hat{\Delta}\phi = -2\pi\phi^5 \left(\rho W^2 - P + \frac{K_{ij}K^{ij}}{16\pi}\right),$$

$$\hat{\Delta}\alpha\phi = 2\pi\alpha\phi^5 \left(\rho h(3W^2 - 2) + 5P + \frac{7K_{ij}K^{ij}}{16\pi}\right) - \hat{\gamma}^{ik}\hat{\gamma}^{jl} h_{ij}^{\mathrm{2PN}}\, \hat{\nabla}_k \hat{\nabla}_l\, U,$$

$$\hat{\Delta}\beta^i = 16\pi\alpha\phi^4 S^i + 2K^{ij}\,\hat{\nabla}_j \left(\frac{\alpha}{\phi^6}\right) - \frac{1}{3}\hat{\nabla}^i\,\hat{\nabla}_k\,\beta^k,$$

$$h_{ij}^{\mathrm{2PN}} = \frac{1}{2}\mathcal{S}_{ij} - 3x^k\,\hat{\nabla}_{(i}\mathcal{S}_{j)k} + \frac{5}{4}\hat{\gamma}_{jm}\,x^m\,\hat{\nabla}_i\left(\hat{\gamma}^{kl}\mathcal{S}_{kl}\right) + \frac{1}{4}x^k\,x^l\,\hat{\nabla}_i\,\hat{\nabla}_j\,\mathcal{S}_{kl} + 3\hat{\nabla}_{(i}\mathcal{S}_{j)} - \frac{1}{2}x^k\,\hat{\nabla}_i\,\hat{\nabla}_j\mathcal{S}_k$$
$$+\frac{1}{4}\hat{\nabla}_i\,\hat{\nabla}_j\,\mathcal{S} - \frac{5}{4}\hat{\nabla}_i\,\mathcal{T}_j - \frac{1}{4}\hat{\nabla}_i\,\mathcal{R}_j + \hat{\gamma}_{ij}\left[\frac{1}{4}\hat{\gamma}^{kl}\mathcal{S}_{kl} + x^k\,\hat{\gamma}^{lm}\,\hat{\nabla}_m\,\mathcal{S}_{kl} - \hat{\gamma}^{kl}\,\hat{\nabla}_k\,\mathcal{S}_l\right],$$

For supernova core collapse and neutron stars: Methods CFC and CFC+ yield similar results!

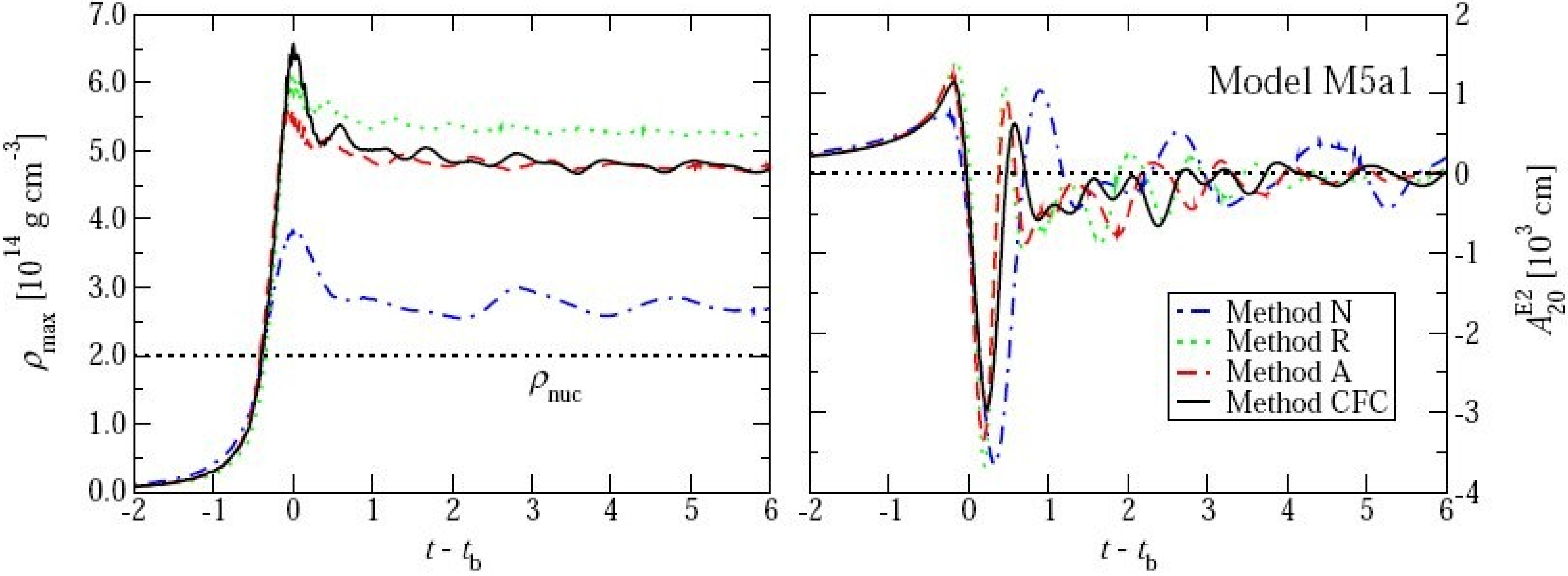
A Model with Slow Rotation and Moderate Compactness

