Moving punctures

that are neither moving, nor punctures

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From Geometry to Numerics workshop
Paris
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Ongoing work following
Hannam, Husa, Pollney, Brügmann, Ó Murchadha, gr-qc/0606099

From geometry to numerics... and on to astrophysics!

- Full black-hole binary evolutions (inspiral, merger, ringdown) are now routine
- Recent results from the Jena group:

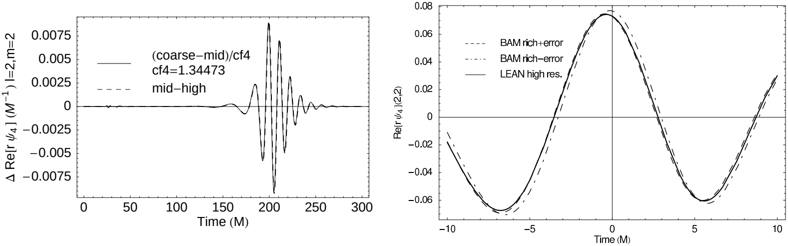
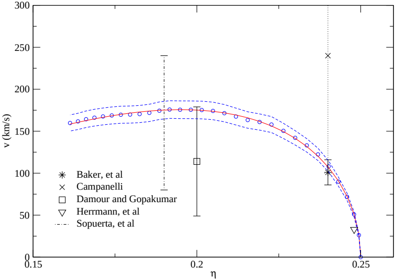


Figure: 4th-order convergence and BAM / LEAN comparison. (gr-qc/0610128)

- Merger time error of 0.2% for $r_0 = 3.257M$. 0.5% for $r_0 = 4M$.
- *No* phase shift applied!
- High-resolution runs take less than 48 hours on LRZ altix cluster.

From geometry to numerics, and on to astrophysics!

- Largest parameter study to date of binary merger evolutions
- Nonspinning unequal-mass binaries with mass ratios of 1:1 to 1:4



Kick velocity vs reduced mass ratio $\eta = m_1 m_2/(m_1 + m_2)^2$. (gr-qc/0610154)

- Maximum recoil velocity of 175.2 ± 11 km s^{-1}.

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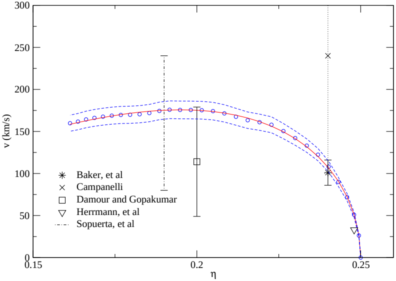

Kick velocity vs reduced mass ratio $\eta = m_1 m_2/(m_1 + m_2)^2$. (gr-qc/0610154)

- Maximum recoil velocity of $175.2 \pm 11 \text{ km s}^{-1}$.

- Now we can
  - Fully explore the physics of BBH mergers
  - Provide waveforms to data analysts

Back to geometry

How to deal with black-hole singularities in a numerical code

- "Excision": Chop them out! (Pretorius, Caltech)
- "Punctures": avoid them. (UTB, Goddard, Penn State, Jena (x2))

Back to geometry

How to deal with black-hole singularities in a numerical code

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The "moving punctures" method is easy to implement and popular

But...

- Are punctures a crude and dirty way to solve the problem?
- Or are they a simple and elegant solution?

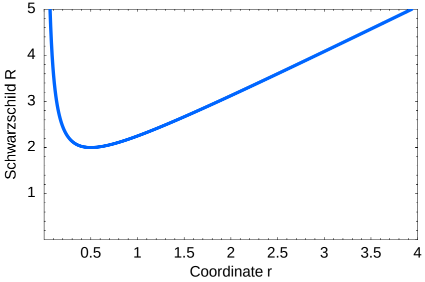
  Attempt to explain how punctures evolve by looking at a Schwarzschild black hole.

