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Spin-orbit interactions in black hole binaries

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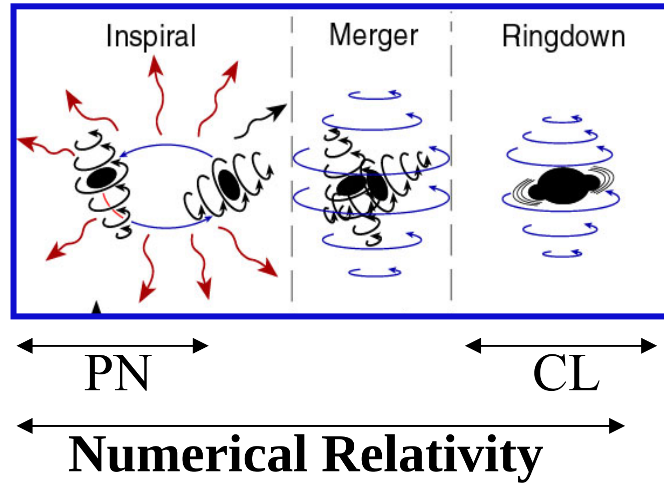
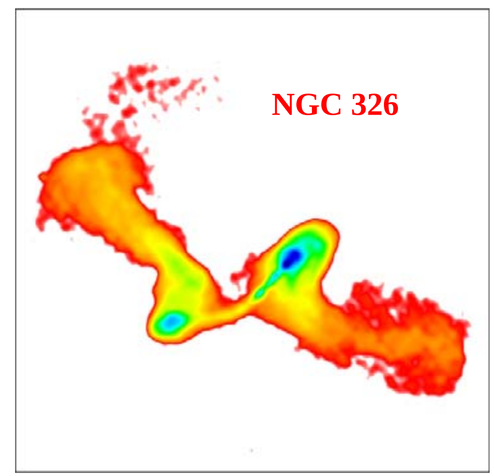
Black hole binary coalescences

- Binary black holes (BBH) of comparable masses are powerful sources of gravitational waves (GW)
- Accurate BBH models (in all phases) are important:
  - Event detection (before GW are detected)
    - Important for LIGO (now taking data at design sensitivity), etc
    - Easier for LISA …
  - Parameter extraction (after GW are detected)
    - Masses, spins, eccentricity of the orbit, etc
- Understanding/testing strong-field gravity in General Relativity (GR)
- Consequences in astrophysics about the formation history of galaxies
  - Recoil (m1≠m2)
    - BH ejection rates from clusters and galaxies
  - Spins
    - Merger population statistics (accretion implies high spin, but mergers at random angles decrease spin)





Spin-flip in X-shaped radio morphologies induced by merger?

Numerical Relativity: 30 years of challenges

1962
(ADM)
3+1 formulation

1964
(Hahn-Lindquist)
2-whormholes

1984
(Unruh)
Excision

1994
(Cook)
Bowen-York initial data

1997
(Brandt-Brügmann)
Puncture initial data
(No Excision)

1999
BSSN evolution system

1999-2000
(AEI/PSU)
Grazing collisions

2004
(Brügmann et al, PSU)
One orbit (corotation)

2005
(Pretorius, Caltech)
***Breakthrough* orbits with harmonic code**

Megaflops — Massive parallel computing resources (flops) — Teraflops — Petaflops

1975-1977
(Smarr-Eppley)
First head-on collision in axysymmetry

1989-1995
(Bona-Masso)
Modified ADM,
(hyperbolicity)

1994-1998
GRAND CHALLENGE

1994-1995
(NSCA-WashU)
Improved head-on collision

LIGO (NSF)

2000-2004
(AEI/UTB-NASA)
Revive crashing codes
Lazarus waveforms!

2000-2002
(Alcubierre, AEI/UNAM)
gauge conditions

2002-2005
(Cornell, Caltech, LSU etc)
1st order formulations
(hyperbolicity!)

2005-2006
(UTB/NASA)
Breakthroughs
Multiple orbits with puncture data

The Lazarus results

Baker, Bruegmann, Campanelli, Lousto, Takahashi, PRL (2001). [gr-qc/0102037]

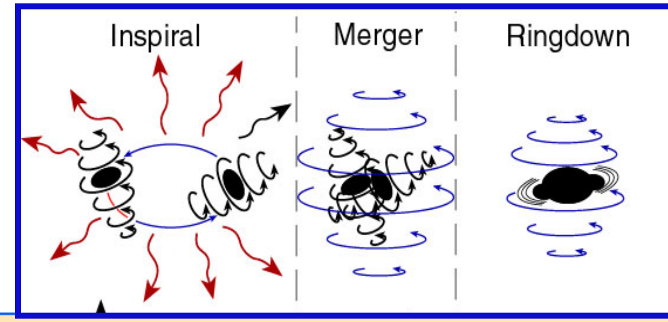
- New hybrid method which uses NR combined with black hole perturbation theory in the ringdown phase

- The first waveforms (for equal-mass, non-spinning BBH) are relatively simple …

- The energy and angular momentum losses during the plunge phase of equal mass non spinning holes are respectively ~ 3% and 15%

- The rotation parameter of the final Kerr hole is $a/M \sim 0.7$ (non-spinning, moderately spinning holes)

- Lazarus: a success, but concerns remain about accuracy (complexity of the interfaces) and the choice of initial data …



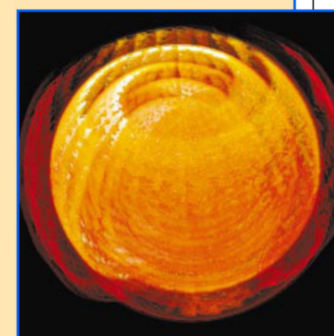
Gravitational physics

Black hole blockbuster

What happens when one black hole hits another black hole? Does it get more black? Seriously, though, such questions are being tackled by scientists seeking to detect gravity waves. These invisible waves are ripples in the fabric of space-time, predicted by Einstein's theory of relativity, but that remain undetected. They are thought to be emitted in copious amounts by colliding black holes, and a new simulation by researchers in Germany (J. Baker *et al. Phys. Rev. Lett.* **87,** 121103; 2001) gives an advance preview.

