Applications of Dynamical Horizons in Numerical Relativity

E. Schnetter[2] B. Krishnan[1] F. Beyer[1]

[1]Max Planck Institut für Gravitationsphysik
Albert Einstein Institut
D-14476 Golm, Germany

[2]Center for Computation and Technology
Louisiana State University
Baton Rouge, LA 70803, USA

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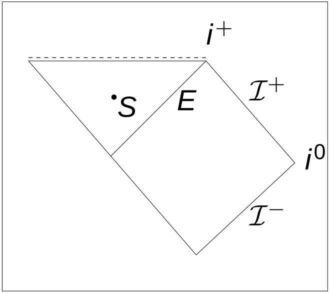
Outline

Definition of a trapped surface

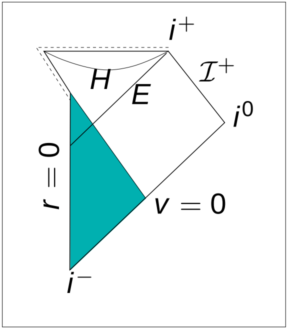
- For a sphere in flat space
  - Outgoing light rays are diverging: $\Theta_{(\ell)} = \tilde{q}^{ab}\nabla_a \ell_b > 0$
  - Ingoing light rays are converging: $\Theta_{(n)} = \tilde{q}^{ab}\nabla_a n_b < 0$
- For a trapped surface, both sets of null rays are converging: $\Theta_{(\ell)} < 0$ and $\Theta_{(n)} < 0$
- Trapped surfaces are signatures of black holes:
  - Existence of trapped surface $\implies$ singularity in future
  - Trapped surfaces lie inside the event horizon
  - For cross sections of stationary EHs $\Theta_{(\ell)} = 0$, $\Theta_{(n)} < 0$
- Future Marginally Outer Trapped Surface (FMOTS):
  - $\Theta_{(\ell)} = 0$, $\Theta_{(n)} < 0$

The trapping boundary

- Trapping boundary is boundary of region containing trapped surfaces
- There are spherically symmetric trapped surfaces right up to the Schwarzschild event horizon



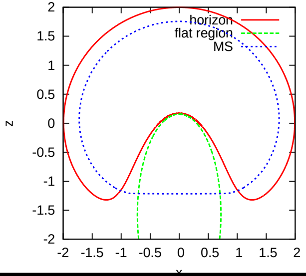
The trapping boundary



- Vaidya: sph. symmetric trapped surfaces only up to H
- Suggestion by Eardley: event horizon is the trapping boundary

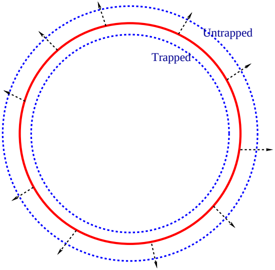
The trapping boundary

- We can look for marginally trapped surfaces on non-symmetric surfaces using apparent horizon finders
- In Vaidya we can push marginally trapped surfaces arbitrarily close to the EH
- Marginal surfaces can also extend into flat region
- Recent analytic proof by Ben-Dov



Evolution of MTSs in Time

- It is observed numerically that MTSs evolve smoothly in time
- Apparent horizons may jump due to outermost condition
- Smooth world tube of MTSs is a Marginally Trapped Tube
- MTT shown to exist if MTS is strictly stably outermost
  - Andersson *et al*, PRL **95** 111102 (2005)



- Linear outward deformation makes S untrapped: $\delta_{f\mathbf{r}}\Theta_{(\ell)} > 0$ for $f \geq 0$
- In practice we look for surfaces with $\Theta_{(\ell)} = \epsilon > 0$ and check that it lies outside S

Definitions

Dynamical Horizon: Spacelike MTT with $\Theta_{(n)} < 0$

- Outermost MTT usually forms a DH
- MTT with $|\sigma_{(\ell)}|^2 \neq 0$ or $T_{ab}\ell^a\ell^b \neq 0$ somewhere are spacelike if they are SSO (Andersson et al.)

