Naked Singularityies and Self-Similarity in Gravitational Collapse

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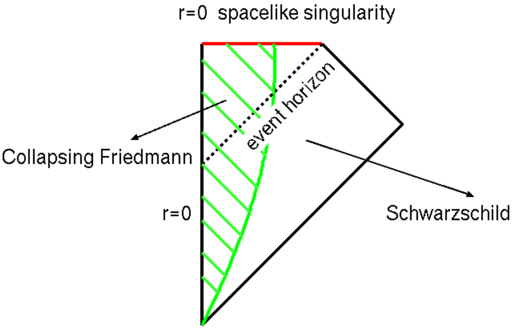
Outline

1. Singularity Formation in Gravitational Collapse
2. Self-Similar Solutions for Gravitational Collapse
   - Self-Similar Solutions
   - Self-Similar Attractor and Cosmic Censorship
   - Unified Picture of Convergence and Critical Phenomena

Singularity Formation in Gravitational Collapse

- The Oppenheimer-Snyder solution
  - The complete collapse of a uniform dust ball
  - Interior: The (time-reversed) Friedmann solution with a dust
  - Exterior: The Schwarzschild solution (vacuum)



  - A spacetime singularity hidden behind the event horizon
  - Globally hyperbolic, i.e., there exists a Cauchy surface.

Conjecture for Gravitational Collapse Singularities

- Naked singularities
  - We cannot apply known physics at singularities. Hence, if a spacetime singularity were observed, it would spoil the future predictability of physics.
  - Or a window into physics beyond general relativity? (e.g. Harada & Nakao 2004)
- Cosmic censorship conjecture (Penrose 1969, 1979)
  - Weak censorship: "A system which evolves, according to classical general relativity with reasonable equations of state, from generic non-singular initial data on a suitable Cauchy-hypersurface, does not develop any spacetime singularity which is visible from infinity"
  - Strong censorship: "... a physically reasonable classical spacetime M ought to have the property ...M is globally hyperbolic ..."
  - reasonable equations of state? generic initial data?
  - Basic assumption to prove the theorems on BH properties, such as no bifurcation, area increase and an event horizon outside an apparent horizon (Hawking & Ellis 1973)

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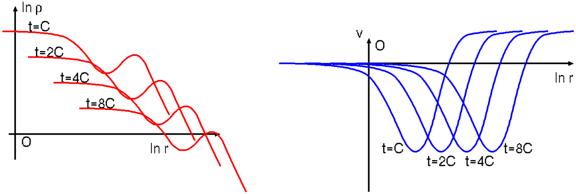
Self-Similar Solutions

- No characteristic scale in gravity
- Easy to obtain: (1+1) PDE reduces to ODEs.
- $\rho(t,r) = t^{-2}\rho_0(r/t)$, $v = v_0(r/t)$



- Describe asymptotic behaviour of more general solutions: e.g. spatially homogeneous solutions (Wainright & Ellis 1997)
- Similarity hypothesis (Carr 1993)
  "... spherically symmetric fluctuations might naturally evolve via the Einstein equations from complex initial conditions to a self-similar form."

Definition of Self-Similar Spacetimes

- Self-similar (homothetic) spacetime
  - Homothetic vector ξ

$$\exists \xi, \quad \mathcal{L}_\xi g_{\mu\nu} = 2 g_{\mu\nu}$$

  - Introducing the coordinates (t, r) such that

$$\xi = t\frac{\partial}{\partial t} + r\frac{\partial}{\partial r},$$

    a nondimensional metric component Q satisfies

$$Q(t, r) = Q(at, ar), \quad \forall a > 0.$$

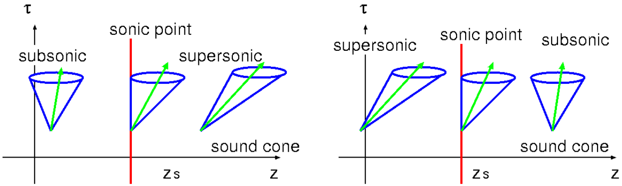
    and hence $Q = Q(r/t)$.
  - The line element:

$$\begin{aligned} ds^2 &= -e^{\sigma(z)}dt^2 + e^{\omega(z)}dr^2 + r^2 S^2(z)(d\theta^2 + \sin^2\theta d\phi^2), \\ z &\equiv \ln|r/(-t)| \end{aligned}$$

  - If $Q(t, r) = Q(at, ar)$ holds only for $a = e^{n\Delta}$ ($\Delta > 0$, $n = 0, \pm 1, \pm 2, \cdots$), this is called discretely self-similar.

