Next steps in the simulation of Einstein equations

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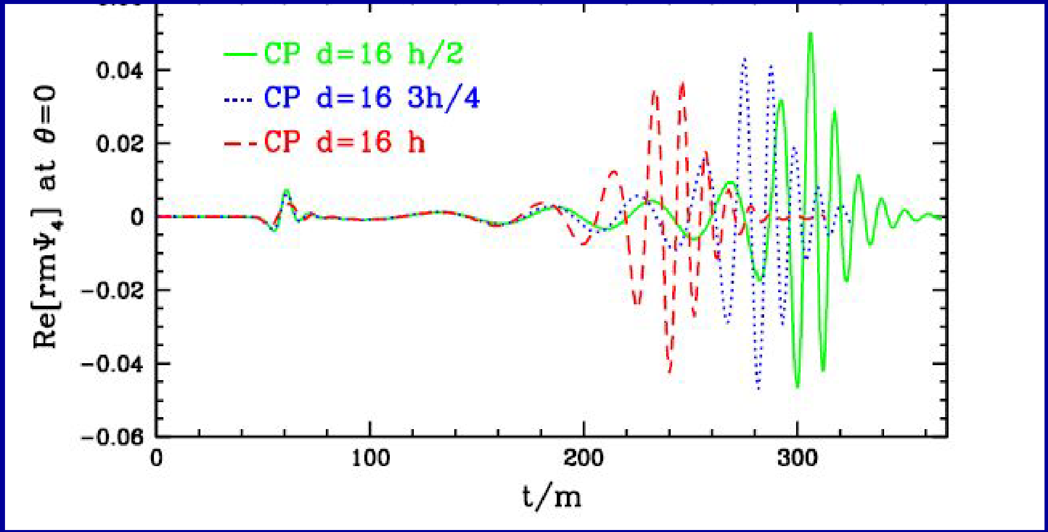
Outline

- Observations from recent simulations
  - What's puzzling, what's 'left'
  - What can be inferred so far
- Next steps & requirements
  - Two driving problems
- Looking ahead, what comes next.

Binary black holes…

- Simulations of bbh's under way. [Pretorius,Baker et.al (NASA), Campanelli et.al. (UTB), Hermann et al. (PSU), Bruegmann et.al. (Jena), Kidder et.al. (Caltech-Cornell) Diener et.al (LSU-AEI)…]
  - Equal mass binaries
  - Different mass binaries (m_1/m_2=1..4)
  - Spinning configurations with spins aligned/antialigned to orbit
  - Collapsing scalar fields

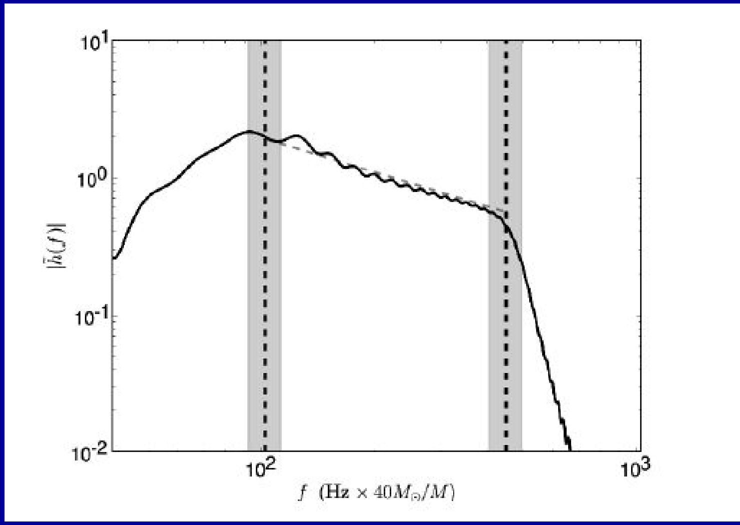


OK… if these are the waveforms… what comes next?

[Pretorius 06]

Main observations?

– Not a 'clear ISCO' at waveform level, subtle one at power spectrum level.