Timelike Membrane: Timelike MTT

- Cannot be the black hole surface
- Inner MTTs might form timelike membranes

Isolated Horizon: Null MTT (BH in equilibrium)

Other cases: MTTs with mixed signature also possible

Basic Properties

- Cross section is topologically S^2
- Area increases along $\hat{r}^a$ for a DH
  - Consequence of $\Theta_{(\ell)} = 0$ and $\Theta_{(n)} < 0$
  - Area increases in time if $t.r > 0$
  - Area decreases for a TLM
- Foliation of DH is unique (Ashtekar & Galloway, gr-qc/0503109)
  - Implies that changing Σ leads to different DH
  - Other restrictions on occurence of MTS in presence of DH
- Event horizon is probably the boundary of the trapped region (Eardley 1998, Schnetter & Krishnan 2006, Ben-Dov 2006)

Multipole Moments

- We are interested in source multipole moments for black hole
- In classical electrodynamics we have charge and current multipole moments for sources
- For a black hole we have mass and angular momentum multipole moments M_n and J_n
- J_0 vanishes by absence of monopole charges (here NUT charge)
- M_0 is mass and J_1 is angular momentum
- In Kerr, M_0 and J_1 determine all higher moments
- In Schwarzschild, only $M_0 \neq 0$

Multipole Moments

- Given rotational Killing vector φ^a construct coordinate system (θ, ϕ) on S
  - $\phi \in [0, 2\pi)$ is affine parameter along φ^a
  - $\zeta = \cos\theta \in [-1, 1]$ is defined by

$$\tilde{D}_a \zeta = \frac{1}{R_S^2} \tilde{\epsilon}_{ba} \varphi^a \, , \qquad \oint_S \zeta = 0$$

- Use spherical harmonics for (θ, ϕ) to define multipoles

$$M_n = \frac{R_S^n M_S}{8\pi} \oint_S \left\{ \tilde{R} P_n(\zeta) \right\} d^2V$$

$$J_n = \frac{R_S^{n-1}}{8\pi} \oint_S P_n'(\zeta) \bar{K}_{ab} \varphi^a R^b \, d^2V$$

Multipole Moments

Properties of the multipole moments

- M_n and J_n are coordinate independent
- They characterize geometry of DH at any given time
- Need only data on MTS to calculate them
- Coincide with corresponding isolated horizon formulae (Ashtekar et. al, CQG **21** 2549 (2004))
- Useful for characterizing rate of approach to Kerr

Brill-Lindquist Initial Data

- Describes head on collision for two BH case
- Σ is $\mathbb{R}^3$ with two "punctures"
- Time symmetric: $\bar{K}_{ab} = 0$
- Conformally flat: $\bar{q}_{ab} = \psi^4 \delta_{ab}$
- $\Delta\psi = 0$, $\psi \to 1$ as $r \to \infty$

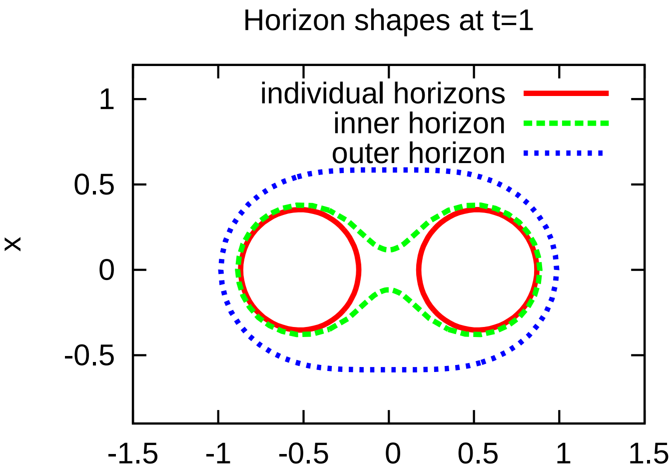
$$\psi = 1 + \frac{\alpha_1}{2r_1} + \frac{\alpha_2}{2r_2}$$

- Single BH case is Schwarzschild in isotropic coordinates
- $m_{ADM} = 2\alpha_1 + 2\alpha_2$ and punctures are asymptotic regions
- Take units such that $m_{ADM} = 1$

