Obstacles in Numerical Calculations

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Paris, November 2006

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Obstacles in Numerical Calculations

General Relativity

Numerical Analysis

Numerical Relativity

Layout

- Hawking Energy
- Ricci tensor and higher derivatives
- Dynamical and Isolated Horizons
- Coordinates

Hawking Energy

- Interesting problem: determine amount of energy contained in the simulated domain
- People usually calculate the ADM mass or related quantities:
  - ADM mass at finite distance
  - as volume integral
  - assuming conformal flatness outside domain
  - approximate Bartnik mass?
- Question: Why don't people calculate the Hawking energy?

Hawking Energy

Simple definition: $E_H = \frac{R}{2}\left(1 - \int \Theta_{(\ell)}\Theta_{(n)}\right)$

Unfortunately, this equation is numerically not well-posed.

Good definition:

$$E_H = \frac{R}{2}\int\left(\sigma\lambda + \bar{\sigma}\bar{\lambda} - \Psi_2 - \bar{\Psi}_2 + 2\Phi_{11} + 2\Lambda\right)$$

[LRR 2004 4]

Hawking Energy

$$E_H = \frac{R}{2}\left(1 - \int \Theta_{(\ell)}\Theta_{(n)}\right)$$

Asymptotic behaviour (large r):

$$\int \Theta_{(\ell)}\Theta_{(n)} \sim 1 - \frac{1}{r}$$

$$E_H \sim r\left(1 - \left[1 - \frac{1}{r}\right]\right)$$

With numerical error:

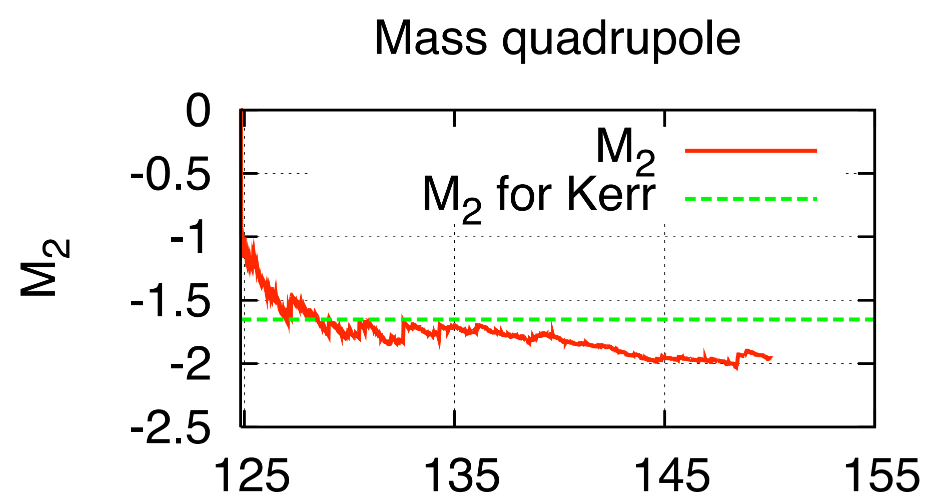
$$\int \Theta_{(\ell)}\Theta_{(n)} \sim 1 - \frac{1}{r} + O(\epsilon)$$