[Baumgarte-Brady-Creighton L.L.-Pretorius-DeVoe (in prep)]

– Waveforms largely independent of eccentricity in I.D. [Buonnano-Cook-Pretorius,Pfeiffer-Scheel-Lindblom…]

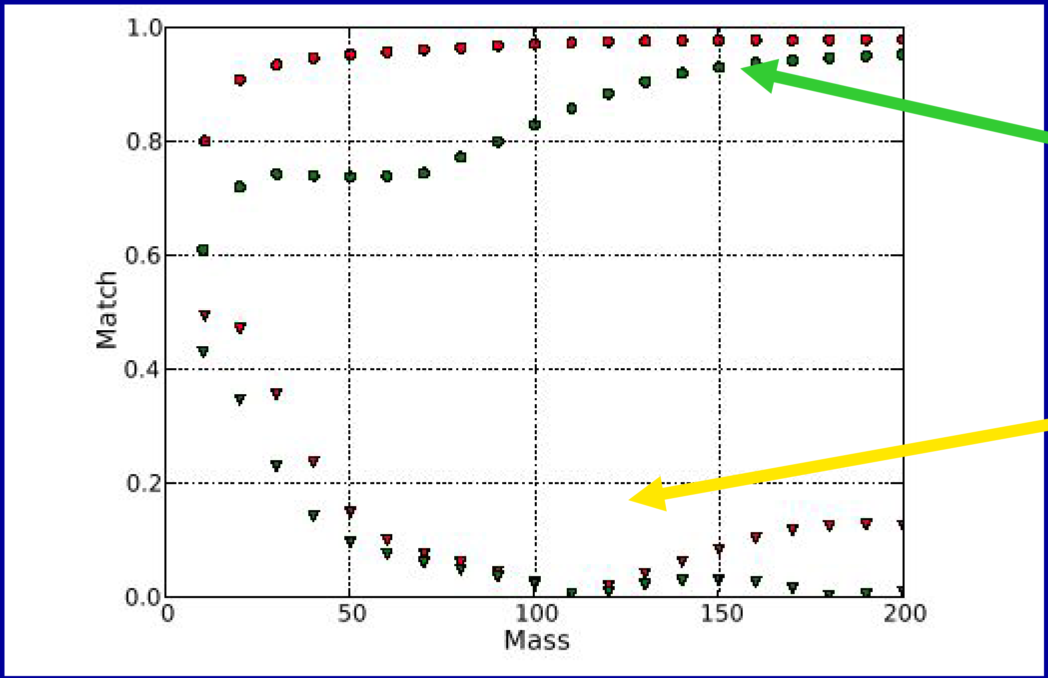
– Quadrupole formula pretty good approximation

– Very good agreement with 3PN approximations

Very good agreement throughout, are we surprised?

further observations/consequences

- Waves & Data analysis
  - We see they differ! Is this 'seen' by data analysis?
  - Matched filter ➔ max{<h,T(t_o)>} over t_o (h : signal, T template)



Really good ?!

Really bad ?!

[Baumgarte-Brady-Creighton
L.L.-Pretorius-DeVoe (in prep)]

- Waveforms seem 'good' for detection…

IS THIS IT?

(mostly) unchartered trails

- ‘Generic’ spin-orbit configurations.
  - Flips and hang-overs likely
  - Qns:
    - do PN and related approxs do well here?
    - Does the QN approx capture main features of the waves?
    - Is there an ‘abrupt’ ISCO in the waves?
  - Even when this is the case:
    - Detection needs are far less demanding than physical interpretation
    - Other systems are awaiting further developments and results from simulations.
      - BH-NS will show significant differences depending on equation of state.

… Two projects along these roads

Infrastructure

- HAD: Distributed adaptive mesh refinement package incorporating
  - True-adaptivity through self-shadow hierarchy (refines/unrefines by monitoring truncation error of the numerical solution without pre-specification of grid structure)
  - Incorporates summation by parts derivative operators, Runge-Kutta time integration, Penalty and Olsson's projection [key ingredients in ensuring stability for generic, linear, 1^{st} order hyperbolic systems, see Tiglio's talk]
  - Incorporates tappered grid approach to ensure stability and order of accuracy are preserved when employing adaptive techniques. [Arbitrary orders in principle, though as present up to 4^{th} order].
  - Both cell-centered and vertex centered grids structures available. Adaptivity fully developed for both types, though not yet fully conservative to round-off beyond unigrid case.
  - In particular : Two GR formulations (ADM-like [Sarbach-Tiglio] & Generalized Harmonic 1^{st} order formulation [Lindblom-Scheel-Kidder-Rinne…]). Two GRHydro formulations [Cell centered or Vertex centered], a MHD code.

