Accurate and realistic black hole-neutron star binaries

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Introduction

The detectors

Detection rates

initial-LIGO (yr^{-1})

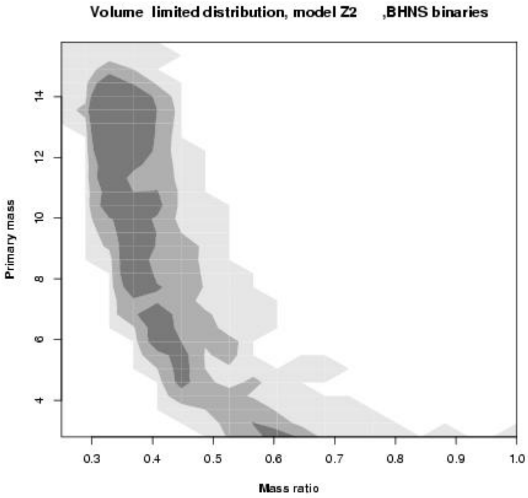
Binary type	Standard Model	All Range
NS-NS	1×10^{-2}	$2 \times 10^{-4} - -7 \times 10^{-1}$
BH-NS	2×10^{-2}	$2 \times 10^{-3} - -7 \times 10^{-2}$
BH-BH	8×10^{-1}	$0 - -2$
Total	8×10^{-1}	$2 \times 10^{-3} - -2$

advanced-LIGO (yr^{-1})

Binary type	Standard Model	All Range
NS-NS	6×10^{1}	$1 - -4 \times 10^{2}$
BH-NS	8×10^{1}	$9 - -4 \times 10^{2}$
BH-BH	2×10^{3}	$0 - -8 \times 10^{3}$
Total	3×10^{3}	$1 \times 10^{1} - -8 \times 10^{3}$

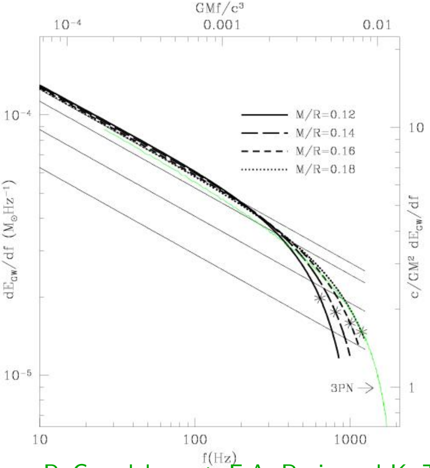
K. Belczynski, V. Kalogera and T. Bulik, Astrophys. J., **572**, 407 (2001)

Binary parameters



T. Bulik, D. Gondek-Rosinska and K. Belczynski, MNRAS, **352**, 1372 (2004)

Constraining the NS compactness $\Xi = \frac{GM}{Rc^2}$

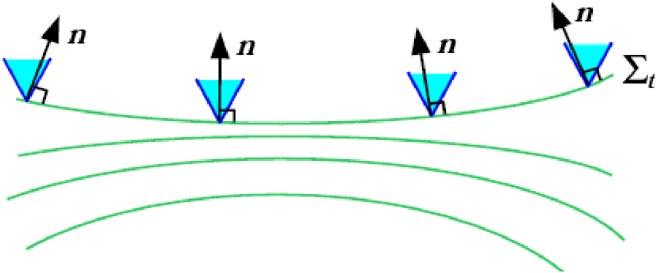


J.A. Faber, P. Grandclement, F.A. Rasio and K. Taniguchi,
Phys. Rev. Lett., **89**, 231102 (2002)

Formalism

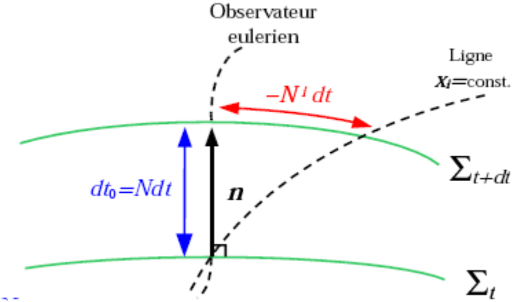
Foliation of space-time

Spacetime is foliated by a family of spatial hypersurfaces



- Coordinate system of Σ_t : (x_1, x_2, x_3).
- Coordinate system of spacetime : (t, x_1, x_2, x_3).

Metric quantities



Various functions

Lapse N, shift $\vec{N}$ and spatial metric γ_{ij}.