Self-Similar Solutions with Physical Matter Fields

- The matter fields are strongly restricted.
  - Perfect fluid with $p = k\rho$: (Sound wave at the speed $\sqrt{k}$)
  - Massless scalar field ϕ: (Scalar wave at the speed 1)
- The EFE reduces to a set of ODEs.
- Sonic point
  - Singular point of the ODEs (not spacetime singularity)
  - Classified through dynamical systems theory technique
  - No information propagates inwardly beyond the sonic point.



ODEs for Self-Similar Solutions with Perfect Fluid

- Perfect fluid with $p = k\rho$ $(0 \le k \le 1)$
- Nondimensional quantities ($G = 1$, r =comoving coordinate)

$$V^2 \equiv e^{2z+\omega-\sigma}, M \equiv \frac{2m}{r}, \eta \equiv 8\pi r^2 \rho, y \equiv \frac{M}{\eta S^3},$$
$$e^{\sigma} = a_{\sigma}(\eta e^{-2z})^{-\frac{2k}{1+k}}, \quad e^{\omega} = a_{\omega}\eta^{-\frac{2}{1+k}} S^{-4}.$$

- The ODEs

$$M' = \frac{k}{1+k}\frac{1-y}{y}M, \quad S' = -\frac{1-y}{1+k}S,$$
$$\eta' = \left[2(1-y) - 2\frac{ky - \frac{1}{4}(1+k)^2 e^{\omega}\eta}{V^2 - k}\right]\eta,$$
$$V^2(1-y)^2 - (k+y)^2 + (1+k)^2 e^{\omega} S^{-2}(1 - y\eta S^2) = 0.$$

- Sonic point: $V^2 = k$

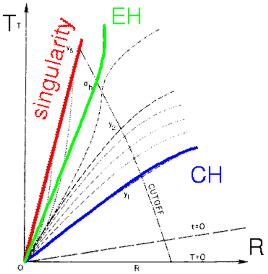
Self-Similar Solutions with Analytic Initial Data

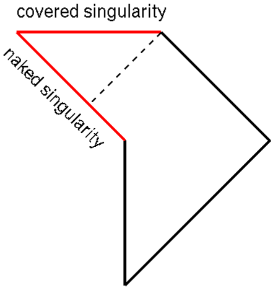
- Analytic (regular) initial data
  - Analytic = Taylor-series expandable with respect to the Riemannian normal and Cartesian coordinates
  - Analyticity at the sonic point ($z = z_s$)
  - Analyticity at the centre ($z = -\infty$)
- Countable number of solutions with analytic initial data
  - Flat Friedmann solution ($0 < k \leq 1$)
  - GR Larson-Penston solution* ($0 < k < 1/3?$)
  - GR Hunter (a) solution* ($0 < k \leq 1$)
  - GR Hunter (b) solution* ($0 < k \leq 1$)
  - ...

  (Solutions with * are obtained numerically.)

Naked Singularity in Self-Similar Collapse

- The GRLP solution
  - Naked singularity forms from analytic initial data for $0 < k < 0.0105$. (Ori & Piran 1987)





- Other self-similar solutions with analytic initial data
  - There exist naked-singular solutions for $0 < k \leq 9/16$. (Ori & Piran 1990, Foglizzo & Henriksen 1993)

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Stability against Regular Mode Perturbation

- Normal mode analysis

$$h(\tau, z) = H_{\rm ss}(z) + \epsilon e^{\lambda\tau} F(z),$$

where $\tau = -\ln(-t)$ and $z = \ln[r/(-t)]$.