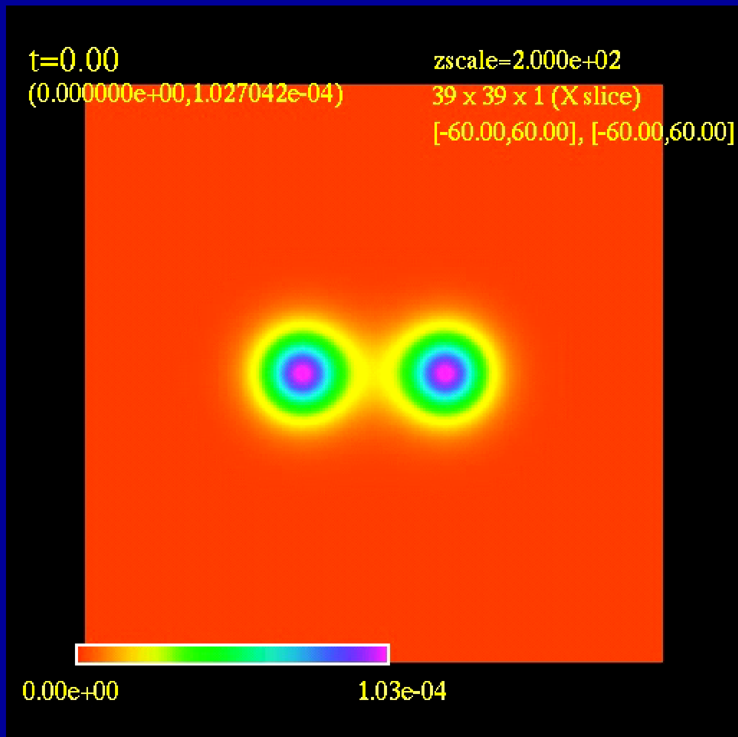
[Liebling,Anderson,Neilsen,Hirschmann,Motl,Olabarrieta,Palenzuela,L.L.]