The metric reads

$$ds^2 = -\left(N^2 - N^i N_i\right) dt^2 + 2 N_i dt dx^i + \gamma_{ij} dx^i dx^j$$

Projection of Einstein equations (without matter)

Type	Einstein	Maxwell
Constraints	Hamiltonian $R + K^2 - K_{ij}K^{ij} = 0$	$\nabla \cdot \vec{E} = 0$
	Momentum : $D_j K^{ij} - D^i K = 0$	$\nabla \cdot \vec{B} = 0$
Evolution	$\frac{\partial \gamma_{ij}}{\partial t} - \mathcal{L}_{\vec{N}} \gamma_{ij} = -2NK_{ij}$	$\frac{\partial \vec{E}}{\partial t} = \frac{1}{\varepsilon_0 \mu_0} \left(\vec{\nabla} \times \vec{B} \right)$
	$\frac{\partial K_{ij}}{\partial t} - \mathcal{L}_{\vec{N}} K_{ij} = -D_i D_j N + N \left(R_{ij} - 2K_{ik}K^k_j + KK_{ij} \right)$	$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \vec{E}$

R_{ij} is the Ricci tensor of γ_{ij} and D_i the covariant derivative associated with γ_{ij}.
K_{ij} is called the extrinsic curvature.

Quasiequilibrium binaries

Assume exact circular orbits

- Only an approximation : no closed orbits in GR.
- Existence of an helical Killing vector.
- In the inertial frame : $\partial_t = \Omega \partial_\varphi$ (the associated shift is N^i).
- In the corotating frame $\Longrightarrow \partial_t = 0$ (the associated shift is β^i)

Additional approximations :

- Maximum slicing $K = 0$.
- Conformal flatness $\gamma_{ij} = \Psi^4 f_{ij}$.
- Asymptotical flatness.

Successfully applied to BBH and BNS.

The equations

5 coupled elliptic equations for the metric fields

- Hamiltonian constraint :

$$\Delta\Psi = -2\pi\Psi^5 E - \frac{\Psi^5}{8}\tilde{A}_{ij}\tilde{A}^{ij}$$

- Momentum constraint :

$$\Delta\beta^i + \frac{1}{3}\bar{D}^i\bar{D}_j\beta^j = 16\pi N\Psi^4 (E+p)\, U^i + 2\tilde{A}^{ij}\left(\bar{D}_j N - 6N\bar{D}_j \ln\Psi\right)$$

- Trace of the evolution equation :

$$\Delta N = 4\pi N\Psi^4 (E+S) + N\Psi^4\tilde{A}_{ij}\tilde{A}^{ij} - 2\bar{D}_i \ln\Psi\,\bar{D}^i N$$

Definition of $\tilde{A}^{ij} = \Psi^4 K_{ij} = \frac{1}{2N}\left(\bar{D}^i\beta^j + \bar{D}^j\beta^i - 2/3\bar{D}_k\beta^k f^{ij}\right)$.

The thin-sandwich approach

Decomposition :

$$\gamma_{ij} = \Psi^4 \tilde{\gamma}_{ij}$$

$$K^{ij} = \Psi^{-4} \left[\frac{(L\beta)^{ij} - \tilde{u}^{ij}}{2N} \right] - \frac{1}{3}\gamma^{ij} K$$

with $(L\beta)^{ij} = \tilde{D}^i \beta^j + \tilde{D}^j \beta^i - \frac{2}{3}\tilde{D}_k \beta^k \tilde{\gamma}^{ij}$.

Freely specifiable variables :

- K, $\tilde{\gamma}^{ij}$ and $\tilde{u}^{ij}$.
- the Hamiltonian constraint $\Longrightarrow$ equation on Ψ.
- the momentum constraint $\Longrightarrow$ equation on β^i.

Extended thin-sandwich

If those initial data are evolved with N and β^i :

- $\partial_t \tilde{\gamma}^{ij} = \tilde{u}^{ij}$.
- $\partial_t K \Longrightarrow$ elliptic equation on N.

Choice of variables for quasiequilibrium

- maximum slicing : $K = 0$
- equilibrium : $\tilde{u}^{ij} = 0$ and $\partial_t K = 0$.
- conformal flatness : $\tilde{\gamma}^{ij} = f^{ij}$.

This system of equations for N, Ψ and β^i is the same as the one derived using the helical symmetry.