Collisions between other astronomical giants, such as galaxies, produce light and other radiation, but black-hole collisions generate only gravity waves. Black hole binaries are thought to emit gravity waves all the time, but only when they collide are the waves strong enough to be detected on Earth. Three detectors are expected to start collecting data soon: the US LIGO and German–British GEO600 projects in 2002, and the Italian–French VIRGO detector in 2003.

To fully simulate the merger of two black holes, the German team merged — appropriately enough — two different approaches for calculating what might happen before and after the collision. In the computer-generated image shown here, spherical shells of intense gravity waves move outwards from the centre of the collision. The authors estimate that 3% of the total mass of the black holes is released as energy by the collision — higher than expected.



W. BENGER, AEI/ZIB

These calculations should provide experimenters with a rough estimate of what to look out for, and guide more advanced simulations that take into account, for example, the likelihood that the black holes are also spinning. Prepare for a glimpse of the darkest corners of the Universe. **Sarah Tomlin**

NATURE|Vol.413|4October 2001|www.nature.com

Why it has been and is so challenging …

- Determination of BBH initial data is highly non-trivial:
  - Elliptical constraints expensive to solve
  - Astrophysically realistic conditions …
- There are a multitude of formalisms (systems) for the evolution equations:
  - Choice of the dynamical variables (1^{st} or 2^{nd} order forms)
  - Role of constraints (e.g. constraints can be added to field equations)
  - Choices of the coordinates or gauges
- The choice of the system has a significant impact on the well-posedness, as well as ability to compute stable (convergent) and accurate solutions
- Black hole interiors:
  - Excision (inner boundary conditions …)
  - Evolved with singularity avoiding slices (`puncture approach')
- Outer boundary conditions:
  - Not known … use Sommerfeld (radiative) boundary conditions for all variables
- Variable grid resolution to handle multiple scales:
  - Resolve the dynamics near the BH horizon as well as gravitational radiation $\rightarrow \lambda_{GW} \sim (10 - 100)M$
  - Units: $c = G = 1 \rightarrow 1\ M \sim 5 \times 10^{-6}\ (M/M_{\odot})$ sec $\sim 1.5\ (M/M_{\odot})$ km
  - Adaptive Mesh Refinement (AMR) Techniques, Higher-order finite difference (HOFD), Pseudo-spectral methods etc

UTB / NASA 2005 the year of the breakthrough: Moving Punctures

Campanelli et al., PRL, 96, 111101 (2006), [gr-qc/0511048]
Baker et al., PRL, 96, 111102 (2006), [gr-qc/0511103]

In late 2005, UTB and NASA Goddard, independently introduced a new approach based on the 3+1 formulation of Einstein's equations, known as `moving punctures':

- Uses conformal BSSN formalism with punctures (no excision)
- Do not split off singular part Ψ_{BL} but *absorb it in the BSSN conformal factor* Φ
  - NASA discretize Φ directly …
  - UTB uses non singular $\chi = \exp(-4\Phi)$
- No corotation, instead punctures move across the grid with new (different in each group) gauge conditions for α & β^i
- High-resolution codes: '4th order + Fisheye' at UTB, AMR at NASA Goddard.
- Enables long term, accurate simulations

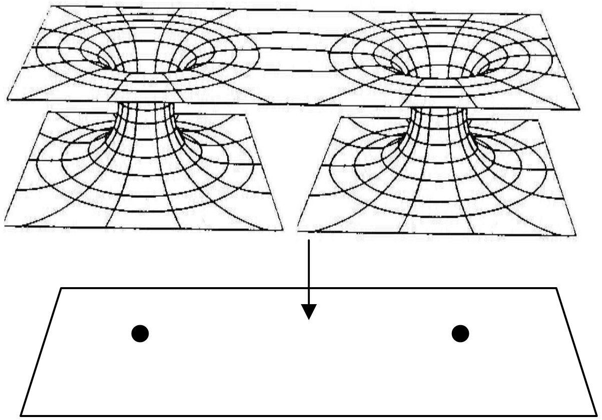
- UTB and NASA move ahead quickly (paper every 2 months in each group):
  - multiple orbits, unequal-mass BBH merger + kicks, spin-orbit effects

- Immediately adopted by other groups: PSU, FAU, Jena, UNAM, AEI, LSU etc
  - At the April 2006 APS meeting an entire session is devoted to `moving punctures'
  - Now not only BBHs but also BH-NS binaries: Shibata-Uryu, Rezzolla et al

From Lazarus to Galileo: the `moving punctures' approach

- Campanelli, Lousto, Marronetti, Zlochower (UTB), PRL (2006)
- Baker, Centrella, Choi, Koppitz, van Meter (NASA Goddard), PRL (2006)

In late 2005, shortly after Pretorius breakthrough results, UTB and NASA Goddard, independently introduced a new approach based on the 3+ 1 formulation of Einstein's equations, known as `moving punctures'



- Punctures (no excision)
- Standard BSSN formulation
- 1+log slicing, modified Γ-driver shifts
- No corotation, *instead allow the punctures to move by absorbing singularities in the BSSN conformal factor Φ*
  - NASA discretize Φ directly …
  - UTB uses non singular χ=exp(-4Φ)
- High-resolution codes ('4th order + Fisheye' or AMR).

$$\gamma_{ab} = (\psi_{BL} + u)^4 \delta_{ab}$$

$$\psi_{BL} = 1 + \sum_{i=1}^{n} m_i/(2r_i)$$

Immediately adopted by many groups: UTB, NASA, PSU, FAU, Jena, UNAM, AEI, LSU etc

- Why the moving punctures work? *Hannam et al (Jena), gr-qc/0606099*
- Strongly hyperbolicity of the system, *Gundlach et al, gr-qc/0604035*

'E pur si mouve' (*Galileo*)