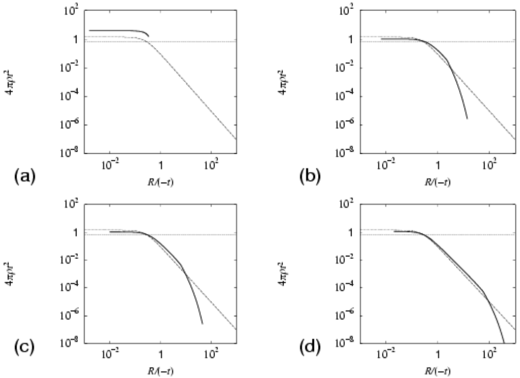
  - Regularity condition imposed both at the centre and the sonic point
  - λ is determined as an eigenvalue problem through the EFE.
- Results (Koike, Hara & Adachi 1995, 1999, Maison 1996, Harada & Maeda 2001, Brady et al. 2002, Snajdr 2006)
  - GRLP: no unstable mode ($0 < k < 0.036$)
  - GR Hunter (a): one unstable mode
  - Other numerical solutions : more than one unstable modes

Stability against Kink Mode Perturbation

- Kink mode perturbation
  - A density gradient discontinuity at the sonic point
  - λ is determined locally through the EFE.
  - The stability is completely determined by the class to which the sonic point belongs as an equilibrium point.
- Results (Harada 2001, Harada & Maeda 2003)
  - Flat Friedmann: Unstable ($0 < k \le 1/3$), Stable ($1/3 < k \le 1$)
  - GRLP: Stable ($0 < k < 0.036$), Unstable ($0.036 \le k < 1/3$)
  - GR Hunter (a): Stable ($0 < k < 0.89$), Unstable ($0.89 \le k \le 1$)

Convergence to the GRLP Solution

- Numerical relativity experiment (Harada & Maeda 2001)



  - Simple Misner-Sharp scheme code with $p = k\rho$ ($k = 0.01$)
  - Dotted = Flat Friedmann, Dotted-dashed = GRLP
  - The central density can reach 10^{10} times the initial value.
- The GRLP solution acts as an attractor.

Confirmation with a Refined Method

- The convergence to the GRLP solution has been confirmed with much more elaborated numerical scheme. (Snajdr 2006)
  - High resolution shock capturing scheme
  - Adaptive mesh refinement: The central density reaches 10^{38} times the initial value or even much higher.
  - Innovative treatment of vacuum: The surface is well controlled.
- Good agreement with the GRLP solution

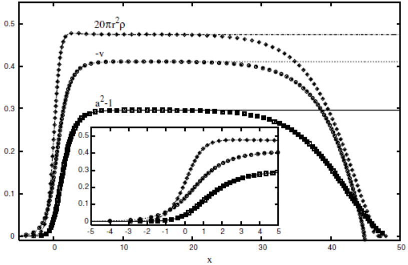


Figure 13. Comparison of the supercritical numerical solution and the GRLP solution for $\Gamma - 1 = 0.01$. The embedded plot shows details of the central region.

Self-Similar Attractor and Cosmic Censorship

- The cosmic censorship will be violated for spherical collapse
  - The GRLP solution is an attractor at least for $0 < k < 0.03$.
  - The GRLP solution describes naked singularity formation for $0 < k < 0.0105$.
  - Therefore, naked singularity is generic outcome of spherical gravitational collapse for $0 < k < 0.0105$.
- The GRLP solution would be unstable against nonspherical perturbation for $0 < k < 1/9$. (Gundlach 2002)

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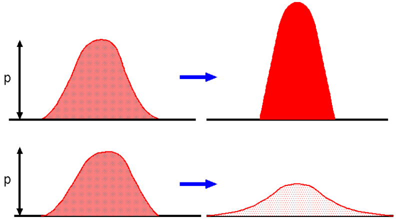
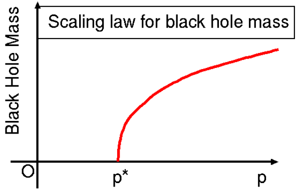
Critical Behaviour in Gravitational Collapse