Now select several interesting models for comparison of various approximation methods.

Model M5a1:

- Regular core mass.
- Almost uniform rotation.
- Slow rotation rate.

$\Longrightarrow$ "Standard" rotating core collapse model.



Method A gives excellent matching with Method CFC (and full general relativity)!

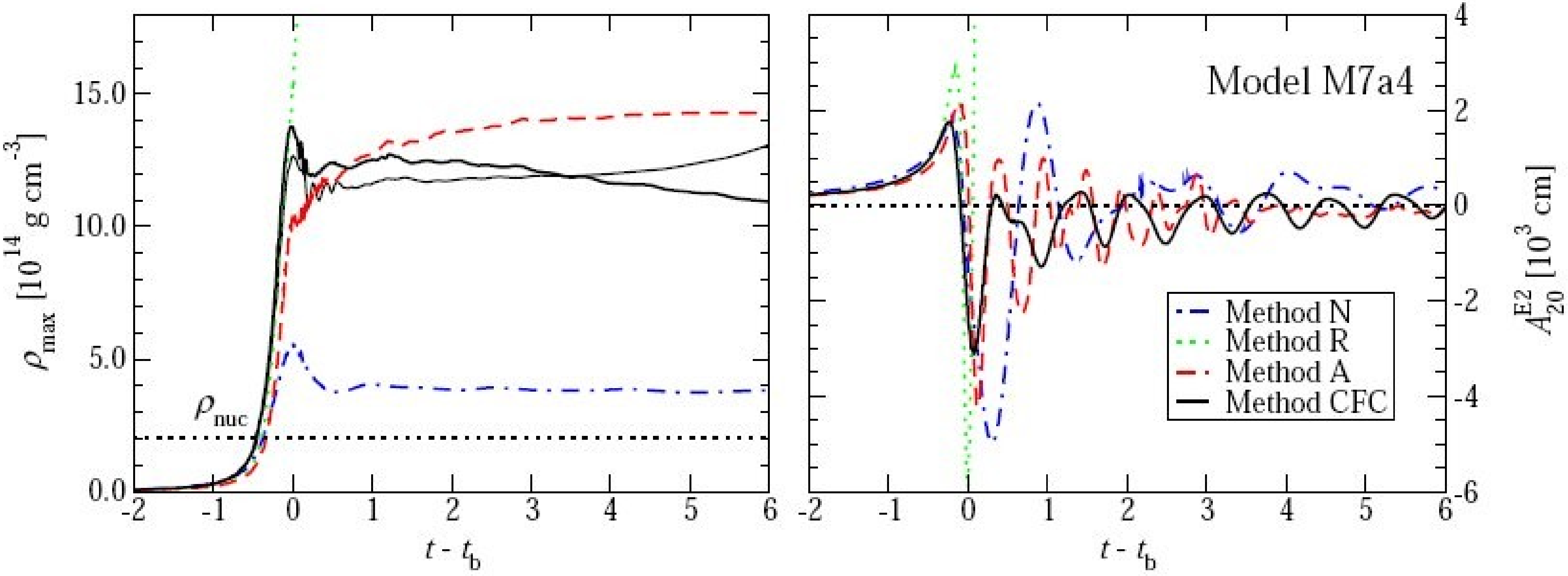
A Model with Moderate Rotation and High Compactness

Move towards a more extreme model.

Model M7a4:

- Medium heavy core mass.
- Almost uniform rotation.
- Moderate rotation rate.