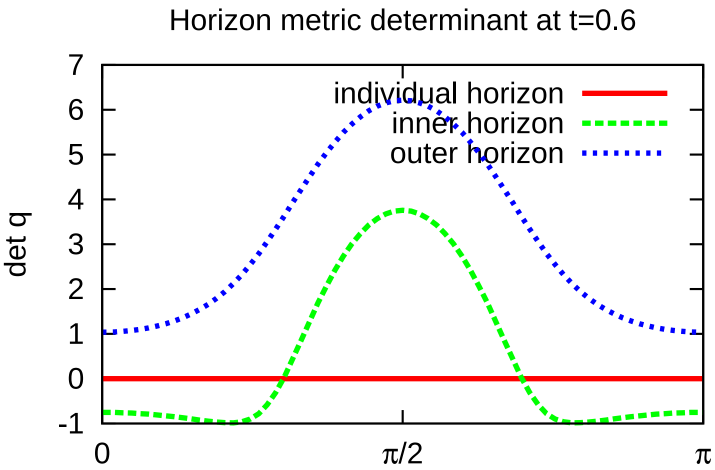
Brill-Lindquist Initial Data

- Equal mass: $2\alpha_1 = 2\alpha_2 = 0.5$
- punctures initially at $z = \pm 0.5$
- Explicit octant symmetry and extent upto $x, y, z = 96$
- 4^{th} order spatial differencing and 3^{rd} order Runge-Kutta
- Use mesh refinement: $h = 1.6$ at boundary and $h = 0.0125$ at horizon
- Horizon diameter contains 32 points initially
- About 10 grid points excised around punctures
- AEI BSSN formulation (inconsistent boundary conditions!)
- $1 + \log$ slicing with $\alpha = 1$ initially; zero shift
- Common MTS forms at $t \approx 0.5$
- We use Jonathan Thornburg's `AHFinderDirect`.