The conformal BSSN system with moving punctures

Modified BSSN system:

$$\partial_0 \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij}, \qquad \partial_0 = \partial_t - \mathcal{L}_\beta,$$

$$\partial_t \chi = \frac{2}{3}\chi\left(\alpha K - \partial_a \beta^a\right) + \beta^i \partial_i \chi,$$

$$\partial_0 \tilde{A}_{ij} = \chi\left(-D_i D_j \alpha + \alpha R_{ij}\right)^{TF} + \alpha\left(K\tilde{A}_{ij} - 2\tilde{A}_{ik}\tilde{A}^k_j\right),$$

$$\partial_0 K = -D^i D_i \alpha + \alpha\left(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2\right),$$

$$\partial_t \tilde{\Gamma}^i = \tilde{\gamma}^{jk}\partial_j\partial_k\beta^i + \frac{1}{3}\tilde{\gamma}^{ij}\partial_j\partial_k\beta^k + \beta^j\partial_j\tilde{\Gamma}^i - \tilde{\Gamma}^j\partial_j\beta^i + \frac{2}{3}\tilde{\Gamma}^i\partial_j\beta^j - 2\tilde{A}^{ij}\partial_j\alpha + 2\alpha\left(\tilde{\Gamma}^i{}_{jk}\tilde{A}^{jk} + 6\tilde{A}^{ij}\partial_j\phi - \frac{2}{3}\tilde{\gamma}^{ij}\partial_j K\right),$$

$$\tilde{\Gamma}^i = -\partial_j \tilde{\gamma}^{ij}.$$

Replace ϕ $(O(\log r))$ with $\chi = e^{-4\phi}$ $(O(r^4))$
$\partial_0 \alpha = -2\alpha K$
$\partial_t \beta^a = B^a$, $\partial_t B^a = 3/4\,\partial_t \tilde{\Gamma}^a - \eta B^a$
$\alpha(t=0) = \psi_{BL}^{-2} \qquad \beta^i = B^i = 0.$

Numerical Code: LazEv

- Modular
  - Cactus-based framework
- Flexible
  - *Mathematica* scripts used to generate C routines (257108 lines)
- Use 4th order finite differencing with MoL integration
  - standard 4th order centered stencils for all derivatives
  - upwinded 4th order stencils for the advection (shift) terms
  - standard 4th order RK for time evolution

Gauge Choices

Gundlach and Martin-Garcia (2006)

	ζ_L	ζ_S	ζ_b	ζ_f	μ_L	μ_S	speeds	strongly hyperbolic for
Campanelli *et al.*	0	1	1	1	$\frac{2}{\alpha}$	$\frac{3}{4}\alpha^{-2}\gamma^{1/3}$	$(0, \pm 1, \pm\sqrt{\mu_L}, -\bar{\beta}_n, \lambda_{2\pm}, \pm\sqrt{\mu_S}$	$\lvert\bar{\beta}\rvert < \lvert(3\mu_L - 4\mu_S)/(3\sqrt{\mu_L})\rvert, \ \lvert\bar{\beta}\rvert < \sqrt{\mu_S}$
Baker *et al.*	0	1	1	0	$\frac{2}{\alpha}$	$\frac{3}{4}\alpha^{-1}\gamma^{1/3}$	$(0, \pm 1, \pm\sqrt{\mu_L}, \lambda_{3\pm}, \lambda_{4\pm})$	$\lvert\bar{\beta}\rvert < \sqrt{4\mu_S/3}, \ \lvert\bar{\beta}\rvert < \lvert\sqrt{\mu_L} - \sqrt{4\mu_S/3}\rvert$
Diener *et al.*	1	1	1	1	$\frac{2\psi_{BL}^{\beta_m}}{\alpha}$	$\frac{3}{4}\frac{\alpha^{p-2}}{\psi_{BL}^{n}}\gamma^{1/3}$	$(0, \pm 1, -\bar{\beta}_n, \lambda_{1\pm}, \lambda_{2\pm}, \pm\sqrt{\mu_S})$	$\mu_L \neq 4\mu_S/3, \ \lvert\bar{\beta}\rvert < \sqrt{\mu_S}$
Herrmann *et al.*	1	1	1	0	$\frac{2}{\alpha}$	$\frac{3}{4}\alpha^{-2}\gamma^{1/3}$	$(0, \pm 1, -\bar{\beta}_n, \lambda_{1\pm}, \lambda_{3\pm}, \lambda_{4\pm})$	$\lvert\bar{\beta}\rvert < \lvert(3\mu_L - 4\mu_S)/(2\sqrt{3\mu_S})\rvert$
Beyer-Sarbach	0	0	0	0	f	GH	$(0, \pm 1, \pm\sqrt{\mu_L}, \pm\sqrt{\mu_S}, \pm\sqrt{4\mu_S/3})$	$\mu_S \neq \mu_L, \ \mu_S \neq 3\mu_L/4$

TABLE I: Parameter values, characteristic speeds, and conditions for strong hyperbolicity for the four "puncture evolution"

NASA-Goddard (2006)

$$\partial_t \beta^i = \frac{3}{4}\tilde{\Gamma}^i + \beta^j \partial_j \beta^i - \eta \beta^i$$

Spinning black-hole binaries: the orbital hang-up

Campanelli, Lousto, Zlochower, PRD. [gr-qc/0604012]

Equal masses, a/m= -0.75 (S- -), 0.0 (S00), +0.75 (S++) with total $J/M^2>1$

Initially $M\Omega = 0.05$ → $T_{orbital} \sim 125M$ (other orbital parameters from 3PN)

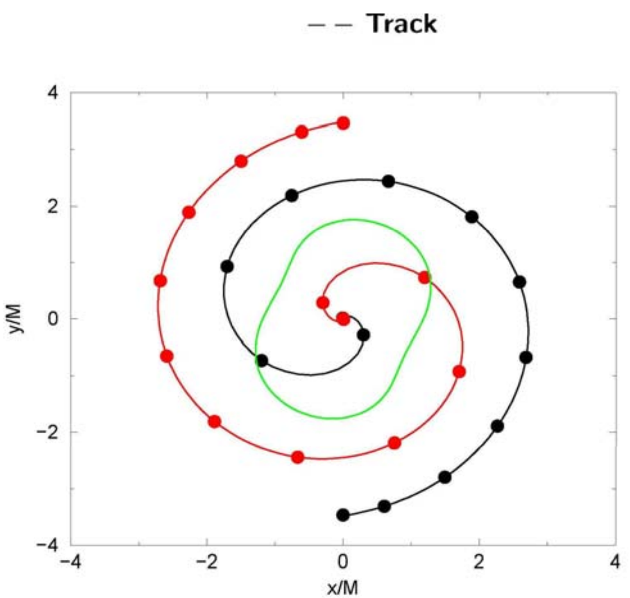


Figure 4: Puncture tracks for the − − configuration.