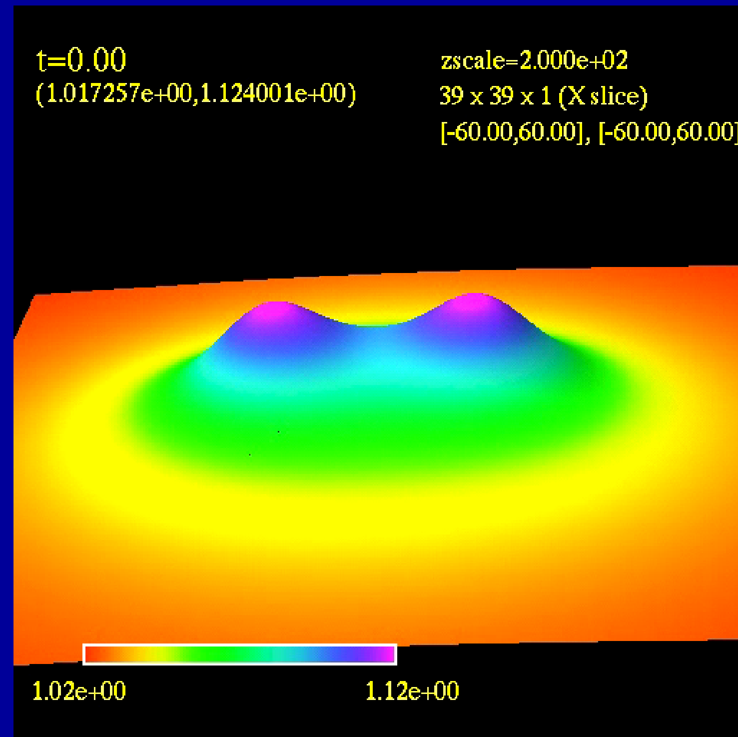
Boson star binaries

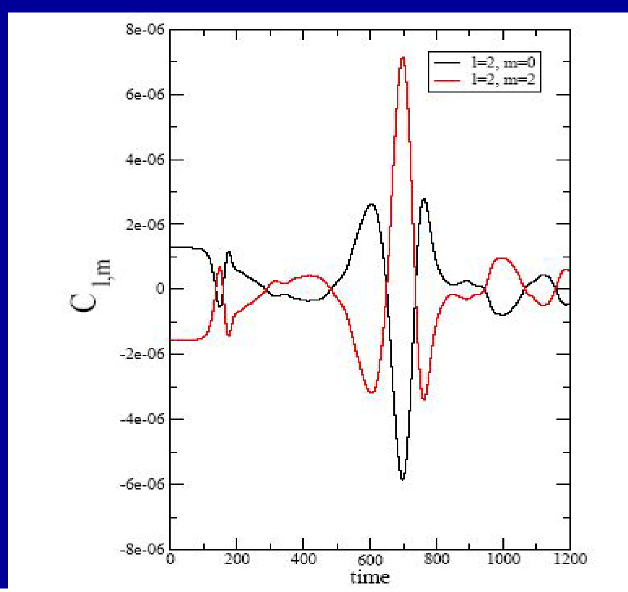
[Palenzuela,LL,Liebling,Olabarrieta]

- Goals:
  - Test infrastructure needed for accurate simulations.
    - Higher order accuracy, adaptive gridding, non-vacuum scenarios, etc.
  - Test ‘conclusions’ drawn from binary black hole simulations
    - Trajectories, radiation describable by (post) Newtonian considerations
- Boson stars,
  - Self gravitating complex scalar fields
  - Employed to obtain a stationary solution of EEs
  - Governed by non-linear wave eqns (no shocks or discontinuities)
  - “Stars” can be defined, have both stable and unstable branches like TOV stars.