Equations for the fluid

Assumptions

- Irrotational flow : $hu^\nu = \nabla^\nu \Phi$, where h is the specific enthalpy, u^ν the 4-velocity and Φ the potential.
- Cold matter $\Longrightarrow$ the quantities n, e and p are function of $H = \ln h$ only.
- Polytropic equation of state : $p = \kappa n^\Gamma$ (in this work $\Gamma = 2$).

Implications :

- One elliptic equation for the potential

$$\xi H \Delta\Phi + \bar{D}^i H \bar{D}_i \Phi = \Psi^4 h \Gamma_n U_0^i \bar{D}_i H + \xi H \left(\bar{D}^i \Phi \bar{D}_i (H - \beta) + \Psi^4 h U_0^i \bar{D}_i \Gamma_n \right)$$

  where $H = \ln h$, $\xi = d \ln H / d \ln n$, $\beta = \ln\left(\Psi^2 N\right)$ and U_0^i is the 3-velocity of the corotating observer

- One integral of motion $hN \dfrac{\Gamma}{\Gamma_0} = \text{const.}$

The black hole

Boundary conditions

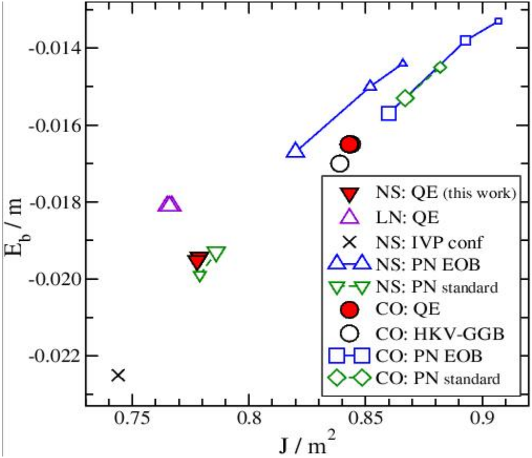
BC are found by imposing apparent horizon conditions on a sphere S with $N|_S = 0$.

- apparent horizon : $\left.\partial_r \Psi + \frac{\Psi}{2r}\right|_S = 0$
- equilibrium : $\beta^i\big|_S = \Omega_r \partial^i_\varphi$

Comments

- $N = 0$ implies a regularization on β^i to ensure regularity of $\tilde{A}^{ij} = 1/2N\left(\bar{D}^i\beta^j + \bar{D}^j\beta^i - 2/3\bar{D}_k\beta^k f^{ij}\right)$
- For a corotating BH : $\Omega_r = 0$.
- Here only irrotational BH $\Longrightarrow$ Ω_r is determined to ensure that the local spin of the BH vanishes.

Influence of the regularization



M. Caudill, G.B. Cook, J.D. Grigsby and H.P. Pfeiffer, Phys. Rev. D, 74, 064011 (2006)

The correction should be even smaller for BH-NS binaries

Additional quantities

Asymptotical flatness $\Longrightarrow$ BC at infinity :

$$N \longrightarrow 1$$

$$\Psi \longrightarrow 1$$

$$\beta^i \longrightarrow \Omega \partial_\varphi$$

Orbital velocity

Ω is determined by equilibrium of the fluid.
The gradient of h is zero at the center of the star.

Numerical methods

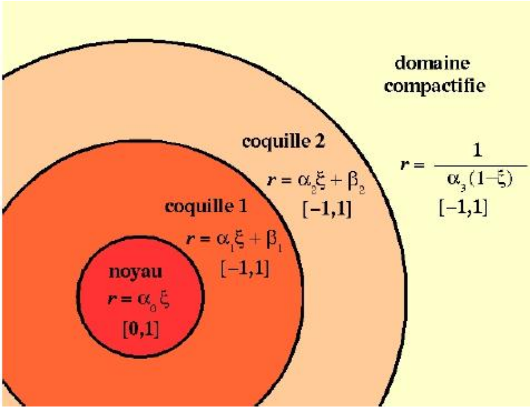
Spectral methods

LORENE `http ://www.lorene.obspm.fr`

Numerical library in C++ to use multi-domains spectral methods

- Cartesian components : $\vec{V} = (V^x, V^y, V^z)$.
- Spherical coordinates around each object : $V^x(r, \theta, \varphi)$.
- The fields are expanded on :
  - trigonometrical polynomials or spherical harmonics for θ and φ.
  - Chebyshev polynomials with respect to r.
- Solving PDEs reduces to matrices inversion.