Puncture initial data

Schwarzschild in isotropic coordinates:

$$ds^2 = -\left(\frac{1-\frac{M}{2r}}{1+\frac{M}{2r}}\right)^2 dt^2 + \left(1+\frac{M}{2r}\right)^4 \left(dr^2 + r^2 d\Omega^2\right).$$
$$R = \psi^2 r.$$

R extends from ∞ to $2M$ (at $r = M/2$), and back to ∞ (at $r = 0$).
Slice connects two asymptotically flat ends; avoids the singularity



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Initial data for a dynamical evolution:

$$\begin{aligned} \tilde{\gamma}_{ij} = \delta_{ij}, \qquad \psi = 1 + \frac{M}{2r} \\ K = 0, \qquad \tilde{A}_{ij} = 0 \\ \alpha = 1, \qquad \beta^i = 0. \end{aligned}$$

With this choice of lapse and shift, there will be nontrivial evolution.

“Fixed puncture” evolutions

The conformal factor diverges at the puncture.
Assume that we keep the wormhole topology during evolution, and write it as

$$\psi = \left(1 + \frac{M}{2r}\right) f,$$

and $\phi = \ln f = 0$ initially. Then evolve $\phi = \ln f$.

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- Sometimes works for single black holes, head-on collisions, orbiting binaries
- Always needs a lot of fine-tuning of gauge parameters.
- By definition, the puncture is always under-resolved.
- Gauge parameters chosen such that $\beta^i = O(r^3)$ at the punctures.
  $\Rightarrow$ even for binaries, punctures are fixed on the grid.
- The evolution does not find a stationary slice. (Reimann and Brügmann, ’04)

Example: "Fixed puncture" evolution of Schwarzschild

Evolve using

- initial data of Schwarzschild in isotropic coordinates (from earlier slide)
- $\alpha = 1$ and $\beta^i = 0$ initially
- BSSN "fixed puncture" reformulation of the 3+1 evolution equations
- $\tilde{\Gamma}$-driver shift evolution
- 1+log slicing, $\partial_t \alpha = -2\alpha K$
  $\Rightarrow$ For a stationary solution, $\partial_t \alpha = 0 \Rightarrow K = 0$, maximal slicing.

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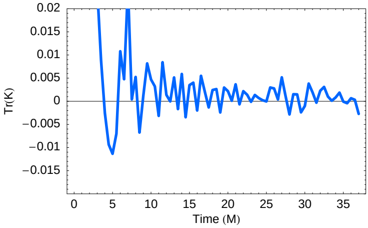
Look at value of $Tr(K)$ on the (outer) horizon $R = 2M$:



“Moving punctures”

Now llet (Goddard)

$$\phi = \ln \psi = \ln \left(1 + \frac{m}{2r}\right),$$

or (UTB)

$$\chi = \psi^{-4},$$

and evolve ϕ or χ. (Don’t assume anything about ψ.)

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BINARY BLACK HOLE PROBLEM SOLVED!

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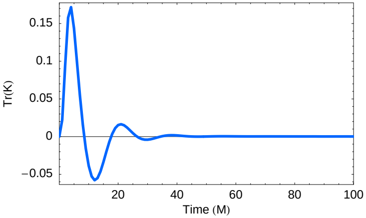
The “moving punctures” package:

- BSSN (with ϕ or χ variables)
- Singularity-avoiding slicing (maximal, 1+log, ...)
- $\tilde{\Gamma}$-freezing shift evolution
- “Puncture” initial data

"Moving puncture" evolution of Schwarzschild

Using "maximal" 1+log slicing, $\partial_t \alpha = -2\alpha K$, and "$\tilde{\Gamma}$-driver" shift evolution.



Reaches stationary (maximal) slice in about $40M$.