Head-on collisions

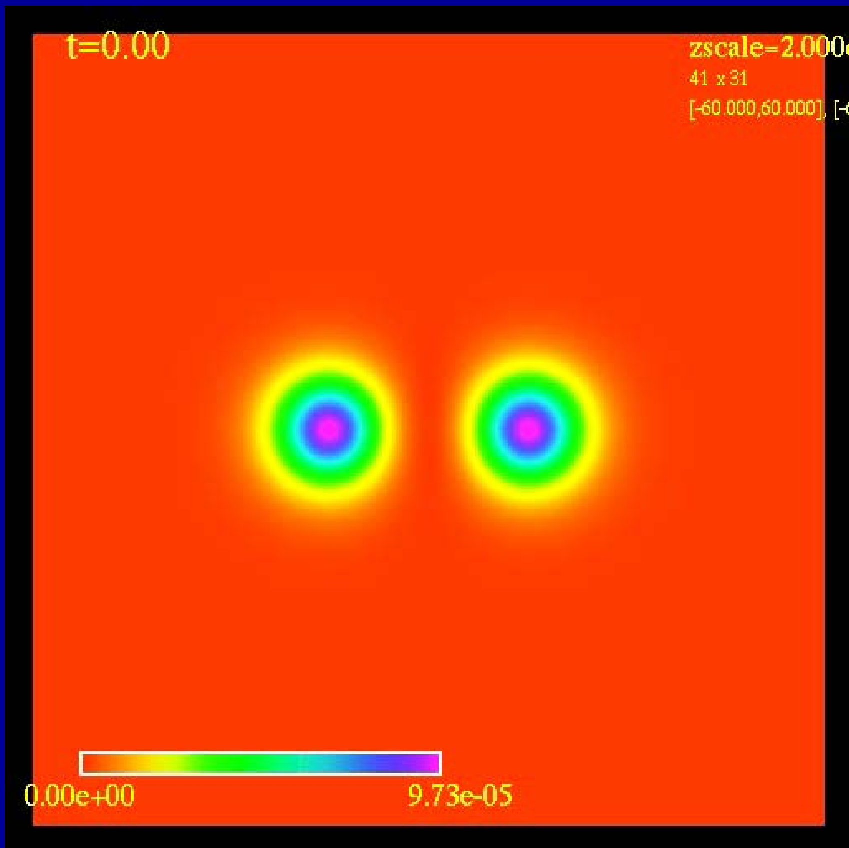


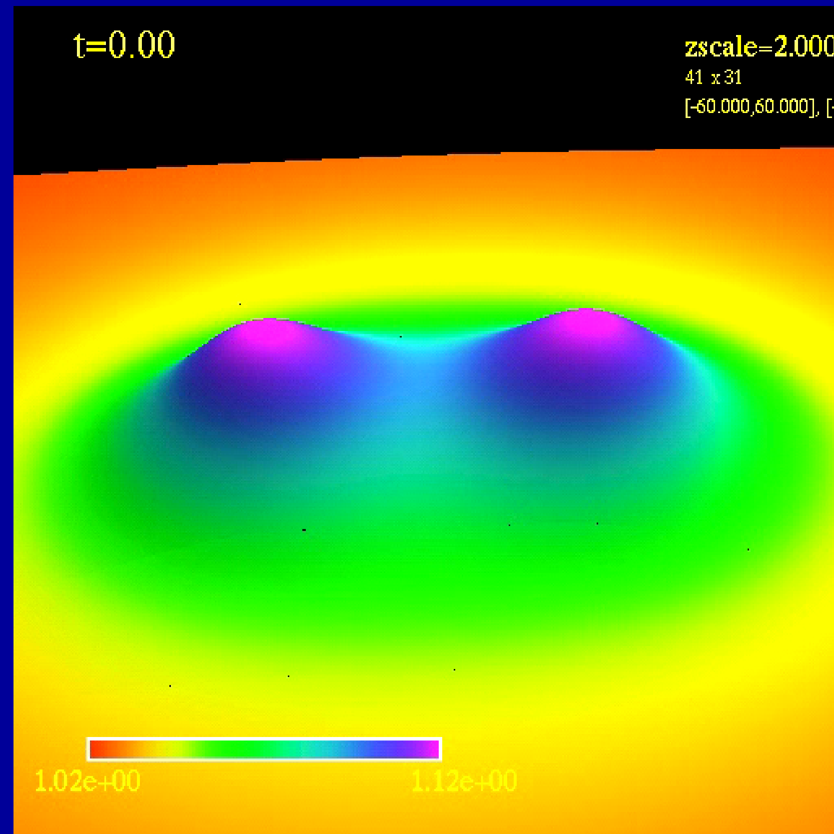




$$\Phi = \Phi_1(x-a) + \Phi_1(x+a)$$

‘other’ cases





$$\Phi = \Phi_1(x-a) + e^{i\delta}\Phi_1(x+a)$$

(P)Newtonian?