$\Longrightarrow$ More extreme rotating core collapse model.



Method R yields (incorrect) collapse the black hole – sufficient resolution is also crucial!

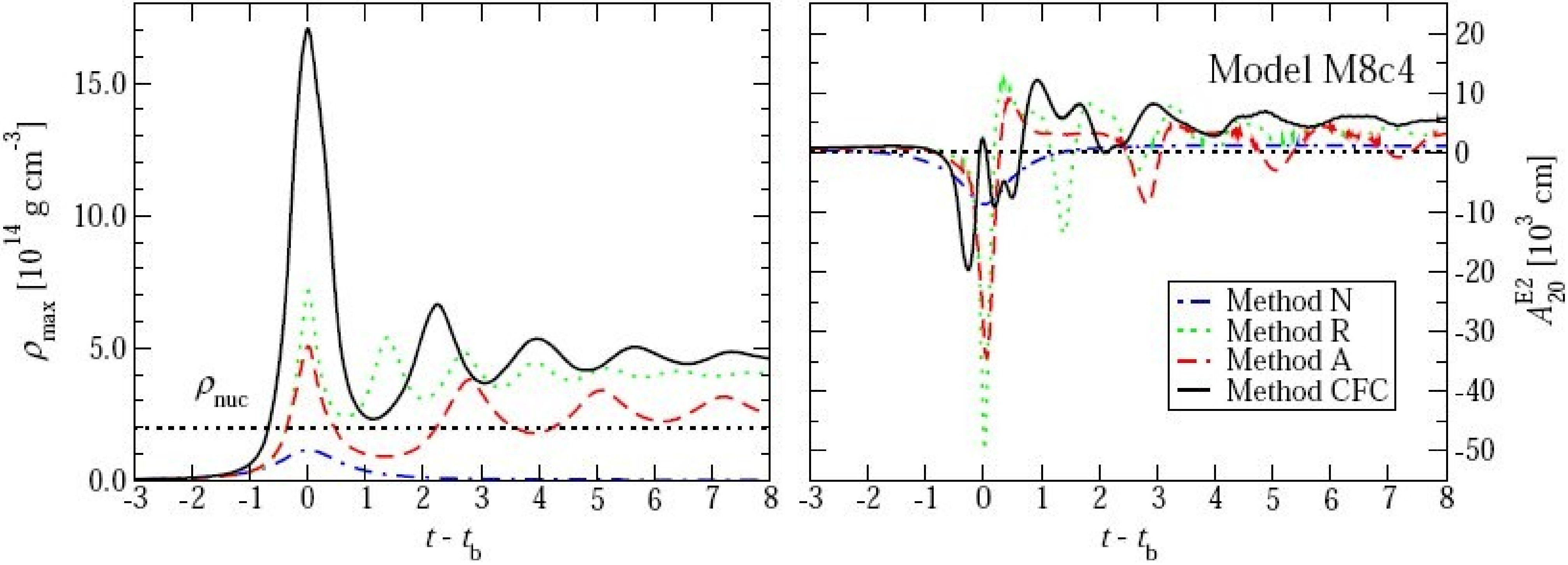
A Model with Strong Rotation and High Compactness

Move towards a more extreme model.

Model M8c4:

- Very heavy core mass.
- Very differential rotation.
- High rotation rate.

$\Longrightarrow$ More extreme rotating core collapse model.



All approximation methods except Methods CFC and CFC+ fail to give good results!

Towards relativistic core collapse simulations with detailed microphysics & neutrino transport

most sophisticated simulations of GR rotational core collapse up to now include: (Ott, Dimmelmeier, et al., 06)

- coupled relativistic gravity (BSSN, CFC) and GRHD
  (Cactus/Carpet/Whisky & CoCoNut codes)