- **Critical phenomena at the BH thereshold**
  - For a generic one-parameter (p) family of initial data sets, there exists a threshold value p^* for the BH formation.
  - A near-critical collapse first approaches a self-similar (critical) solution and deviates away eventually.
  - $M_{\rm BH} \propto |p - p^*|^\gamma \quad$ for $\quad p \approx p^*$
- **Matter fields**
  - Massless scalar field (Choptuik 1993)
  - Perfect fluid with $p = k\rho$ (Evans & Coleman 1994, Neilsen & Choptuik 2000, Brady et al. 2002, Snajdr 2006)





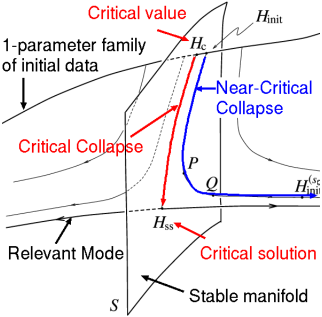
Critical Behaviour as Intermediate Behaviour

- Renormalisation group approach (Koike, Hara & Adachi 1995)
  - The critical solution is a fixed point with a single unstable mode.

$$h(0,z) = H_{\rm init}(z) = H_{\rm c}(z) + \epsilon F(z), \quad \epsilon = p - p^*$$

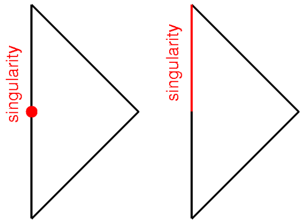
$$h(\tau, z) \approx H_{\rm ss}(x) + \epsilon e^{\lambda\tau} F_{\rm rel}(z) \quad (\lambda > 0) \quad \text{for large} \quad \tau$$

$$M_{\rm BH} = O(r) = O(e^{-\tau}) \propto |p - p^*|^{\gamma}, \quad \gamma = 1/\lambda$$



Critical Behaviour and Cosmic Censorship

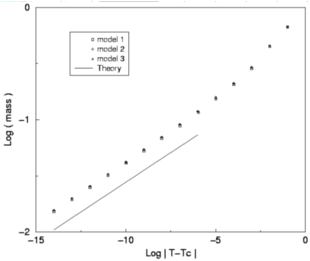
- BH threshold as a naked singularity
  - Intuitively, an arbitrarily small BH can be regarded as a naked singularity because the curvature strength scales as $1/M^2$ for BHs.
  - The Choptuik critical solution, which is discretely self-similar, actually has a naked singularity. (Gundlach & Martin-Garcia 2003)

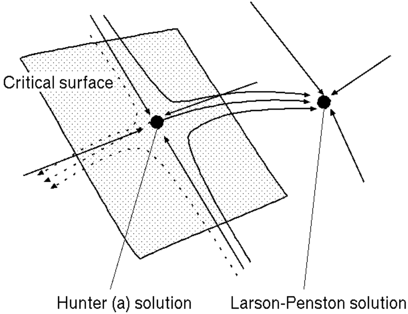


- However, this naked singularity is realised as a consequence of exact fine-tuning and hence nongeneric.

Newtonian Gravitational Collapse

- Isothermal gas model in Newtonian gravity: $p = c_s^2 \rho$
  - Self-similar solutions with analytic initial data: a homogeneous collapse, Larson-Penston, Hunter (a), (b) ...
  - There exist both the convergence and critical phenomena. The LP solution acts as an attractor, while the Hunter (a) solution acts as a critical solution. (Maeda & Harada (2001), Harada, Maeda & Semelin (2003))





Summary

- Numerical relativity reveals the properties of spacetime singularities.
- Self-similar solutions can describe asymptotic or intermediate behaviour of more general solutions.
- The cosmic censorship will be violated within spherical symmetry.
- A unified picture of convergence and critical phenomena is obtained.
- Both the convergence and critical phenomena will be seen in a large class of scale-free and nonlinear systems.