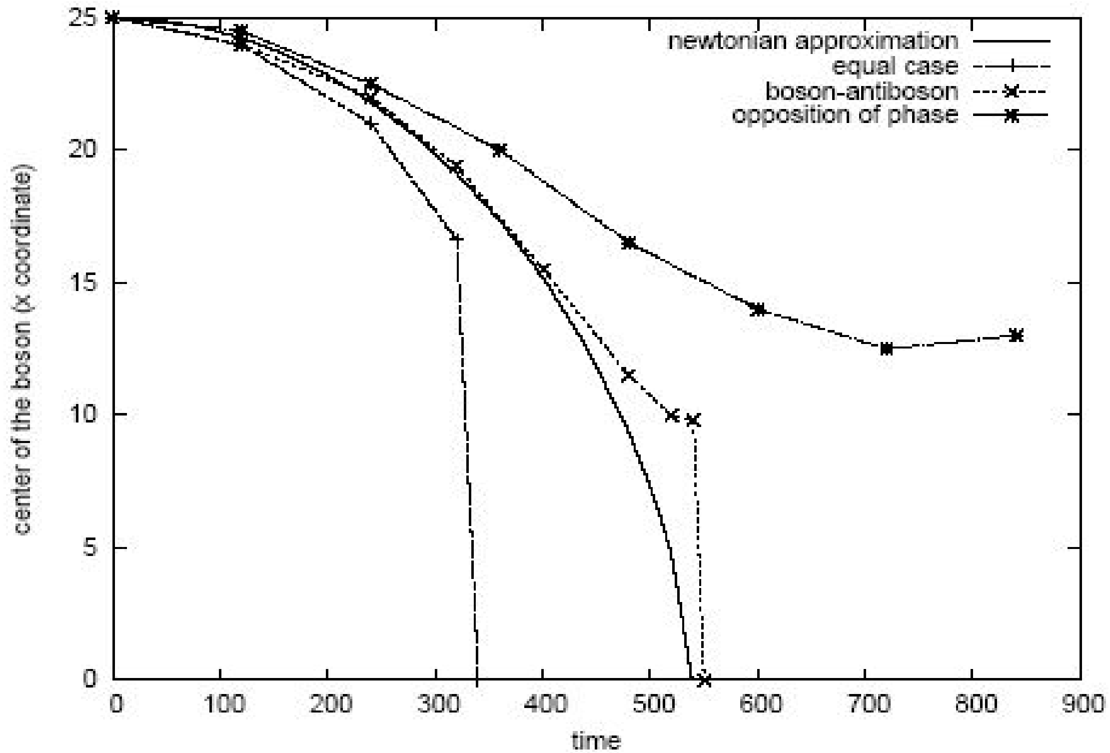
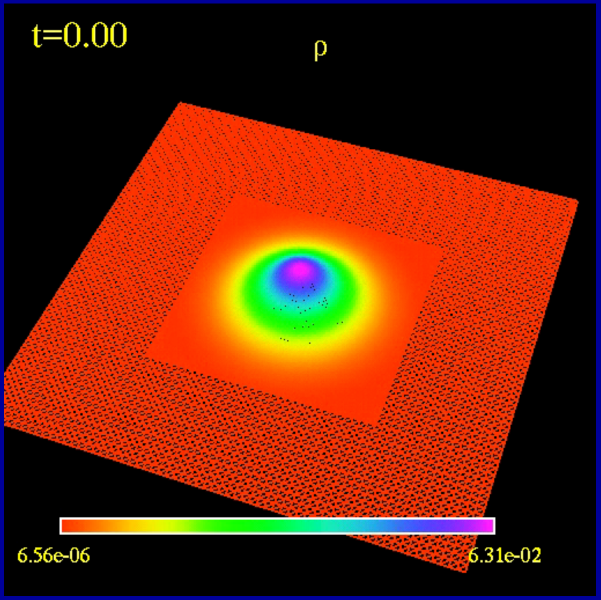


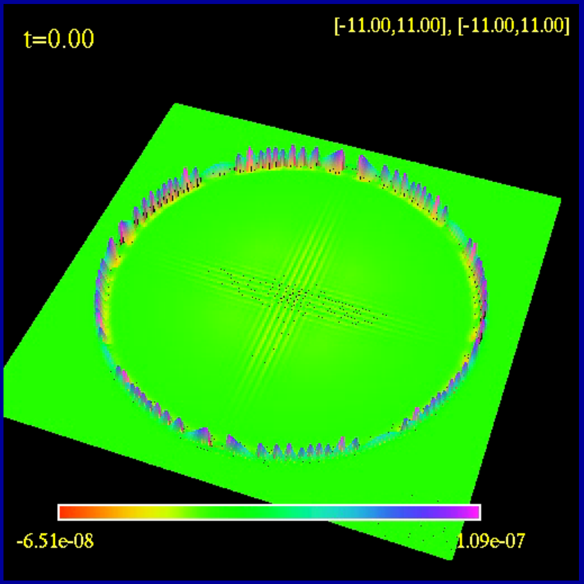
FIG. 10: The x coordinate of the center of the boson star as a funtion of time for the different cases and the newtonian approximation. The position of the boson is identified with the maximum of the Noether density

A step after another

[Anderson,Olabarrieta,Motl,Neilsen,Hirschmann,LL]

- GR + (M) Hydro
  - Finishing debugging phase & head-to-head comparison for cell-centered vs. vertex centered approaches.
    - Latter, give-up conservation to round-off level but tension!
      - In GR we have no conservation….why bother?
      - Newtonian binaries do not behave well unless conservation + adapted variables are adopted [incidently…worries for BH binaries as well!]
    - TOV stars handled similarly well with both approaches so far, though not all schemes able to do this for cell-centered schemes.