Spherical domains



Numerical coordinates $(\xi, \theta, \varphi) \longrightarrow$ Physical coordinates (r, θ, φ).

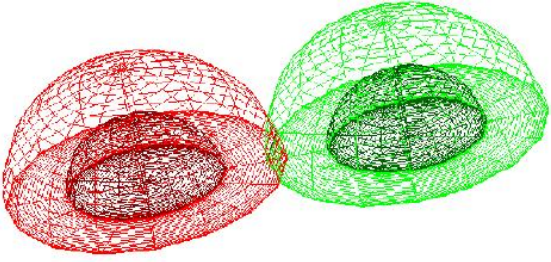
Binary systems

Sources are concentrated around the two objects :

$$\Delta f = S \Longrightarrow \begin{cases} \Delta f_1 & = & S_1 \\ \Delta f_2 & = & S_2 \end{cases}$$

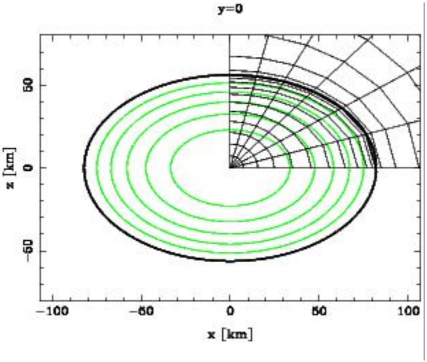
with $S = S_1 + S_2$. The splitting is not unique.

Two sets of domains are used.



Additional properties of the domains

- Spatial infinity is part of the numerical domain by use of $u = 1/r$ in the external domain.
- The grid of the NS is adapted to the surface of the star, where $H = 0$, to avoid discontinuities that would cause the spectral methods to lose their accuracy.



Regularity and boundary conditions are easily enforced.

Iterative scheme

Some parameters must be tuned during the course of the iteration to fulfill some additional conditions :

- Ω_r to get $S_{\rm BH} = 0$.
- h_c to converge to a given baryon mass for the NS $M_{\rm b}$.
- the radius of the BH to get a given irreducible mass $M_{\rm irr}$.
- the position of the rotation axis so that the linear momentum vanishes $P_{\rm tot} = 0$.

Typical run

- The system is solved by iteration until the fields converges to 10^{-7}.
- Resolution : $\mathrm{N}_r \times \mathrm{N}_\theta \times \mathrm{N}_\varphi = 33 \times 21 \times 20$
- 8 domains for each objects.

Comparisons and results

Extreme mass ratio limit

K. Taniguchi, T.W. Baumgarte, J.A. Faber and S.L. Shapiro, Phys. Rev. D, 72, 044008 (2005)

Assumptions

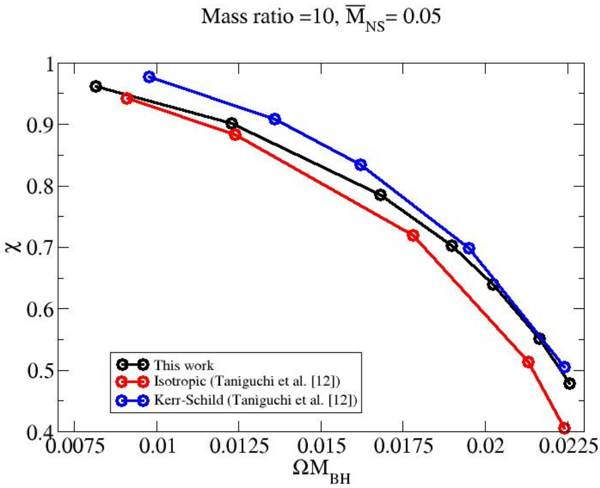
- BH background is fixed.
- Solve only for the NS.

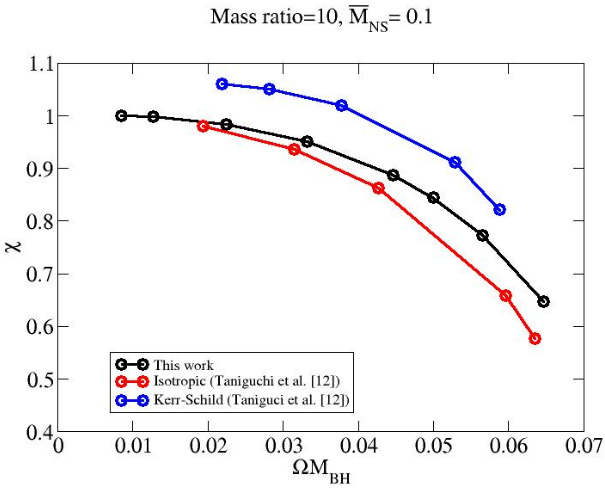
Properties

- Decomposition not unique.
- No access to the binding energy of the binary.
- Should work for high mass ratios.