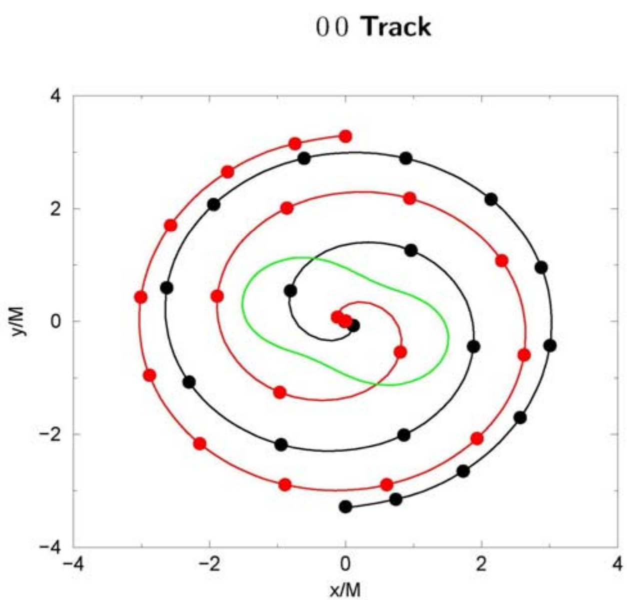


Figure 5: Puncture tracks for the 0 0 configuration.

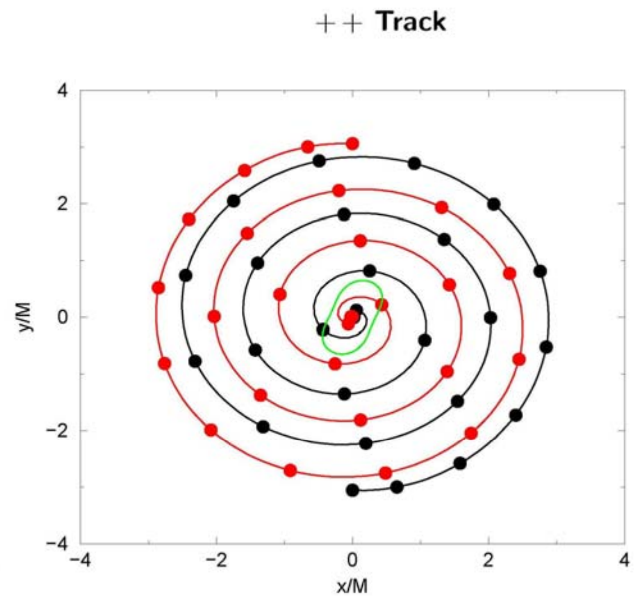


Figure 6: Puncture tracks for the + + configuration.

Spin-orbit coupling effects:

- S - - (unaligned) case: early merger ~ 1 orbit → a/M=0.44
- S00 (non-spinning) case: complete ~1.75 orbits → a/M=0.68
- S++ (aligned) case: hang-up ~ 3.2 orbits → a/M=0.89
- Extrapolating to maximal individual spins → a/M=0.97