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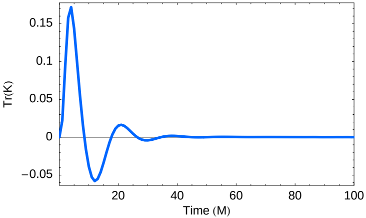

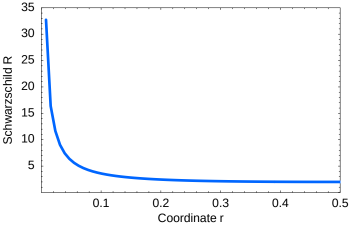

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Evolution of Schwarzschild $R(r)$: $T = 0$.

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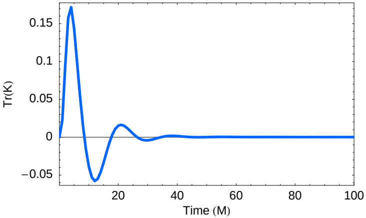

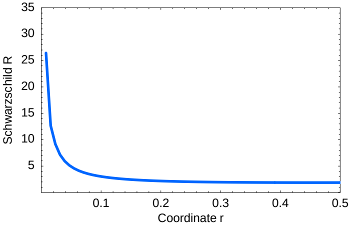

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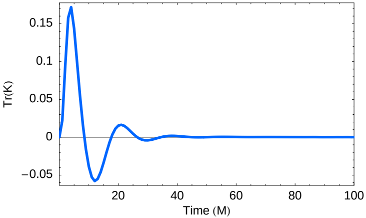
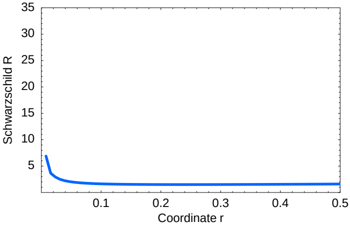
Evolution of Schwarzschild $R(r)$: $T = 1M$.

“Moving puncture” evolution of Schwarzschild

Using “maximal” 1+log slicing, $\partial_t \alpha = -2\alpha K$, and “$\tilde{\Gamma}$-driver” shift evolution.



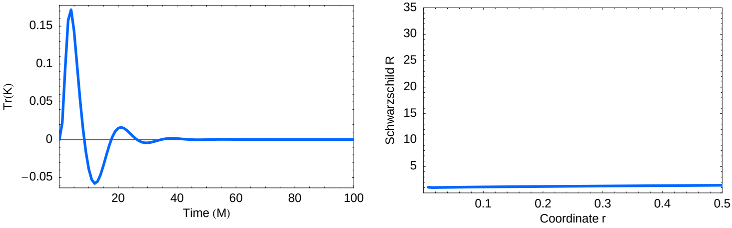


Reaches stationary (maximal) slice in about $40M$.

Evolution of Schwarzschild $R(r)$: $T = 2M$.

"Moving puncture" evolution of Schwarzschild

Using "maximal" 1+log slicing, $\partial_t \alpha = -2\alpha K$, and "$\tilde{\Gamma}$-driver" shift evolution.



Reaches stationary (maximal) slice in about $40M$.

Evolution of Schwarzschild $R(r)$: $T = 3M$.

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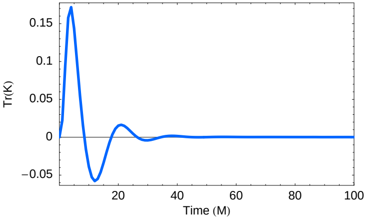

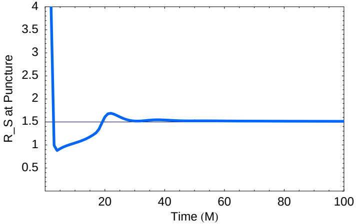

Reaches stationary (maximal) slice in about $40M$.

Evolution of Schwarzschild $R(r = 0)$: slice ends at $R = 3M/2$.

Slice loses contact with other asymptotically flat end.

This should not be a surprise...