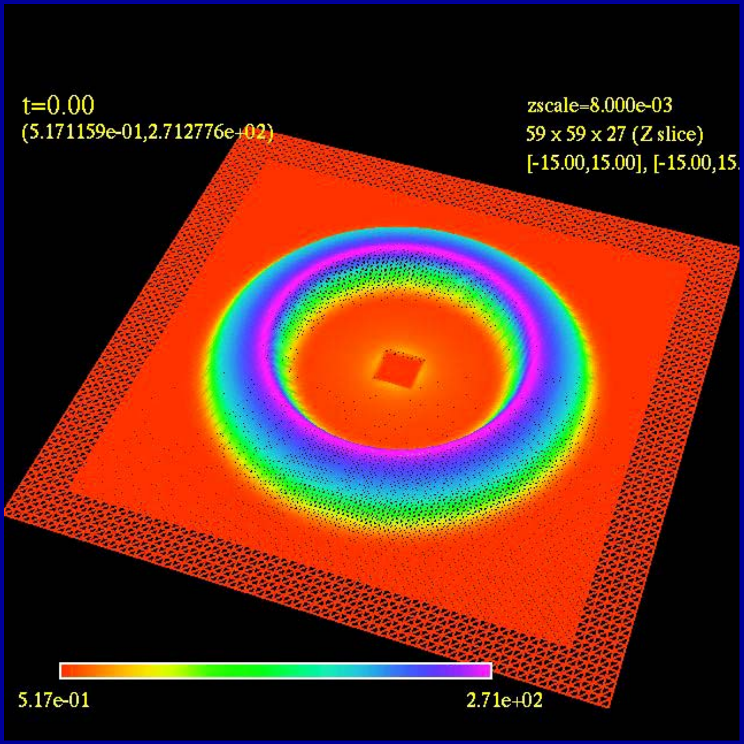




BH-NS & related problems

- Code must be able to tackle:
  - Binary objects [early stages]. Differentially rotating members [ongoing collaboration with J. Novak]
  - Disruption of star
  - Disk [Megevand,Anderson,LL] and possible jet formation [Novak's data for magnetized stars, [Anderson-Novak Neilsen-Hirschmann-LL].

- ID, both binary black holes and preliminary BH-NS simulations [Bishop,LL,Winicour,Gomez,Maharaj] indicate 'reasonable' is enough to tackle the problem. Grandclement-Gourgholom data to be employed.



So… what's next

- Firm-up current observations
  - Resolution (adaptivity + higher order)
  - Cleaning up systematics. Eg, current radiation extraction assume needed structure translates cleanly to finite distances/time-like worldtubes.

$$g_{uu} = 1$$
$$g_{uA} = 0$$
$$g_{AB} / R^2 = F^2 q_{AB} + c_{AB} / R$$

$$g_{tt} = 1$$
$$g_{tA} = 0$$
$$g_{AB} / R^2 = F^2 q_{AB} + c_{AB} / R$$

If F not 1, radiation formulae must be modified [Winicour 82]

If first conditions not satisfied, spurious time influence & coordinate dependence induced

  - Boundaries are a factor, extractions at ~30-50M, boundaries much farther out. CPBC+'physically' motivated bdry conditions needed [see Tiglio's, Winicour's,Rinne's,Buchman's, Lindblom's talks]

What's next after next…

- Can simulations be pushed (much) further in time?… *unlikely!*
- Why? Recall total computational time ~ $1/h^4$. Error ~ t^q h^m
  - Suppose a computation took T_A time, with error E and covering n_A orbits
  - Earlier orbits, PN expression holds, n ~ $t^{5/8}$
  - Time for n_B orbits with ***same total error*** ➔ $T_B = T_A (n_B/n_A)^{8/5+6q/m}$
  - If q=1, T_B=T_A * {4.8, 3.2, 2.7,2.4, …1.6} [for 2nd,4th,6th…..,'spectral' orders]

- Examine 'ISCO' break-up by exploring early stages before merger and monitoring h(f)
- Examine 'generic' spin-orbit configurations.
- If current observations stand➔ worrisome/problematic status of affairs!
  - We'd need tremendous resources to 'extract' non-linear features
  - Are non-linear features so 'mundane'… it'd be a first time!
- Non-vacuum scenarios hold a definitive promise.