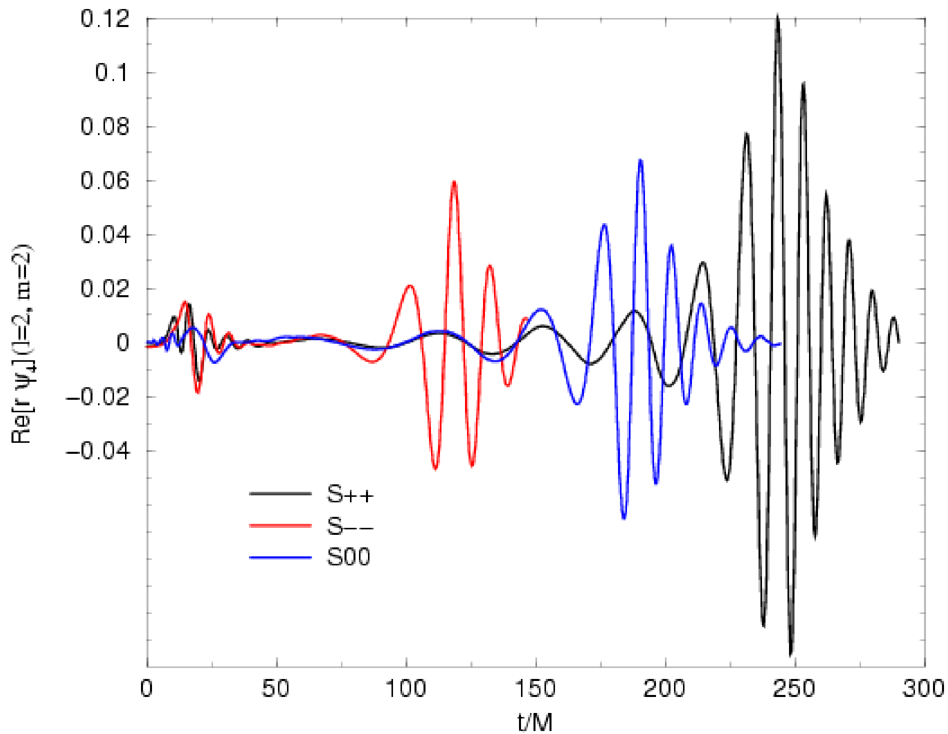


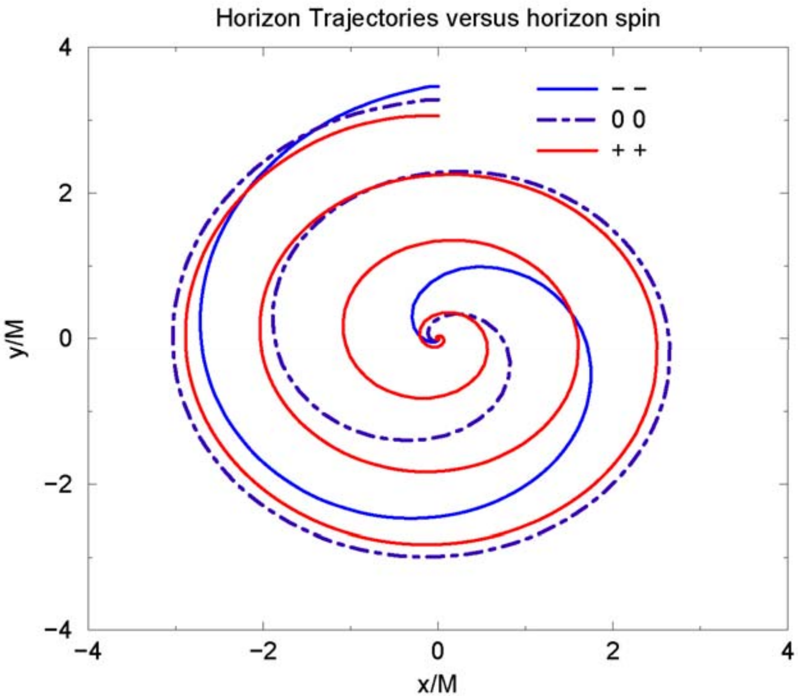
Table 1: Results of the evolution as determined from the waveform and the remnant horizon.

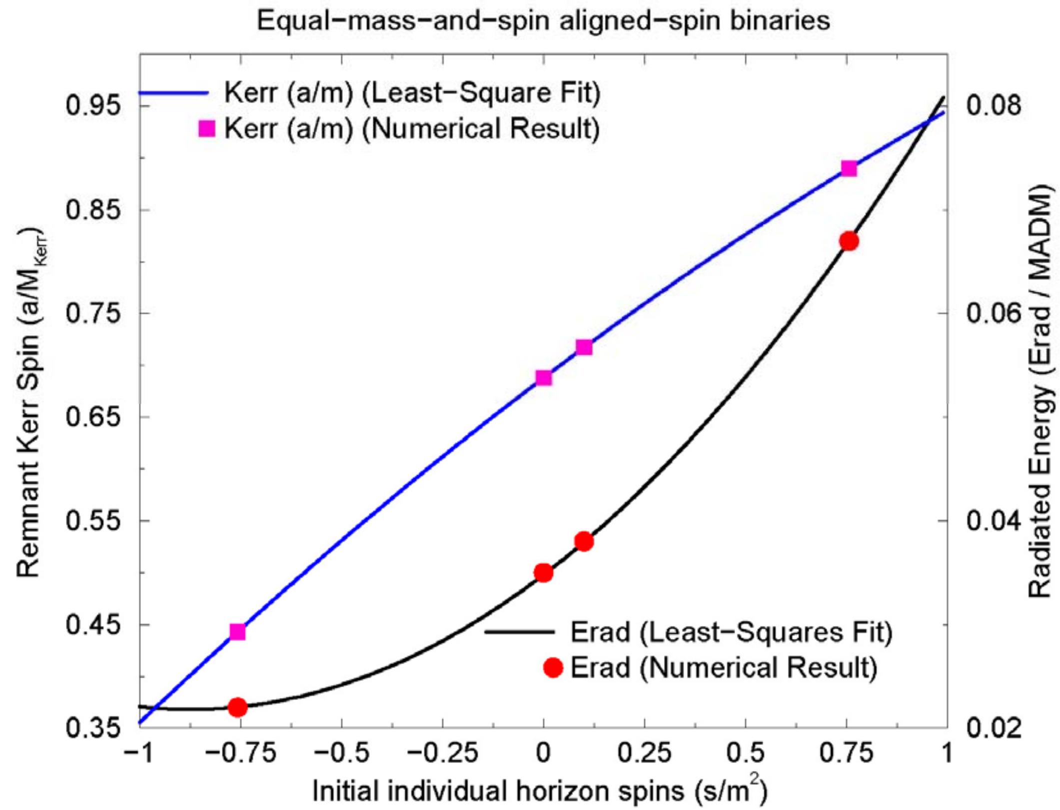
Config	E_{rad}/M_{ADM}	J_{rad}/J_{ADM}	T_{cah}/M	$a/M_{\mathcal{H}}$
$++$	$(6.5 \pm 0.1)\%$	$(33.8 \pm 1.5)\%$	≈ 232	0.892 ± 0.002
$0\,0$	$(3.51 \pm 0.01)\%$	$(26.9 \pm 0.1)\%$	≈ 161	0.688 ± 0.001
$--$	$(2.1 \pm 0.1)\%$	$(26 \pm 2)\%$	≈ 105	0.44 ± 0.01

Extrapolating to maximal individual spins we get $a/M^2 = .976$ (linear) and $a/M^2 = .952$ (quadratic).

The cosmic censorship is respected … unfortunately!

The effect of spins …





- Gravitational radiation and merger time are strongly affected by the value and direction of each individual BH spins (Campanelli, Lousto, Zlochower, gr-qc/060412, astro-ph/0608275)

- Note that the GW energy emitted for highly spinning binaries (with aligned spins) can increases by almost a factor 3, while inspiral last at least twice as long as in the non spinning case …

Fittings

$$(a/M_H)|_R = 0.6879 + 0.2952\,((a/m)|_I) - 0.0374\,((a/m)|_I)^2\,,$$

$$\frac{E_{rad}}{M} = 0.0348 + 0.0297\,((a/m)|_I) + 0.0170\,((a/m)|_I)^2\,,$$

TABLE VI: The measured remnant horizon specific spin $(a/M_H)|_R$ and energy radiated E_{rad}, as well as the predicted remnant horizon specific spin $(a/M_H)|_{pred}$ and energy radiated $E_{rad}/M|_{pred}$ (based on a least-squares fit) for quasi-circular, equal-mass, equal-spin binaries with initial specific spins $(a/m)|_I$.