- Estabrook *et. al.*, PRD 7 (1973) 2814, derive an analytic maximal slicing of Schwarzschild for all time.
- $t = 0$ limit: Schwarzschild spatial metric with $\alpha = 1$ and $\beta^i = 0$.
- This is the starting point for our numerical evolution!

$$\begin{aligned} \gamma_{rr} &= \left(1 - \frac{2M}{r}\right)^{-1} \\ \beta^r &= 0 \\ \alpha &= 1. \end{aligned}$$

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- Estabrook *et. al.*, PRD 7 (1973) 2814, derive an analytic maximal slicing of Schwarzschild for all time.
- $t \to \infty$ limit: slice ends on a cylinder of radius $R = 3M/2$!

$$\begin{aligned} \gamma_{rr} &= \left(1 - \frac{2M}{r} + \frac{C^2}{r^4}\right)^{-1} \\ \beta^r &= \frac{\alpha C}{r^2} \\ \alpha &= \sqrt{1 - \frac{2M}{r} + \frac{C^2}{r^4}}, \end{aligned}$$

with $C = 3\sqrt{3}/4$.

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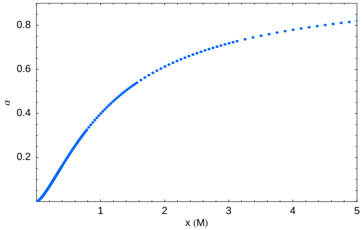
\end{aligned}
$$

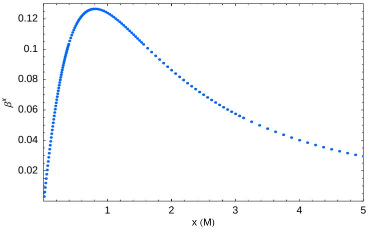
with $C = 3\sqrt{3}/4$.

- In evolution, $\psi \sim \frac{M}{2r} \Rightarrow \psi \sim \sqrt{\frac{3M}{2r}}$.
- With "new 1+log": slice ends at $R = 1.3M$.
(Hannam, Husa, Pollney, Brügmann, Ó Murchadha, gr-qc/0606099.)

“Cylindrical” initial data

- Take the $t \to \infty$ limit of the Estabrook *et. al.* solution
- Map to conformal coordinates in which $\psi \sim \sqrt{\frac{3M}{2r}}$ at the puncture.
- (Solve the Hamiltonian constraint for ψ with a 1D code.)
- Reconstruct $\alpha, \beta^i, \tilde{A}_{ij}$, in these conformal coordinates.
- Now we have stationary data





Cylindrical data movie: promotional shots

Look at close-up of $\tilde{g}_{xx}$ during evolution.
(It should remain at $\tilde{g}_{xx} = 1$.)

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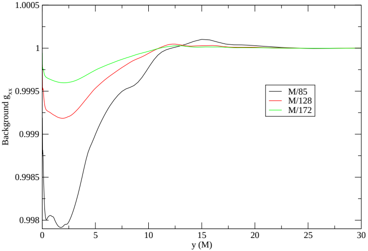

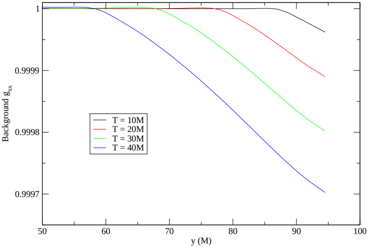

At puncture, 0.2% error after $20M$ for resolution $M/85$.
At boundary, 0.01% error per $20M$ of evolution.

An excellent environment to test and study the moving-puncture approach.

Conclusions

- "Moving punctures" quickly cease to be punctures
- The numerical solutions are well-resolved and accurate
- Puncture evolutions find the stationary solution in $\sim 40M$.
- Black holes move on the numerical grid, with their singularities elegantly avoided

Next steps

- Look for stationary 1+log/maximal puncture slicing of Kerr (the final state of BBH evolutions!)
- What happens when matter and radiation are present?
- Construct "cylindrical" data for binaries.