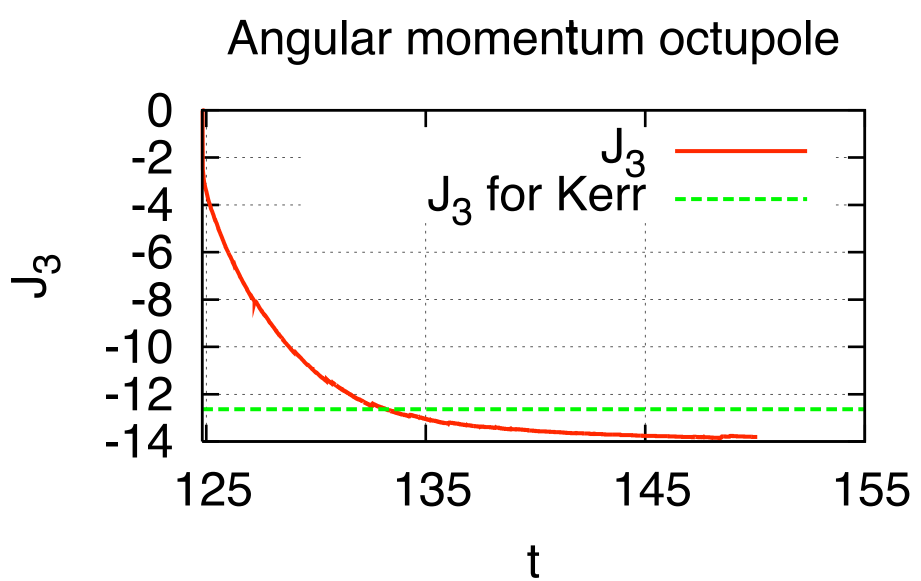
$$\bar{E}_H \sim r\left(1 - \left[1 - \frac{1}{r} + O(\epsilon)\right]\right)$$

$$\bar{E}_H \sim E_H + O(r\epsilon)$$

Noise through Derivatives

- Numerical simulations contain noise. Derivatives amplify noise.
- Formally, d^n/dx^n loses n orders of accuracy
- Empirically, higher than second derivatives are difficult (... with current methods)
- In 3+1 D, resolution is always a problem





Noise through Derivatives

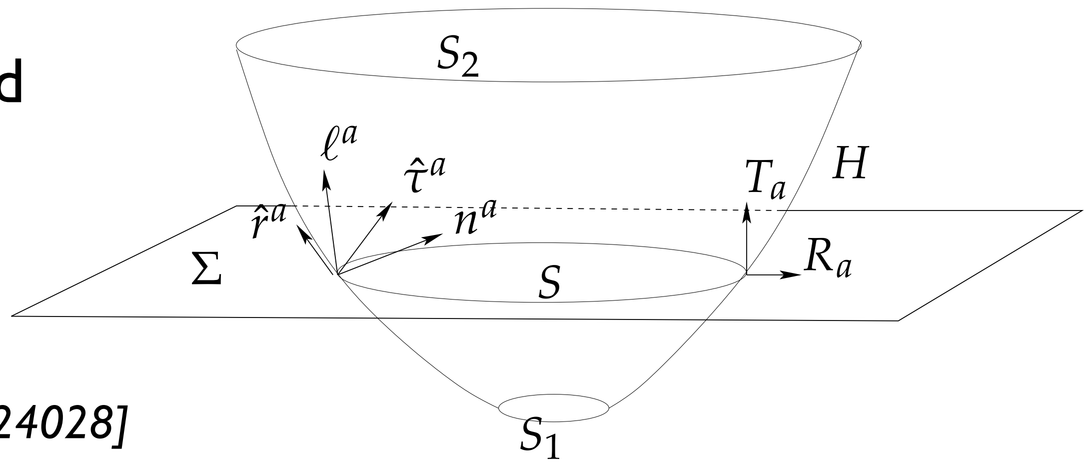
- Goal: Define angular momentum on non-axisymmetric horizons
- Requires: Find a generalisation of a Killing vector field on a horizon
- Idea (Ashtekar?): Use isocontour lines of a 2-scalar on the horizon
- Problem: This would require at least n=4 derivatives
- Which would therefore not work in spacetimes with matter

From DH To IH

- Intuitively, a dynamical horizon will become "more and more null" at late times, becoming isolated "at late times".

- Mathematically, this transition from spacelike to null is not smooth, and does not happen.

- Numerically, the horizon will be indistinguishable from a null surface at some time, and the transition must be handled.