$(a/m)\|_I$	$(a/M_H)\|_R$	$(a/M_H)\|_{pred}$	E_{rad}/M	$E_{rad}/M\|_{pred}$
-0.757	0.443 ± 0.001	0.4430	$(2.2 \pm .01)\%$	2.20%
0.000	0.688 ± 0.001	0.6878	$(3.5 \pm 0.1)\%$	3.48%
0.1001	0.717 ± 0.001	0.7169	$(3.8 \pm 0.1)\%$	3.79%
0.757	0.890 ± 0.001	0.8900	$(6.7 \pm 0.2)\%$	6.70%
-1.0	* * *	0.355	* * *	2.2%
$+1.0$	* * *	0.946	* * *	8.1%

Spin-orbit interactions in black hole binaries

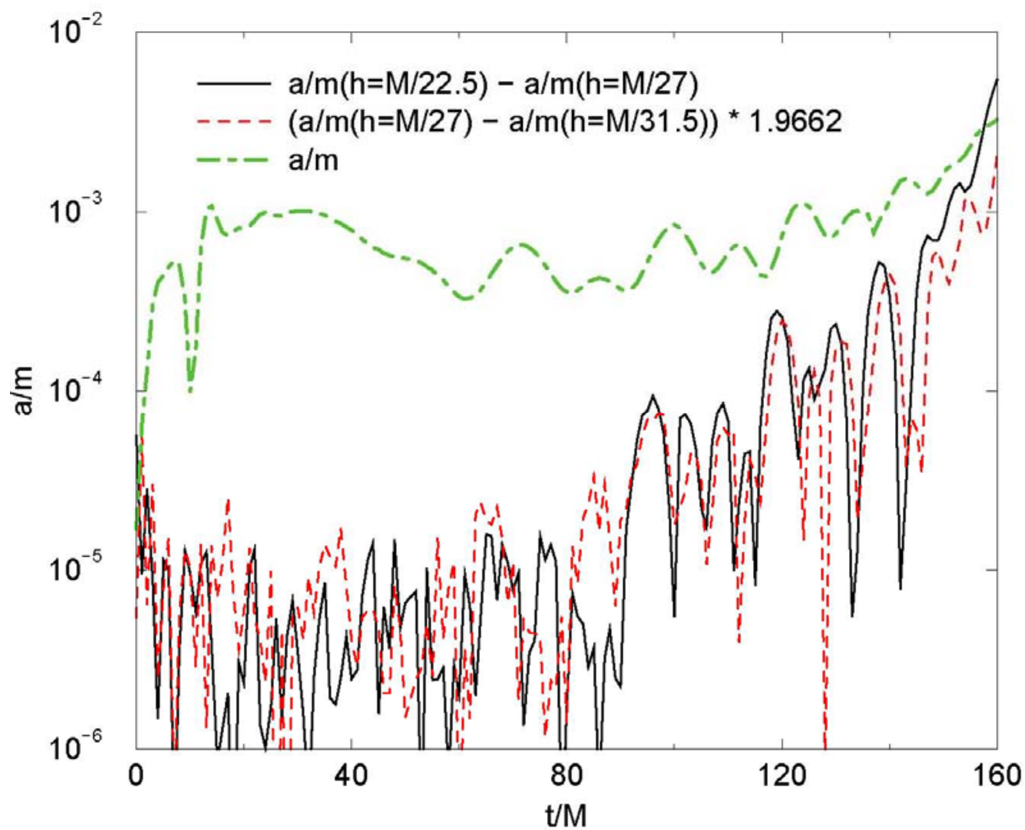
Campanelli, Lousto, Zlochower, PRD [astro-ph/0608275]

Can tidal effects spin-up the holes to the orbital frequency, or equivalently lock the spins of the holes to a corotation state? Tidal effects stronger in the merger stage …

We calculate the spin-up of the holes with the isolated horizon algorithm developed by Dreyer et al, PRD (2003) [qr-qc/0206008]:

$$S = \frac{1}{8\pi}\oint_{AH}(\varphi^a R^b K_{ab})\,\mathrm{d}^2V.$$

We can measure spins of the order of a/M~10^{-3} with an accuracy of 1% or better for L≥4.5M and of 20% for L~3M

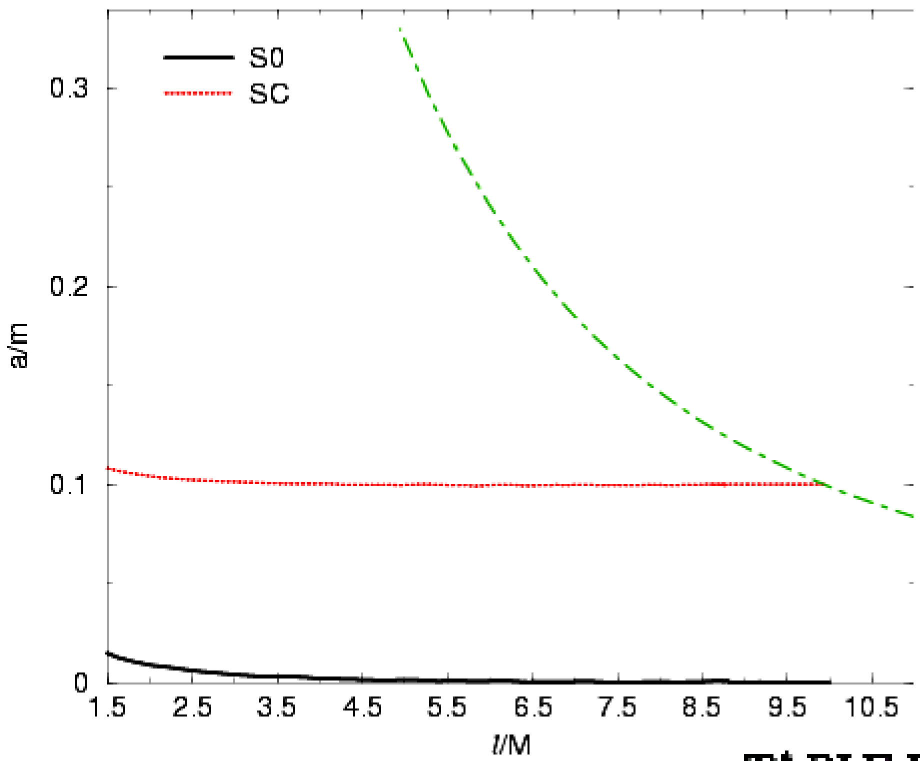


Spin-up of the individual black-hole horizons in the S00 (0 initial spin)

