Small-scale structures in dispersive MHD

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Abstract

Magnetohydrodynamics with Hall effect (Hall-MHD) allows one to take into account scales of the order of the ion inertial length and the dispersive character of media like the Earth magnetosheath.

Weakly nonlinear, quasi-monochromatic Alfvén waves propagating along an ambient magnetic field in such a medium can be subject to transverse instabilities leading to the formation of intense magnetic filaments. This "filamentation" phenomenon has been first predicted by amplitude equations of nonlinear Schrödinger type and then observed in numerical simulations of the Hall-MHD equations using a spectral inviscid code. We observed in particular that for sufficiently weak initial amplitudes the wave collapses and no saturation regime is reached at the resolutions of 128^3 actually available. The problem has then been reconsidered using a finite-differences AMR (Adaptive Mesh Refinement) code which allows an effective resolution of 512^3 in the neighbors of the filaments. The simulations was then proceeded until the destabilization of these filamentary structures. The magnetic field intensity saturates but singularities of gradient type, associated with intense current sheets and plasma acceleration tend to develop. Apart from the specific interest in the phenomenon, this problem constitutes a severe test case for the AMR codes, because of the formation of localized intense structures resulting from a strongly dispersive dynamics.

The model : Hall-MHD

The magnetohydrodynamics with Hall effect (Hall-MHD) gives a fluid description of magnetized plasmas taking into account scales of the order of the ion-inertial length $d_i = c/\omega_{pi}$, at which the dynamics of ions and electrons separates and the medium becomes dispersive. This is considered a more accurate model than the usual MHD to describe some astrophysical media like the Earth magnetosheath.

$$\begin{aligned}
&\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0\\
&\rho(\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}) = -\frac{\beta}{\gamma}\nabla \rho^\gamma + (\nabla \times \mathbf{b}) \times \mathbf{b}\\
&\partial_t \mathbf{b} = \nabla \times (\mathbf{u} \times \mathbf{b}) - \frac{1}{R_i}\nabla \times \left(\frac{1}{\rho}(\nabla \times \mathbf{b}) \times \mathbf{b}\right)\\
&\nabla \cdot \mathbf{b} = 0
\end{aligned}$$

Adimensionalization in the presence of an ambient field of magnitude $\mathcal{B}$:

$$v_A = \frac{\mathcal{B}}{\sqrt{4\pi\rho_0}}, \quad \Omega_i = \frac{e\mathcal{B}}{m_i c}, \quad \beta = \frac{c_s^2}{v_A^2}$$

Units fixed by choosing a value for $R_i = L\frac{\Omega_i}{v_A} = \Omega_i T$

Parallel-propagating Alfvén waves

Finite-amplitude circularly-polarized Alfvén waves propagating along the ambient magnetic field $\mathcal{B}\hat{x}$ represent an exact solution of Hall-MHD equations:

$$\begin{aligned}
&b_y - \sigma i b_z = -\frac{\omega}{k}(u_y - \sigma i u_z) = B e^{i(kx-\omega t)}\\
&b_x = 1, \qquad \rho = 1, \qquad u_x = 0, \qquad\qquad \sigma = \pm 1(\mathrm{RH/LH})
\end{aligned}$$

Dispersive character of Hall-MHD Alfvén waves :

$$\omega = \frac{\sigma k^2}{2R_i} + k\sqrt{1 + \left(\frac{k}{2R_i}\right)^2} \qquad\qquad k^2 = \frac{\omega^2}{1 + \sigma\frac{\omega}{R_i}}$$

Filamentation of Alfvén waves

In appropriate regimes for the plasma β and the wavenumber, parallel circularly-polarized Alfvén waves in Hall-MHD are unstable to transverse perturbations. This instability leads to a concentration of the wave energy into tubes parallel to the ambient field, with a local increase of the wave amplitude (Alfvén-wave filamentation)[1, 2].