Results : χ, measure of the deformation of the star, as a function of Ω

- $\chi = 1$ for a spherical star.
- $\chi \rightarrow 0$ at the tidal disruption limit (a cusp is forming).


Mass ratio =10, $\overline{M}_{NS}$= 0.05
χ
1
0.9
0.8
0.7
0.6
0.5
0.4
0.0075
0.01
0.0125
0.015
0.0175
0.02
0.0225
ΩM_{BH}
This work
Isotropic (Taniguchi et al. [12])
Kerr-Schild (Taniguchi et al. [12])

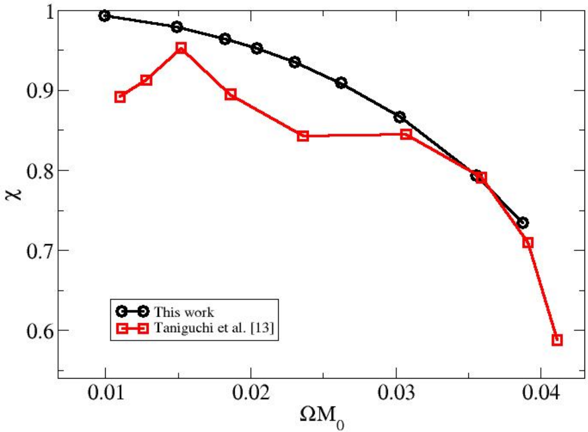

Mass ratio=10, $\overline{M}_{NS}$= 0.1
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
χ
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
ΩM_{BH}
This work
Isotropic (Taniguchi et al. [12])
Kerr-Schild (Taniguci et al. [12])

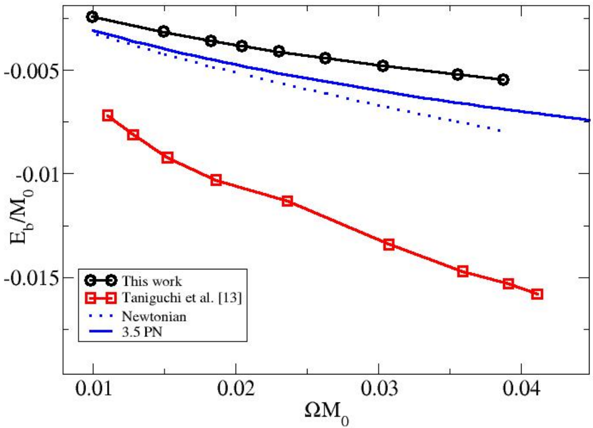
Low compactness limit, mass ratio = 5

K. Taniguchi, T.W. Baumgarte, J.A. Faber and S.L. Shapiro, Phys. Rev. D, 74, 041502 (2006)

Same hypothesis as this work but

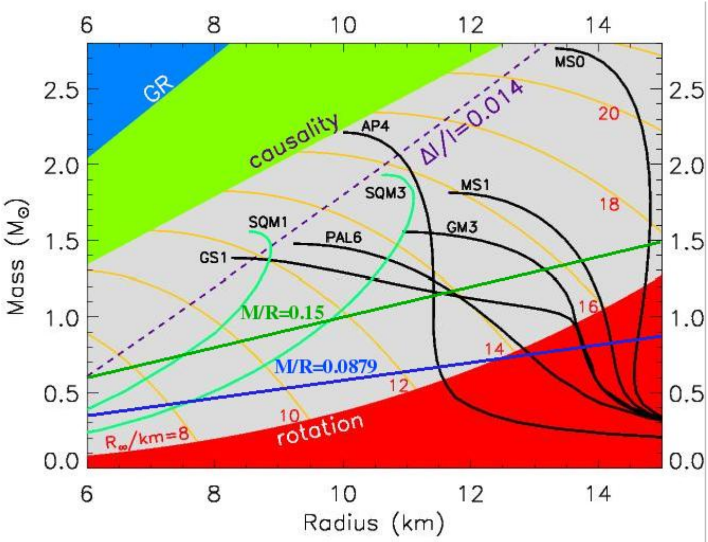
- "freely" specifiable variables from Kerr-Schild.
- Irrotationality imposed only to first order $\Omega_r = \Omega$.
- Better BC on the horizon $N \neq 0$.
- Similar but independent numerics.
- Neutron star has a very low compactness : $\Xi = 0.0879$