Name	S/M^2	Y/M	P/M	J/M^2	$M\Omega$	m_p/M	l/M
S00	0.000	3.280	0.1336	0.876	0.0500	0.4848	10.01
S0.1	0.025757	3.2534	0.132957	0.917	0.0500	0.483115	9.93

Spin-orbit interactions in black hole binaries

Campanelli, Lousto, Zlochower, PRD [astro-ph/0608275]

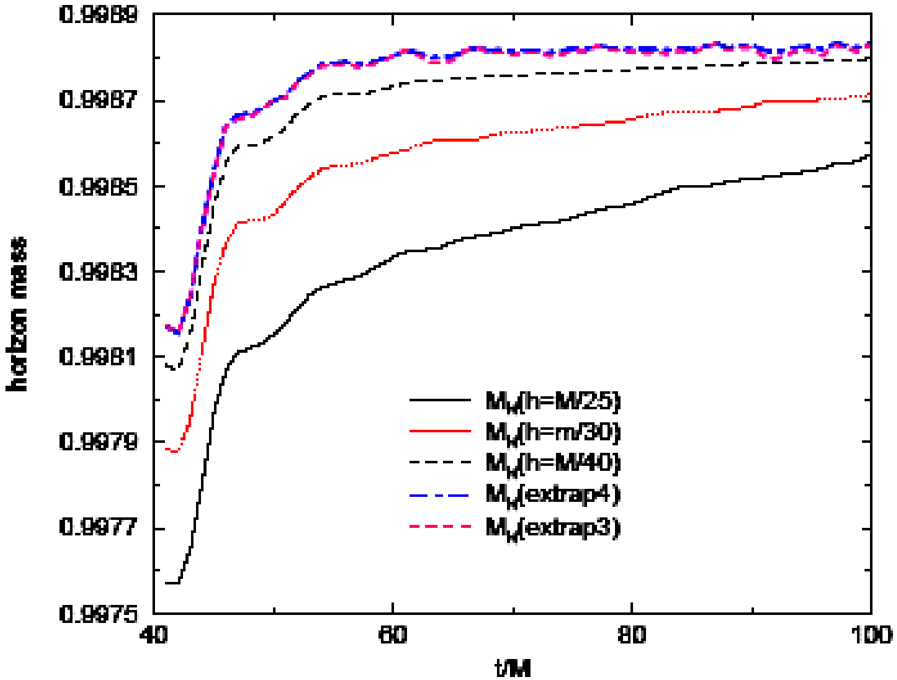


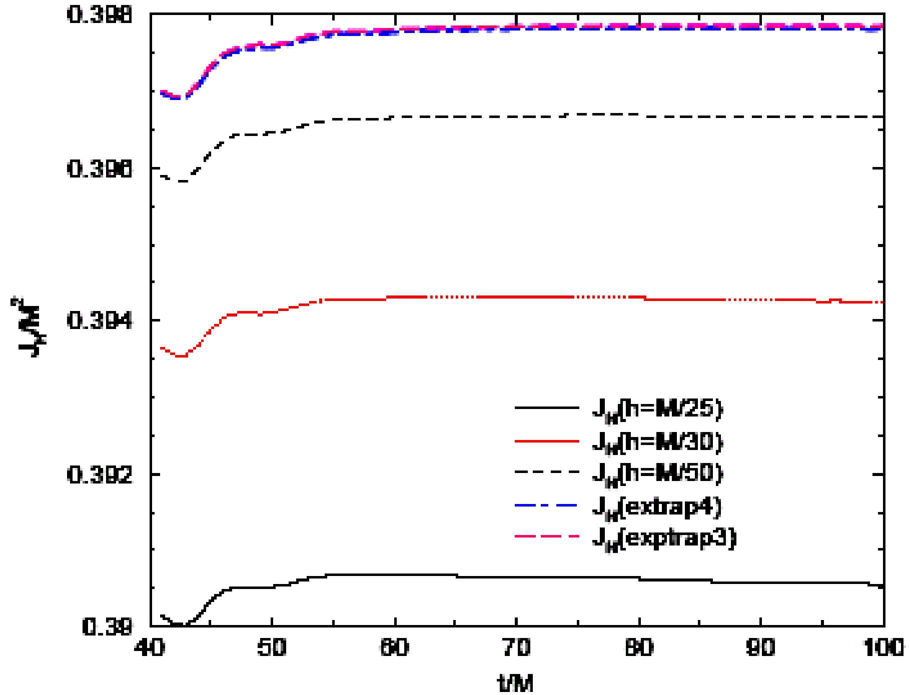
The values that we obtain for the spin-up of the binary holes (in the S00 and S0.1 cases) are two order of magnitude smaller that those expected for a corotation state!

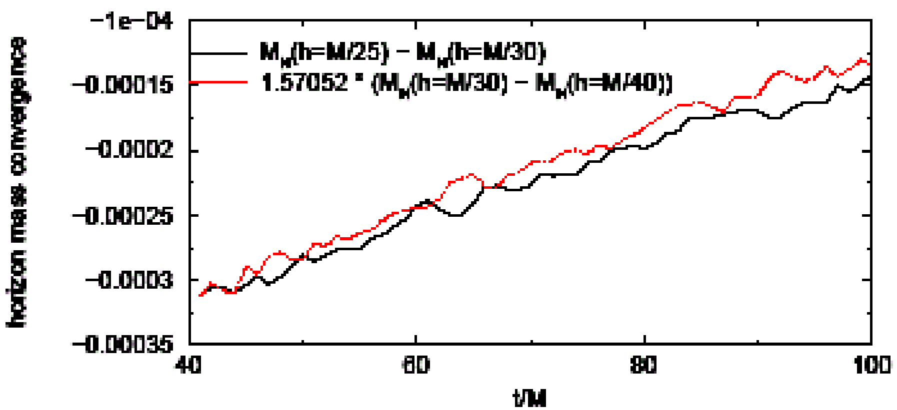
TABLE III: Merger time, T_{CAH}/M, for the S0 and SC configurations versus resolution. Horizon searches were performed every $0.3M$ and $0.2M$ for the S0 and SC configuration respectively.