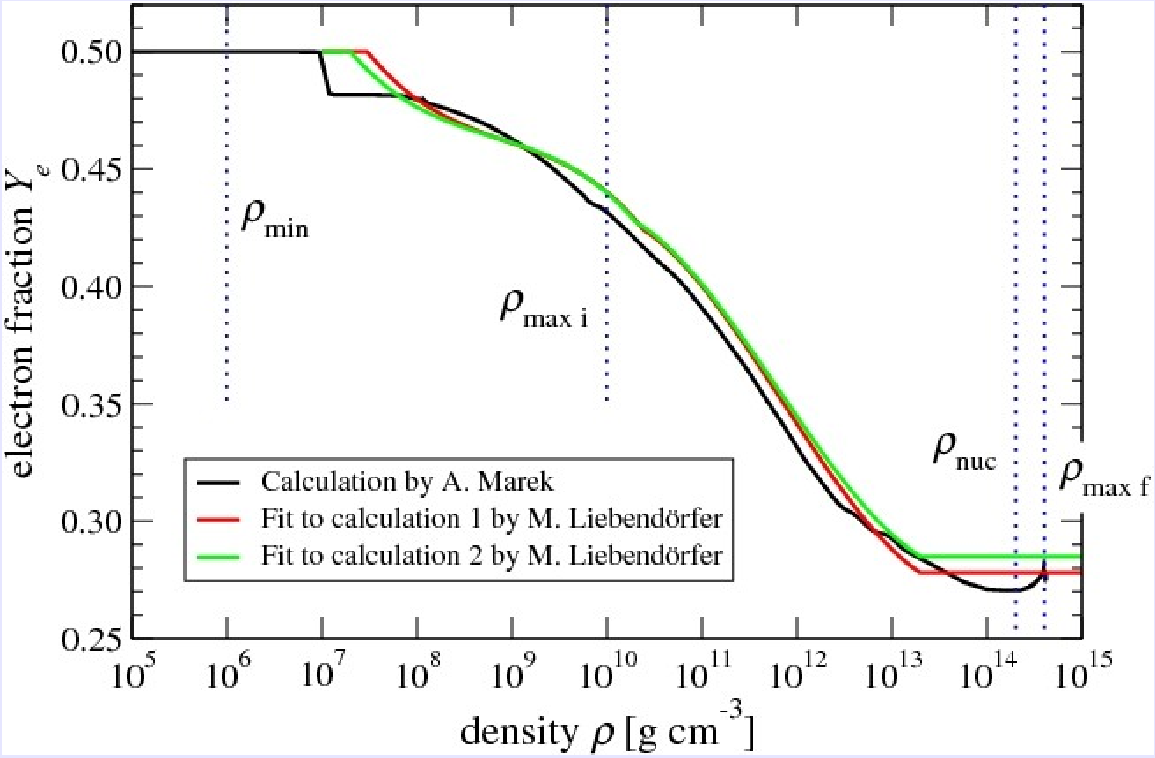
- tabulated microphysical EOS (Shen et al., '98; Marek et al., '05)

- Newtonian quadrupole formula for GW signal

- parameterized, approximate
  treatment of deleptonization
  (Liebendörfer '05: 1D!!)