1
0.9
0.8
0.7
0.6
χ
0.01
0.02
0.03
0.04
ΩM0
This work
Taniguchi et al. [13]



-0.005
-0.01
-0.015
E_b/M_0
0.01
0.02
0.03
0.04
ΩM_0
This work
Taniguchi et al. [13]
Newtonian
3.5 PN

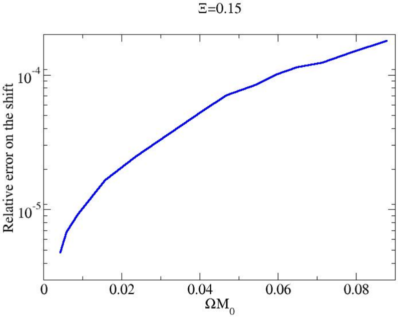
Unrealistic compactness

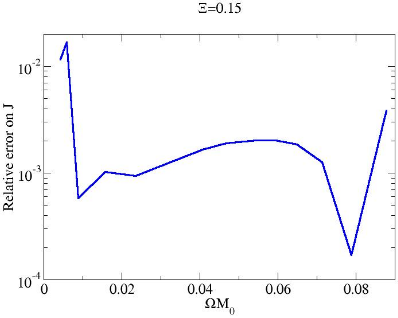


New results

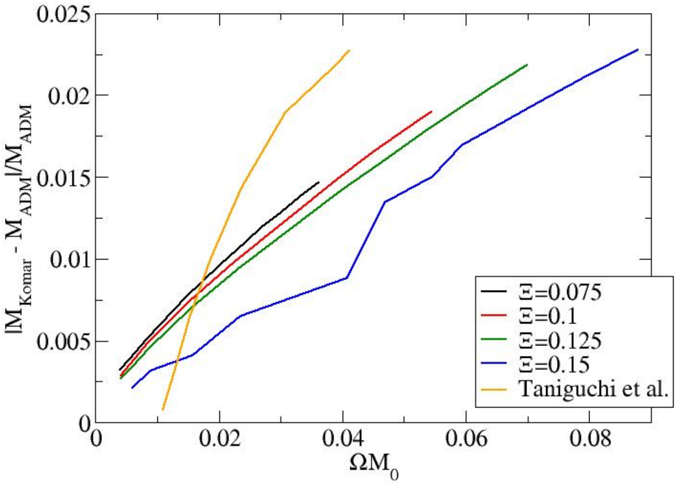
- Mass ratio 5
- Compactness of the NS increased by varying κ in the EOS.
- Sequences of constant M_{irr} and M_{b}.
- Compactness up to $\Xi = 0.15$ (moderate value, not very high).