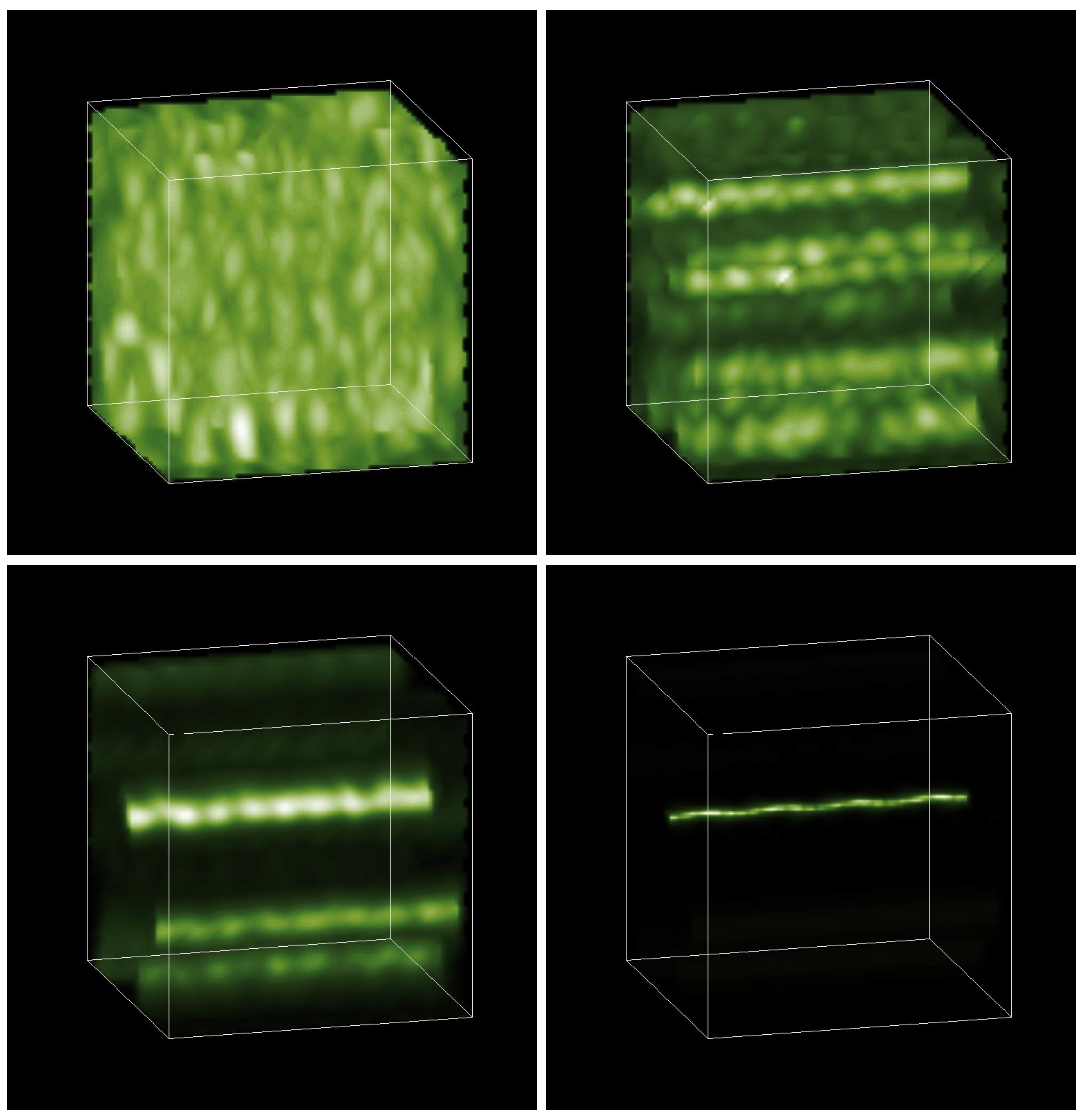

Figure 1: Simulation of Alfvén-wave filamentation with a spectral code: the energy of the wave concentrates into magnetic filaments (t=25,200,500,640)

The NLS asymptotic equation. Wave collapse

Finite-amplitude Alfvén waves in Hall-MHD belong to the general class of weakly nonlinear dispersive waves. In this context, a 2D Nonlinear Schrödinger (NLS) equation can be derived to describe the modulation of the complex wave envelope $B\ :\ \epsilon B e^{i(kx-\omega t)} = b_y \pm i b_z$ on times $\sim \epsilon^{-2}$ at large scales $\approx 1/\epsilon$ in a plane transverse to propagation.

$$i\partial_\tau B + \alpha \Delta_\perp B + \gamma |B|^2 B = 0$$

The NLS equation allows to recover the filamentation phenomenon until a wave collapse occurs, with the formation of a finite-time singularity for the wave amplitude, associated to the breaking of the asymptotics.

Wave collapse in 3D Hall-MHD? Which possible mechanism for saturation?