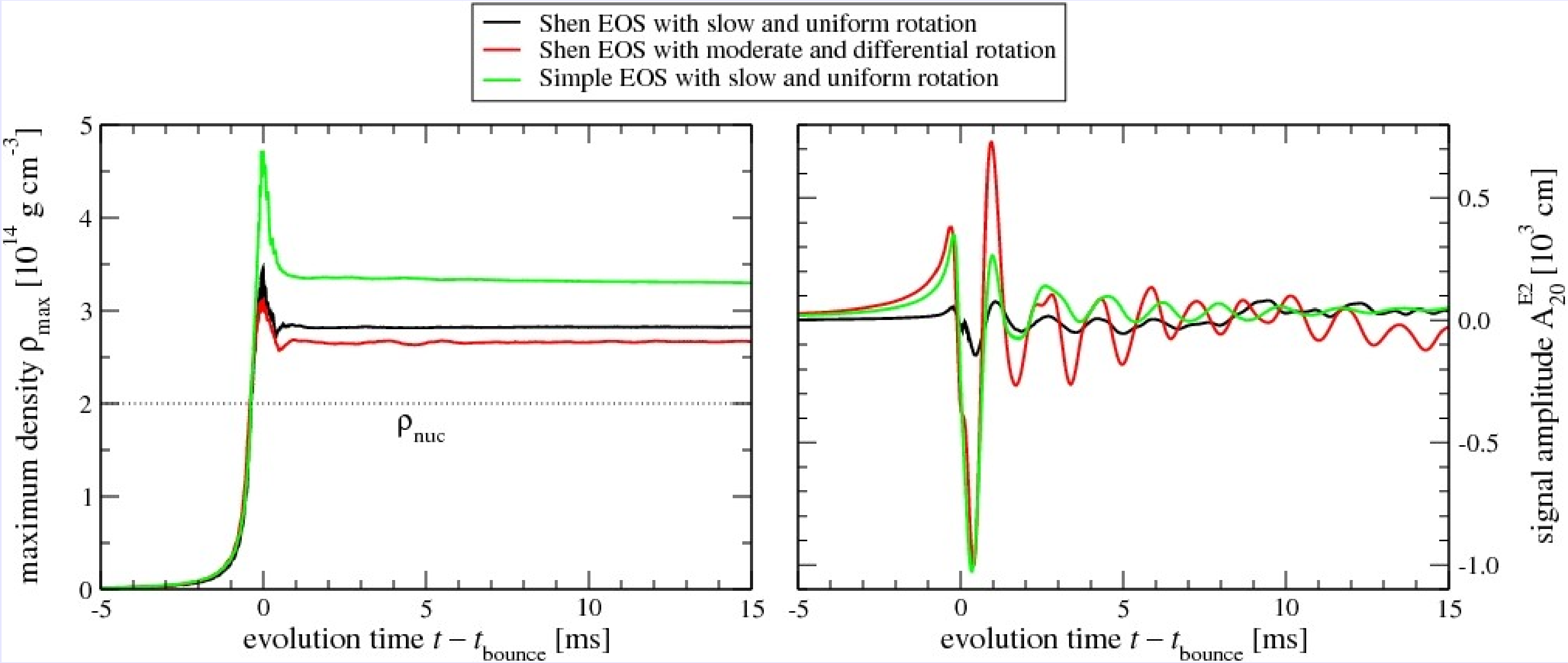
$Y_e \sim \rho$ and ρ_{trap}

fine until shortly after bounce!



Simple vs. microphysical EOS (Ott, Dimmelmeier, et al., 06)

slow & (almost) uniform rotating progenitor („best“ stellar evolution)