Regularization of the shift

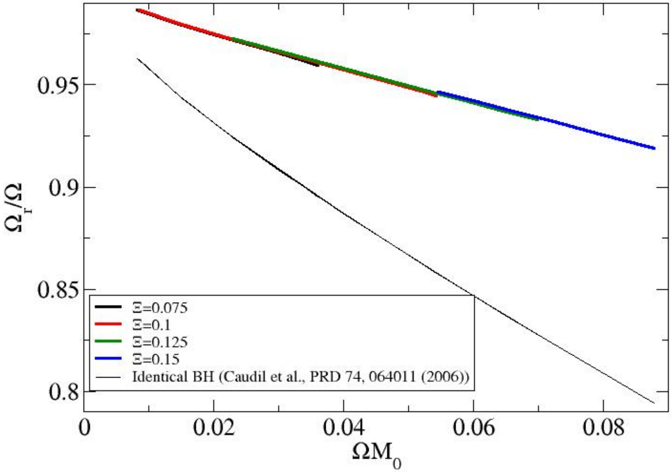




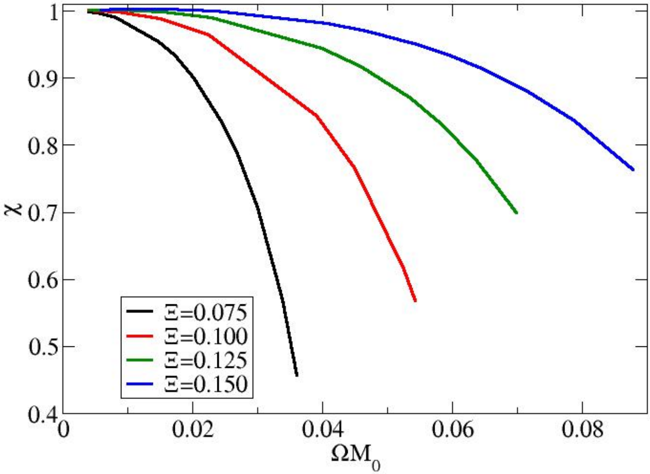
Viriel theorem



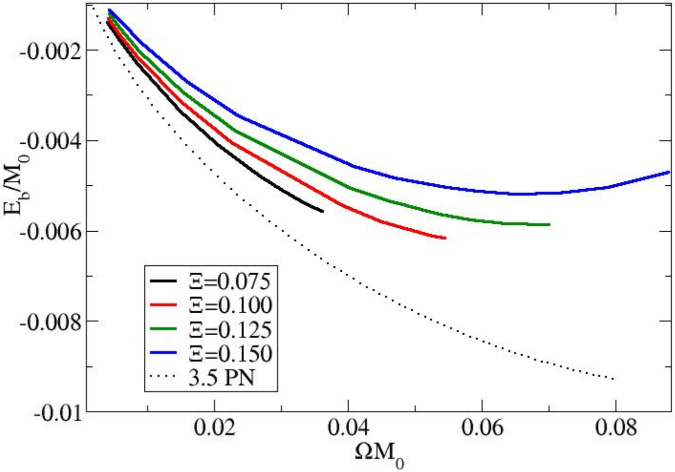
Local rotation rate



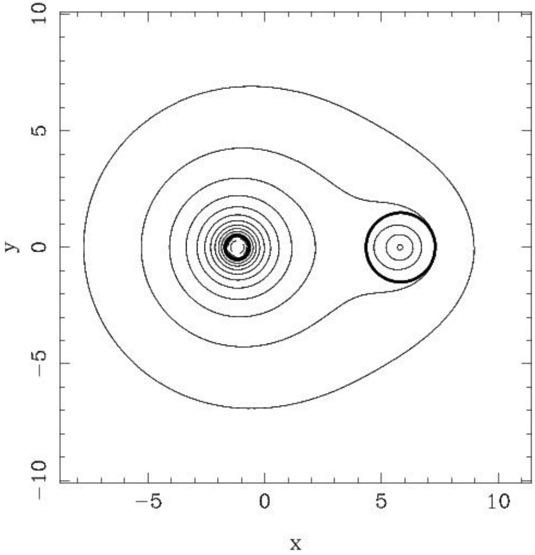
Deformation



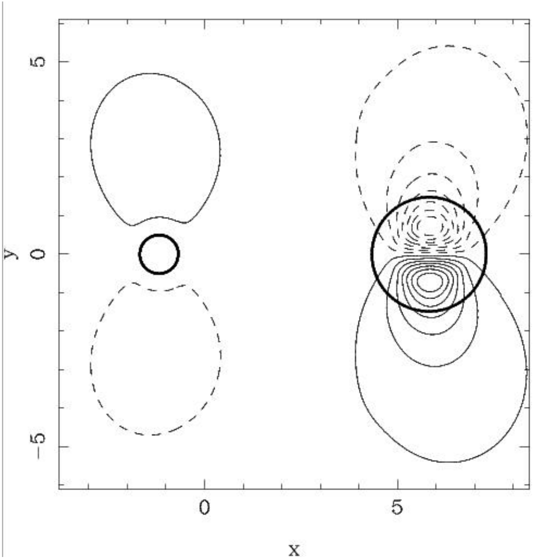
Binding energy



N in the orbital plane, $\Xi = 0.15$



$\tilde{A}^{XX}$ in the orbital plane, $\Xi = 0.15$



Conclusions

Results

- Accurate code for NSBH binaries.
- Good agreement with other works for high mass ratios.
- Somewhat good agreement for low compactness.
- First results for realistic NS.

Future work

- Remove $N|_S = 0$.
- Compute the deviation from conformal flatness.
- Explore the parameter space (EOS, mass ratios).
- Perform time evolution of the data.
- Deduce implications on the emitted GW.