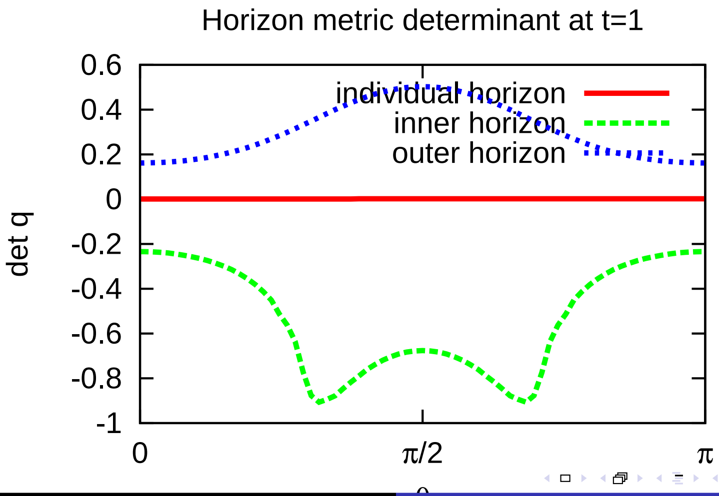
Horizon Shapes



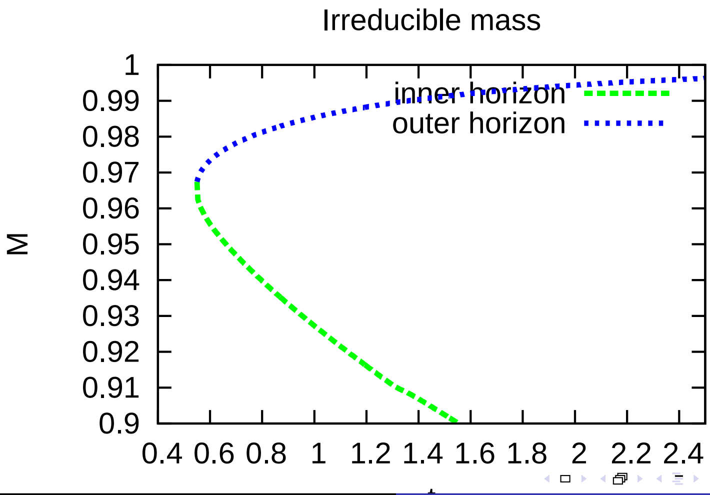
Signature of MTT



Signature of MTT

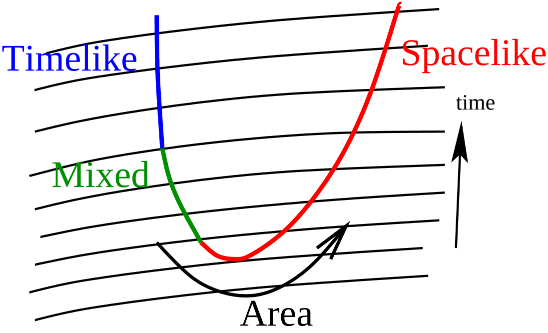


Horizon Mass



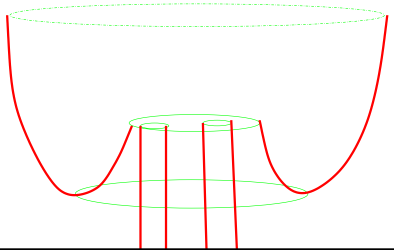
Behavior of the MTTS

- Outer MTT is spacelike and growing
- Individual MTTs are essentially isolated
- Inner MTT is initially spacelike but soon becomes timelike
- Inner MTT has decreasing area

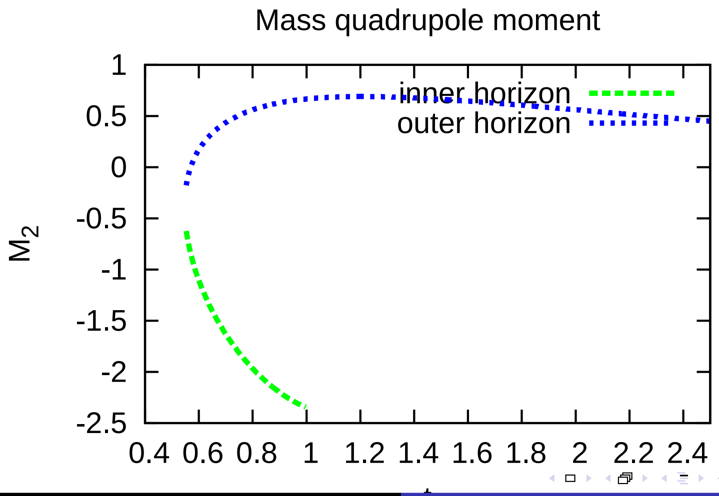


Behavior of the MTTS

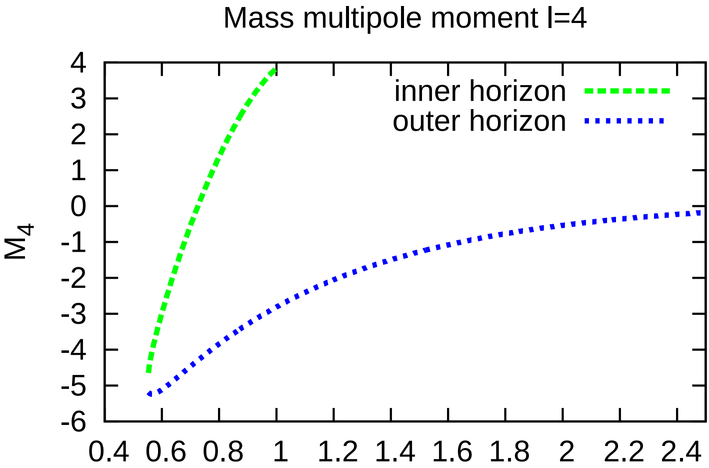
- What is the geometry of the full MTT – “inverted pair of pants”?
- Lose track of inner MTT because of resolution and because it may not be star shaped
- How, if at all, does the inner MTT merge with the individual horizons?
- Area is monotonic radially outwards