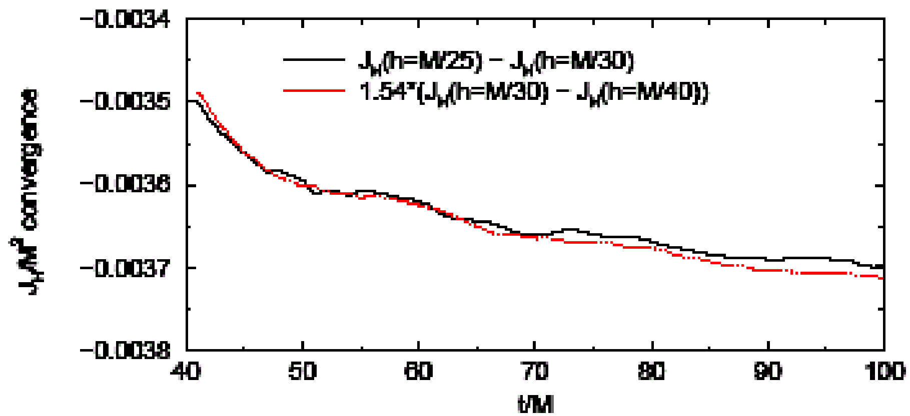
resolution	S0	SC
M/22.5	160.7 ± 0.3	168.6 ± 0.2
M/27	166.0 ± 0.3	174.2 ± 0.2
M/31.5	168.3 ± 0.3	176.6 ± 0.2
extrapolation	172 ± 2	179 ± 2

Accuracy of the method: Spinning BHs (from rest)









Accuracy of the Method

Case	resolution	E_{rad}/M_{ADM}	J_{rad}/J_{ADM}
HS++	M/25	$(0.16 \pm 0.02)\%$	$(2.37 \pm 0.02)\%$
	M/30	$(0.136 \pm 0.009)\%$	$(1.44 \pm 0.01)\%$
	M/40	$(0.123 \pm 0.004)\%$	$(0.832 \pm 0.003)\%$
	extrapolation	$(0.118 \pm 0.002)\%$	$(0.546 \pm 0.002)\%$
MS++	M/25	$(0.065 \pm 0.004)\%$	$(1.038 \pm 0.008)\%$
	M/30	$(0.063 \pm 0.003)\%$	$(0.7698 \pm 0.0040)\%$
	M/35	$(0.062 \pm 0.002)\%$	$(0.6567 \pm 0.0069)\%$
	extrapolation	$(0.060 \pm 0.002)\%$	$(0.4986 \pm 0.0086)\%$
OS++	M/22.5	0.0616 ± 0.0032	0
	M/25	$(0.0595 \pm 0.0025)\%$	0
	M/30	$(0.057 \pm 0.002)\%$	0
	extrapolation	$(0.054 \pm 0.002)\%$	0

Campanelli, Lousto & Zlochower, PRD74:084023,2006

Conclusions

- Remarkable progress in last year:
  - Moving punctures approaches (UTB and NASA) quickly adopted with very minor changes by several groups, including PSU, FAU, Jena, LSU, AEI, and UNAM
  - A variation of the harmonic approach (Pretorius) now adopted by Caltech/Cornell groups (adapted to 1st order formulation, spectral code etc)
- Waveforms for equal-mass non-spinning BBH merger appear to be 'universal'
  - The merger is relatively insensitive to small changes of the initial data parameters!
  - Not true for the orbital dynamics (small ellipticity in all initial data)
- Multiple orbits (five-ten) are necessary to explore overlapping with PN results
  - Accuracy in the phase important …
  - Work in progress at UTB/FAU to built PN initial data for puncture evolution
- We also started to explore the parameter space:
  - NASA, PSU, Jena (FSU), etc $\longrightarrow$ unequal-mass BBH mergers
  - UTB, etc $\longrightarrow$ spinning BBH mergers
- Most groups are now limited by:
  - Computational resources …
  - Sophisticated software algorithms to improve accuracy (AMR)
- Numerical relativity is finally entering a golden age of applications!

Supercomputers:

UTB used a 70-node Linux cluster, Funes, built at the beginning of 2004, thanks to the support of a NASA University Research grant and NSF grid computing projects. Each node is dual Pentium Xeon 3.2 Ghz processors with 8 Gb of RAM, 2 x 120 Gigabyte hard drives, and is interconnected through a gigabit network.

NASA used Columbia, the fourth fastest supercomputer in the world, which consists of a 10,240-processor SGI Altix system comprised of 20 nodes, each with 512 Intel Itanium 2 processors, interconnected with InfiniBand network. Columbia has 440 terabytes of Fibre Channel RAID storage and running a Linux operating system.

NUMERICAL RELATIVITY CHALLENGES

'target' waveforms for Data Analysis containing *all three stages* of binary coalescence may be too expensive to compute starting from very large separations and a large parameter space:

- Simulation costs (equal mass, non spinning, AMR) ~1000 CPU hours, 18GByte of RAM
- Simulation costs (non-equal mass, non-spinning, AMR) > 5,000 CPU hours
- Simulation costs (non-equal mass, spinning, AMR) > 40,000 CPU hours

- Construct hybrid analytical/numerical models e.g. waveform:
  - Match PN inspiral waveforms with the NR waveforms over a region (more than a period long) where both the calculations are presumably valid …
  - How consistent is the matching procedure?
  - NR accuracy is important to validate PN theory →