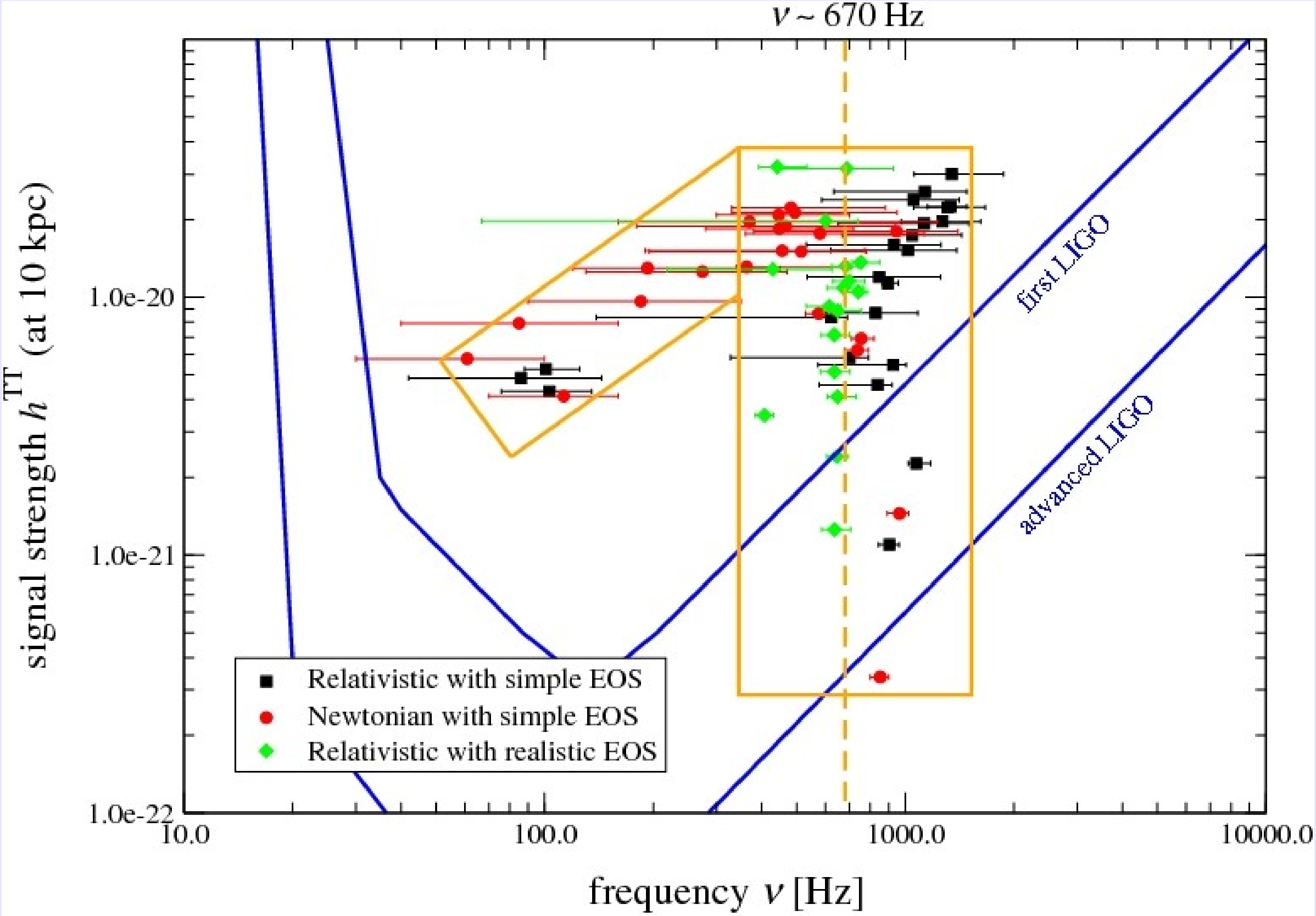


simulations with microphysical EOS:

- influence of rotation on dynamics & GW signal less pronounced
- no longer multiple centrifugal bounces & type-II GW signals
  (for very rapid rotators: type-I collapse dynamics & GW signal)

Detection prospects of GW from core collapse (to NS)

- bounce signal of a galactic supernova detectable by current detectors
- microphysical EOS: GW signal frequency range significantly narrower



low frequency GW signals (i.e. multiple centrifugal bounces) are suppressed in simulations with GR & a microphysical EOS!

(Ott, Dimmelmeier, et al., 06)

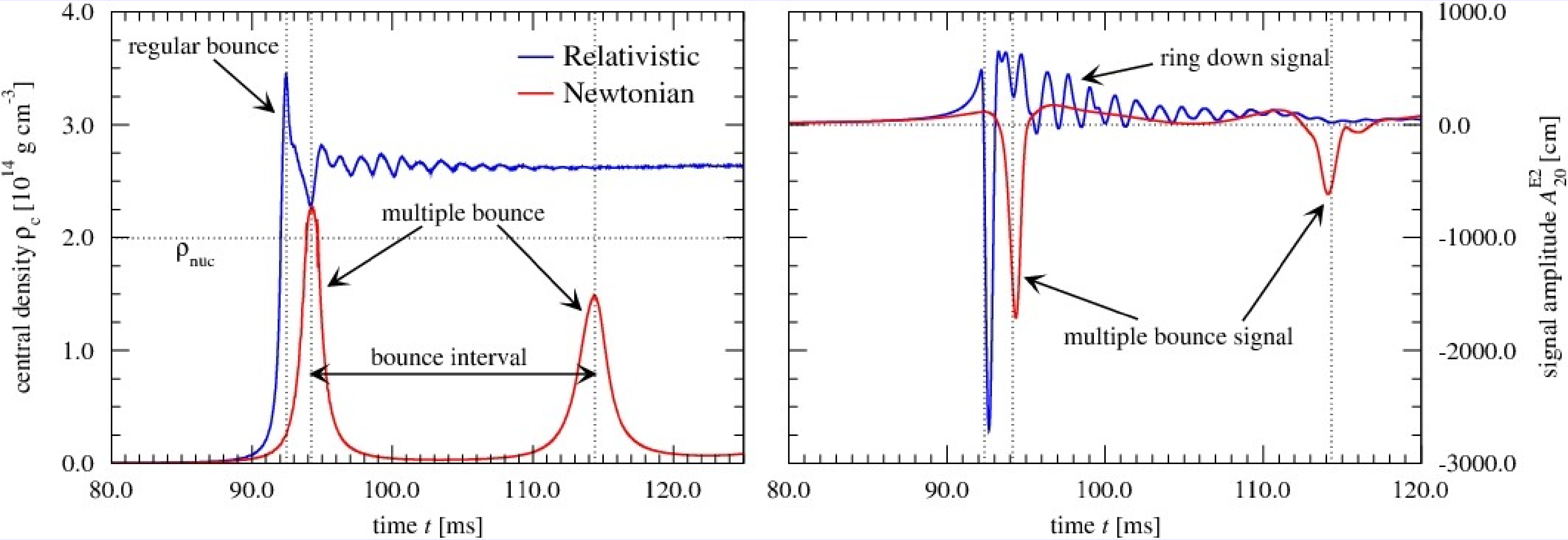
Cause of suppression: relativistic gravity?