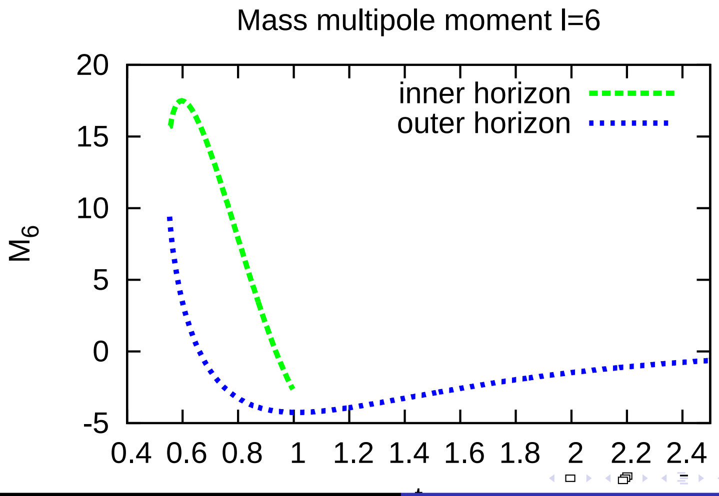
Mass Quadrupole Moment



Mass Multipole M_4



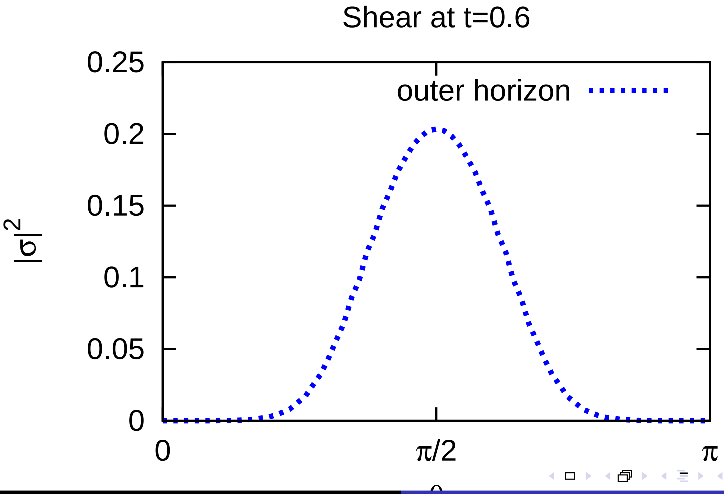
Mass Multipole M_4



Behavior of Multipole Moments

- All J_ns vanish
- $M_n = 0$ for odd n
- All higher moments for outer horizon vanish asymptotically
- But inner MTT does not seem to approach Schwarzschild

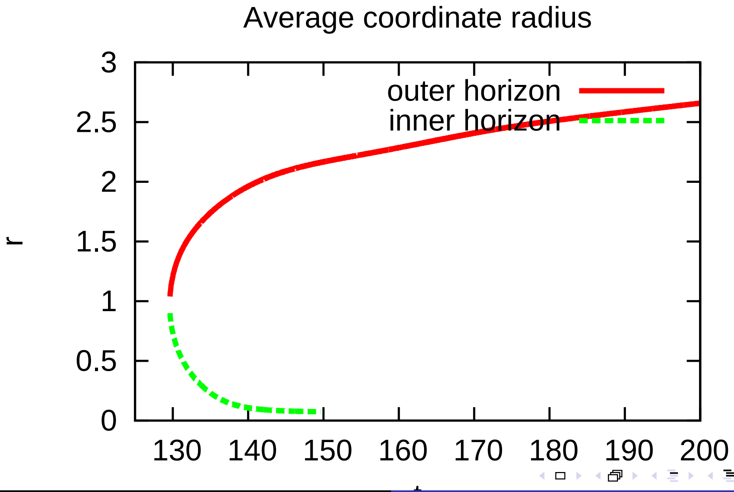
Shear



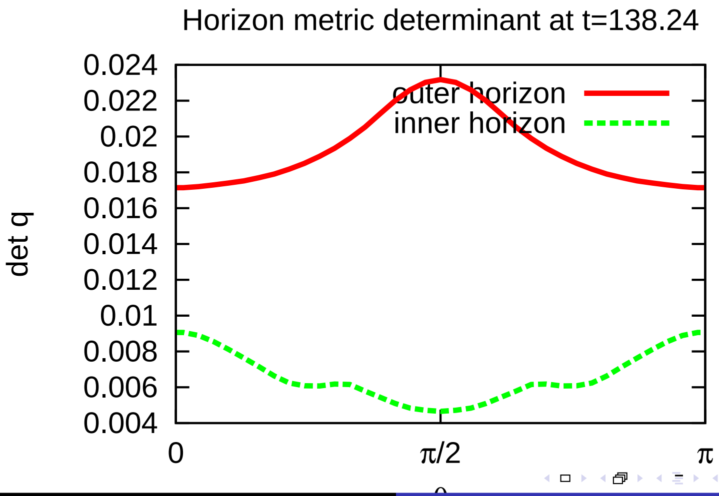
Axisymmetric gravitational collapse

- Uniformly rotating perfect fluid
- Used `Whisky`
- $K = 100$, $\Gamma = 2$ polytrope ($p = K\rho^{\Gamma}$)
- Model D4 in Baiotti et al (PRD, 2005): $M_{NS} = 1.86 M_{\odot}$, $\rho_c = 1.934 \times 10^{15}\,\mathrm{g\,cm^{-3}}$, and $J_{NS} = 0.543 M_{NS}^2$.
- Ratio of polar to equatorial coordinate radius is 0.65
- Rotational frequency 1295.34 Hz
- Equatorial radius 14.22 km
- Configuration is dynamically unstable – reduce pressure to induce collapse
- We use Jonathan Thornburg's `AHFinderDirect`.