[PRD 74 024028]

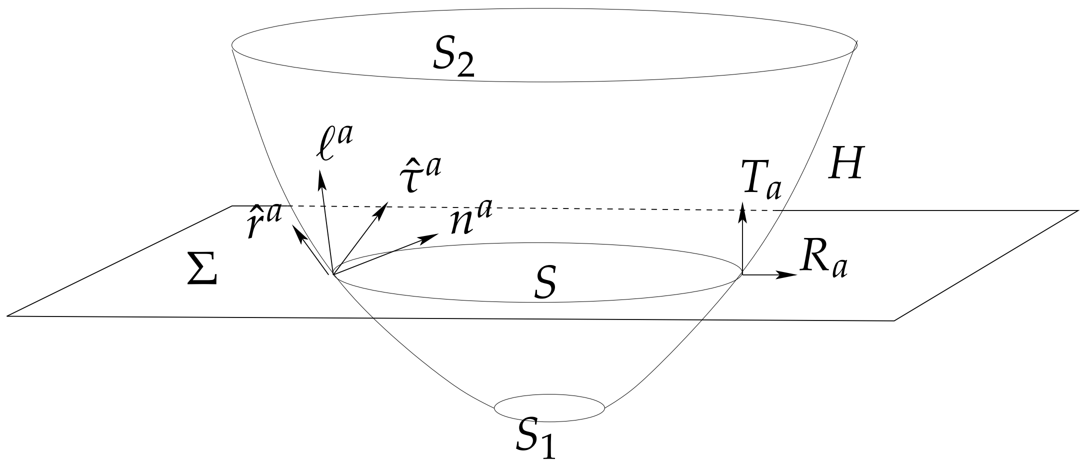
From DH To IH

Relation between normals:

$$\ell = T + R \qquad n = T - R$$

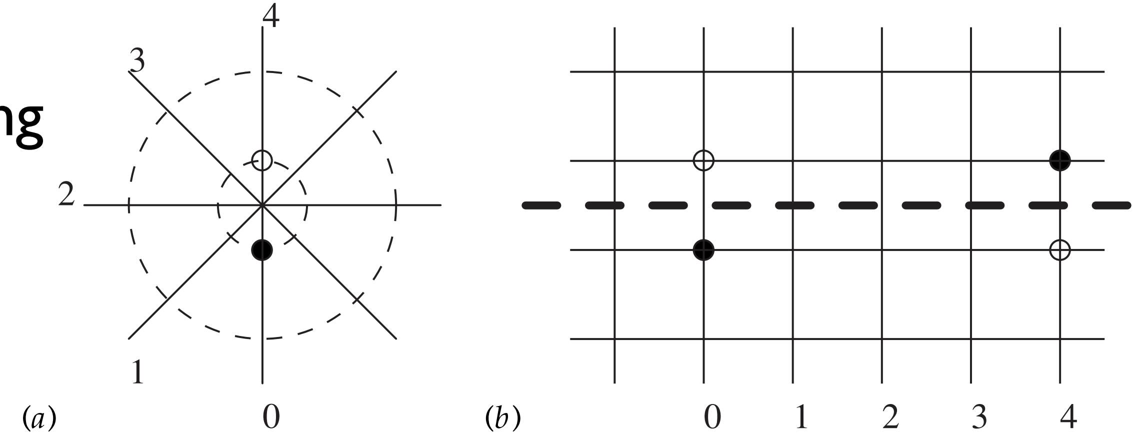
$$\hat{\ell} = \hat{\tau} + \hat{r} \qquad \hat{n} = \hat{\tau} - \hat{r}$$

$$\ell = \alpha\hat{\ell} \qquad n = \hat{n}/\alpha$$



Coordinates

- In numerical work, everything is expressed in terms of coordinates (basis, gauge):
  - domain (grid points)
  - tensors (components)
- Transformations between domains (e.g. from a 3D hypersurface to a 2D surface) require interpolation, which is inaccurate
- Coordinate systems can have singularities; handling multiple maps requires much additional work



[CQG 20 4719]

Coordinates

- In a 3+1 time evolution, the foliation is determined by the gauge conditions, which is chosen according to stability properties

- No one (afaik) has analysed a 3+1 spacetime in a foliation different than the given one

- There would be interesting questions: In a different slicing,
  - how do the trapped surfaces look? what is the total trapped region?
  - do extracted waves change much?
  - do different codes converge pointwise?

Final Thoughts

- There are also some tasks which are easier numerically:
  - Represent arbitrary functions
  - Solve ODEs
  - Integrate (over a given domain)
- I don't want to be blinded by my numerical glasses