Newtonian study of the collapse of rotating polytropes (Zwerger & Mueller, '97) repeated in relativistic gravity

(Dimmelmeier, Font & Muller, '02; Dimmelmeier et al., 05; Cerda et al., '05; Shibata & Segikuchi, '05, '06)

relativistics effects: deeper potential --> larger bounce densities, more compact PNS

less multiple centrifugal bounces (less type II GW signals)



Cause of suppression: microphysical EOS?

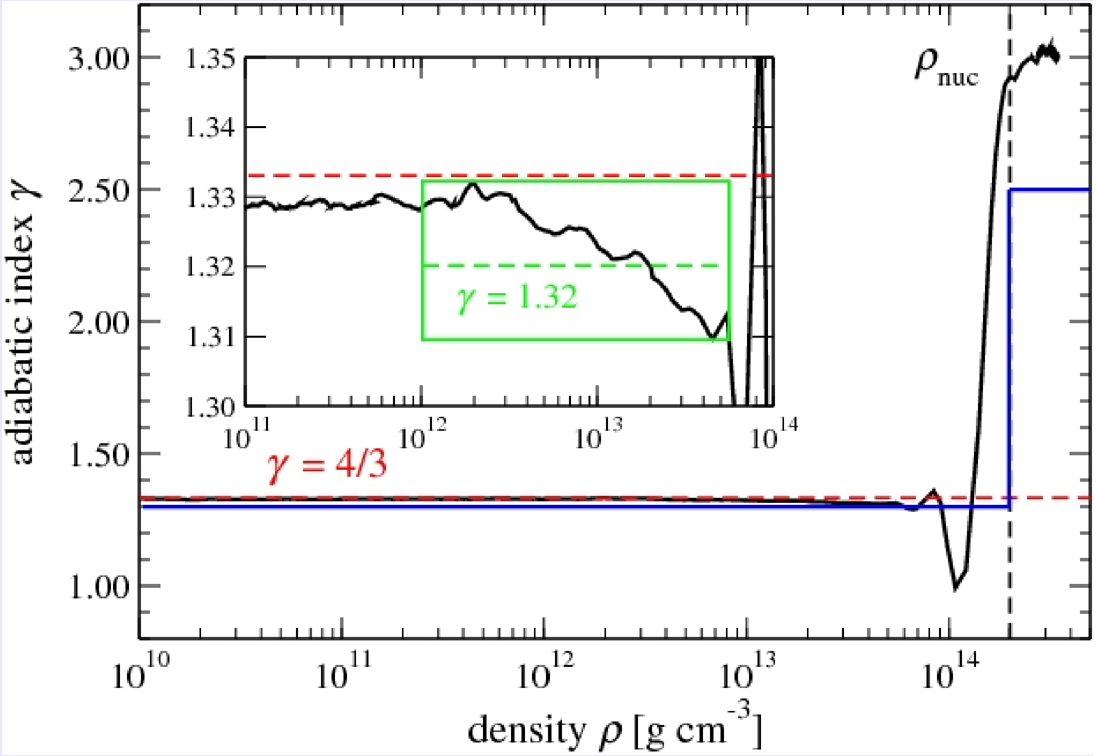
- neglect deleptonization and adopt adiabatic index for simplified EOS according to microscopic EOS

- simple EOS (piecewise polytropic + thermal part to mimic shocks)

  $\gamma < 4/3$ for $\rho < \rho_{nuc} = 2\ 10^{14}$g/cm^3

  $\gamma = 2.5$ for $\rho > \rho_{nuc}$

  similar to microphysical EOS



- centrifugal bounce: (below ρ_{nuc})