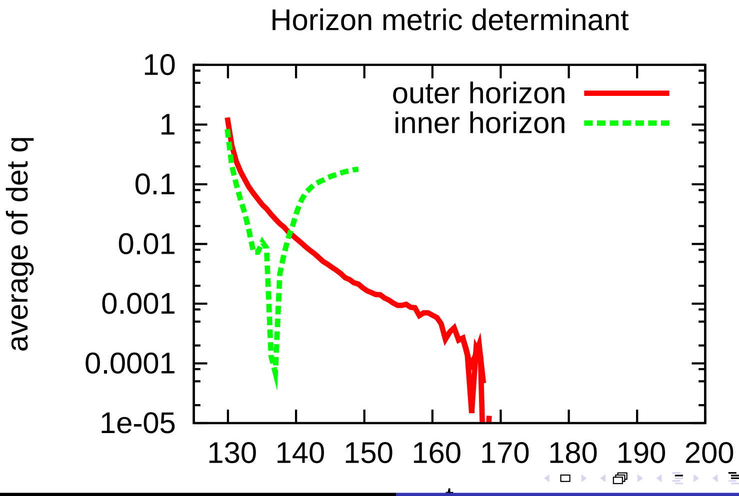
Horizon Shapes



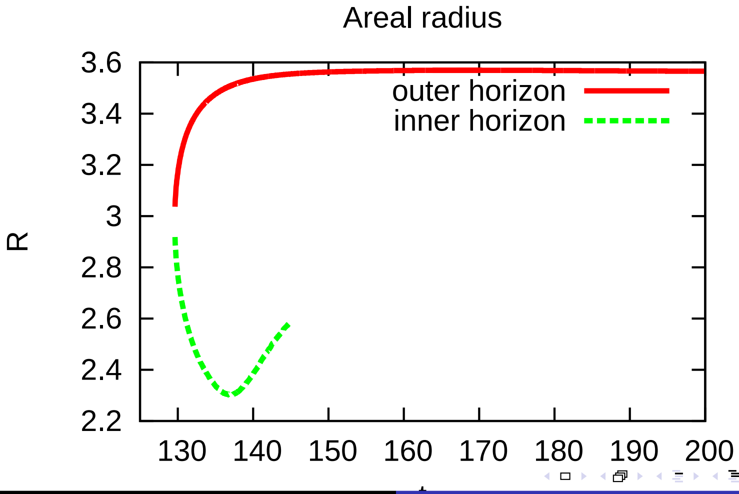
Signature of the MTT



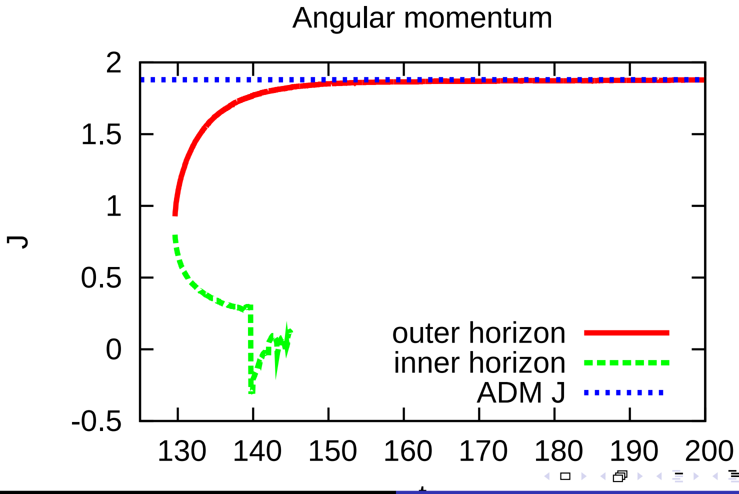
Signature of the MTT



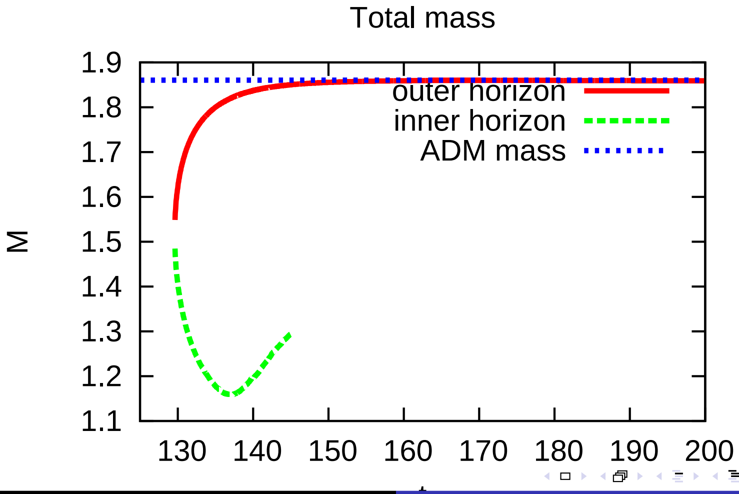
Horizon Area



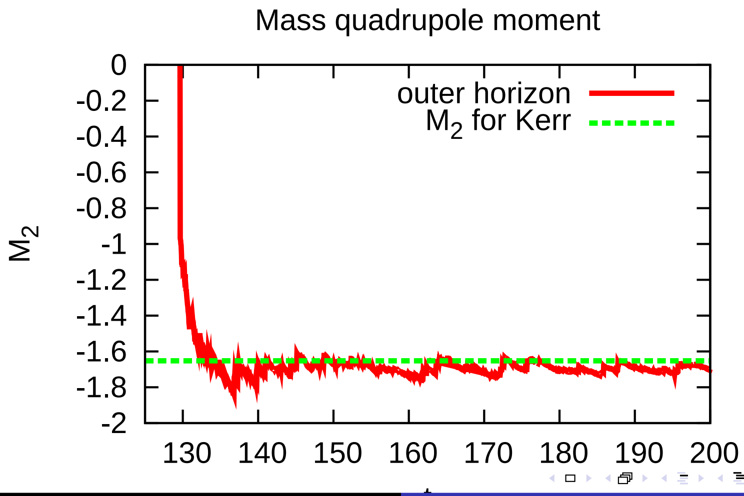
Horizon Angular Momentum



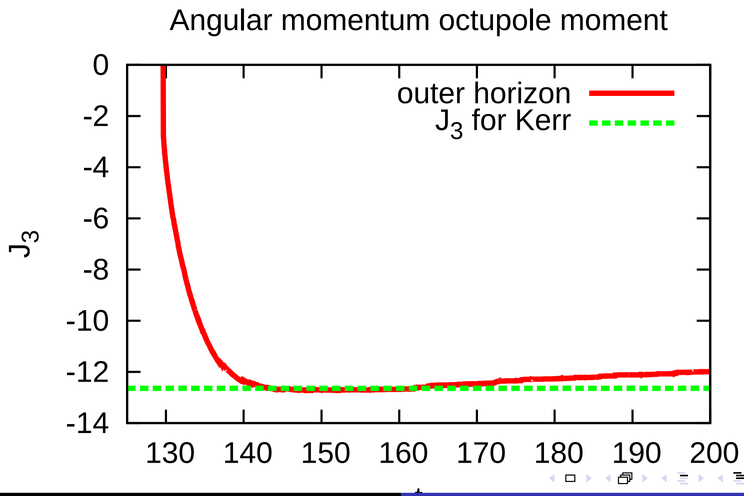
Mass


Total mass
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
M
130
140
150
160
170
180
190
200
t
outer horizon
inner horizon
ADM mass

Mass Quadrupole Moment M_2



Angular Momentum Multipole Moment J_3



Summary

- We have looked at various types of trapped surfaces, multipole moments, etc. in some example simulations
- Shows how a black hole grows and settles down to a Kerr horizon
- Can be applied in current stable evolutions
    - Are non-spinning punctures really non-spinning?
    - Observe spin-orbit coupling?
    - How far is initial data from Kerr?
    - . . .