In some regimes, Alfvén wave filamentation is weak and the magnetic amplitude saturates. In other regimes instead the filamentation is so strong that the available numerical resources do not allow to reach a possible saturation stage with a spectral code. The question arises about the possibility of wave collapse towards a finite-time singularity, or the emergence of some saturation mechanism not described by the asymptotic NLS equation.

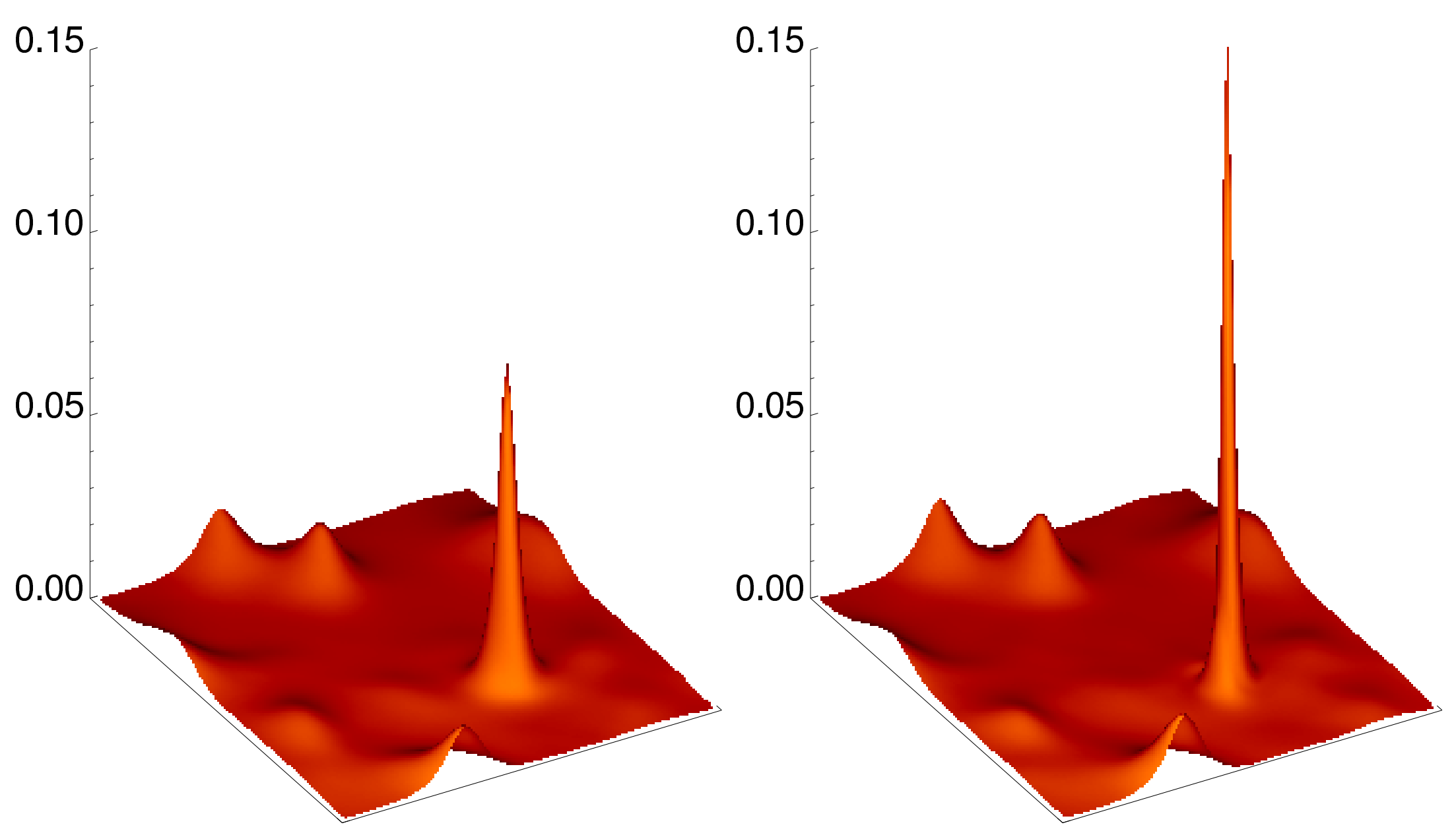

Figure 2: Spectral simulation of filamentation in Hall-MHD at 128^3 : wave intensity in a magnetic filament at t=640 (left), t=660 (right). No insights about saturation emerged so far, apart from a longitudinal modulation not described by NLS

AMR simulation of late-stage filamentation

An Adaptive Mesh Refinement (AMR) finite-differences code[3] has been used to investigate the possible saturation of magnetic filaments. Local refinement around the tube allows to follow the late-stage of filamentation more easily than with a spectral code. The equivalent maximal resolution reached is 512^3.