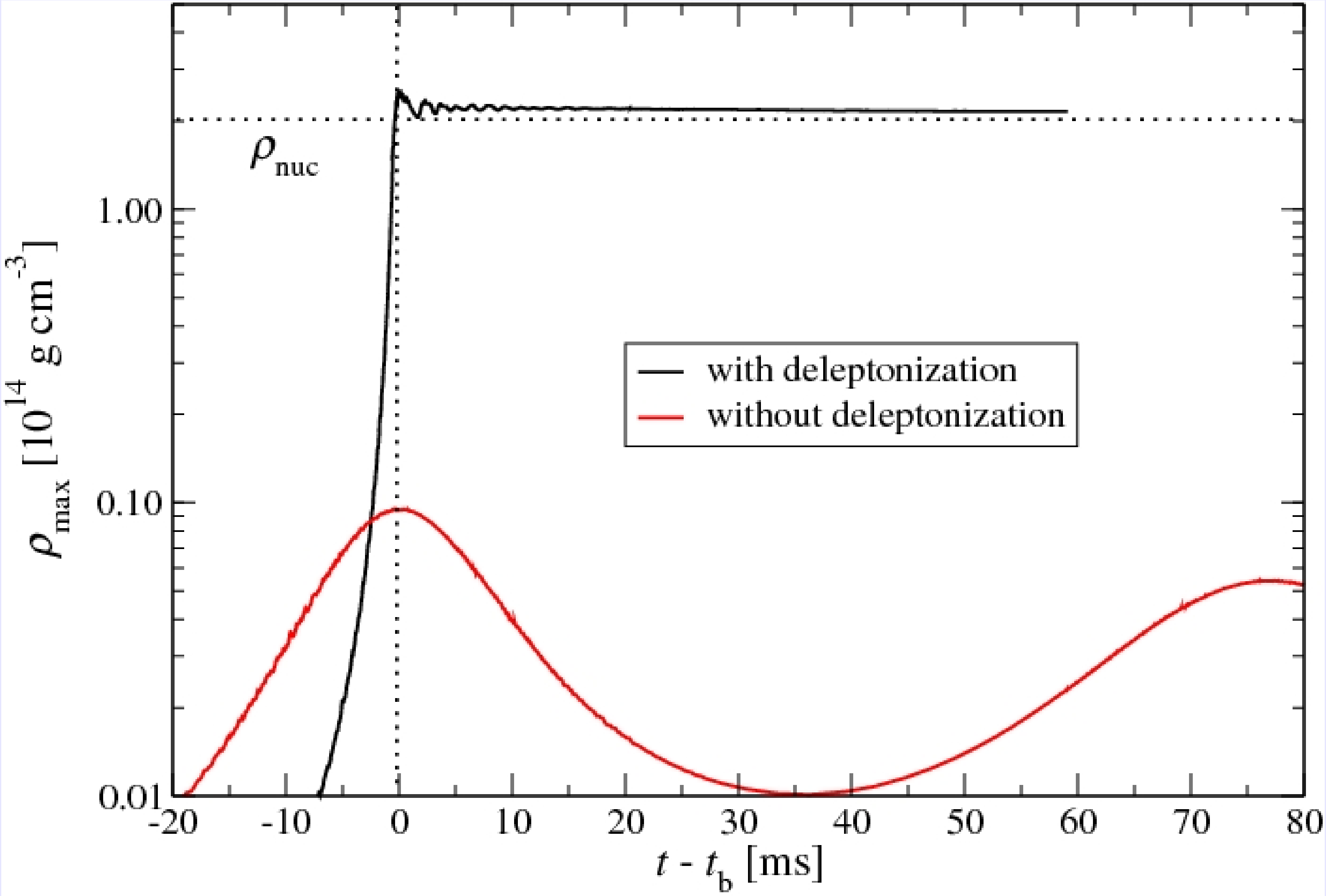
  γ is always very close to 4/3

---> approximating microphysical EOS with simple EOS (with γ=1.32) predicts correct collapse & GW signal type!

Cause of suppression: deleptonization?

- when including effects of deleptonization: no centrifugal multiple bounce found! (neither in Newtonian nor relativistic gravity)

  (Dimmelmeier et al., in prep.)



Conclusions

Modelling neutrino driven core collapse supernovae requires detailed treatment of transport physics coupled to multidimensional hydrodynamic flow

Relativistic gravity important for explosion mechanism & GW signal Its effects can be well modelled (for not too extreme models) by means of an effective relativistic potential (during collapse to NS)

Models of (rotational) core collapse including a microphysical EOS, some treatment of the core's deleptonization, and (approximate) relativistic gravity show no multiple centrifugal bounce

Open questions:
GR post-bounce evolution & GW signal
GR & magnetic fields (collapsars)
BH formation from core collapse