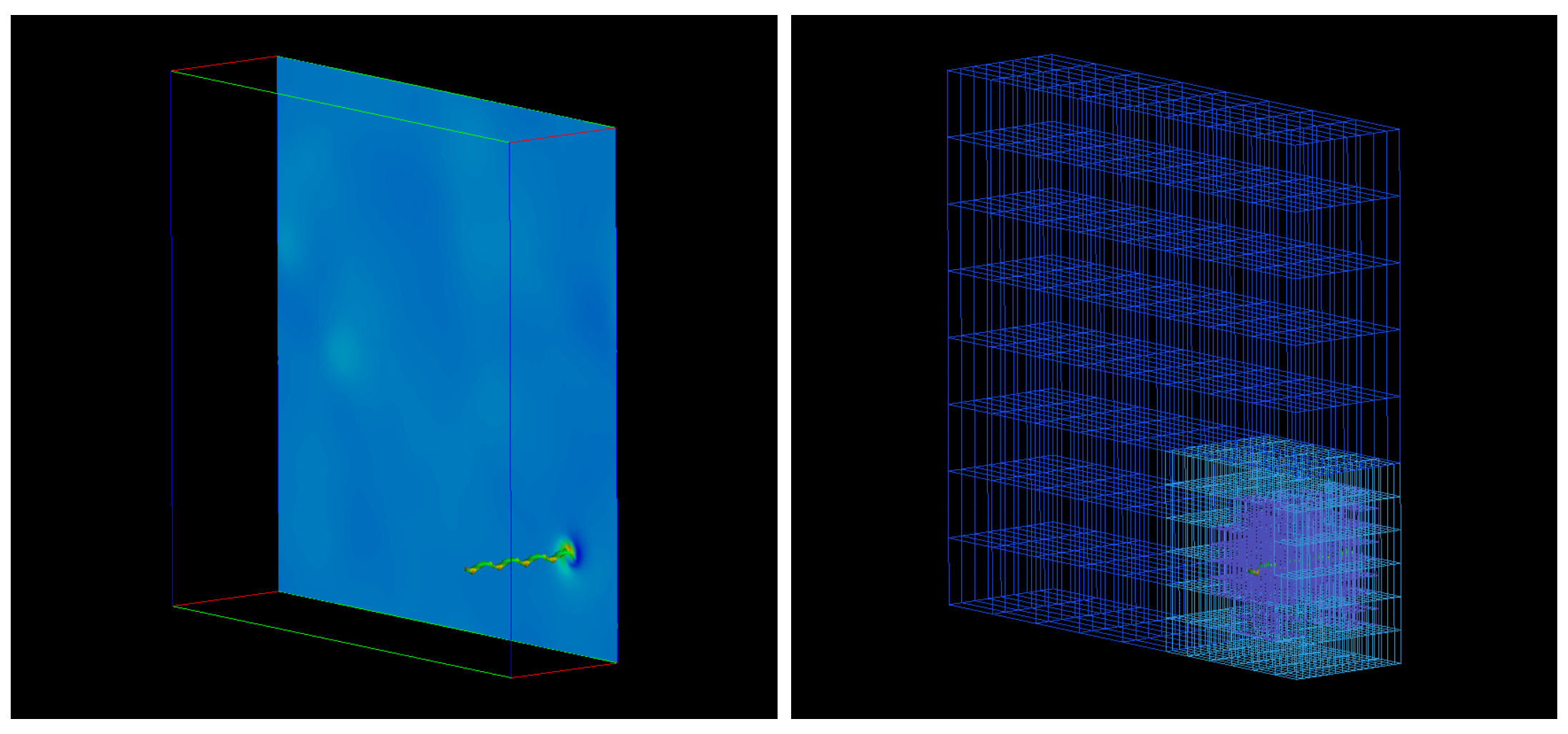

Figure 3: AMR simulation of filamentation: isosurface of the magnetic amplitude $|\mathrm{b}_\perp|$ (left) and locally refined mesh (right) at t=667.5

AMR simulations show that the magnetic filaments are longitudinally modulated and tend to an helical shape with a flattened section.

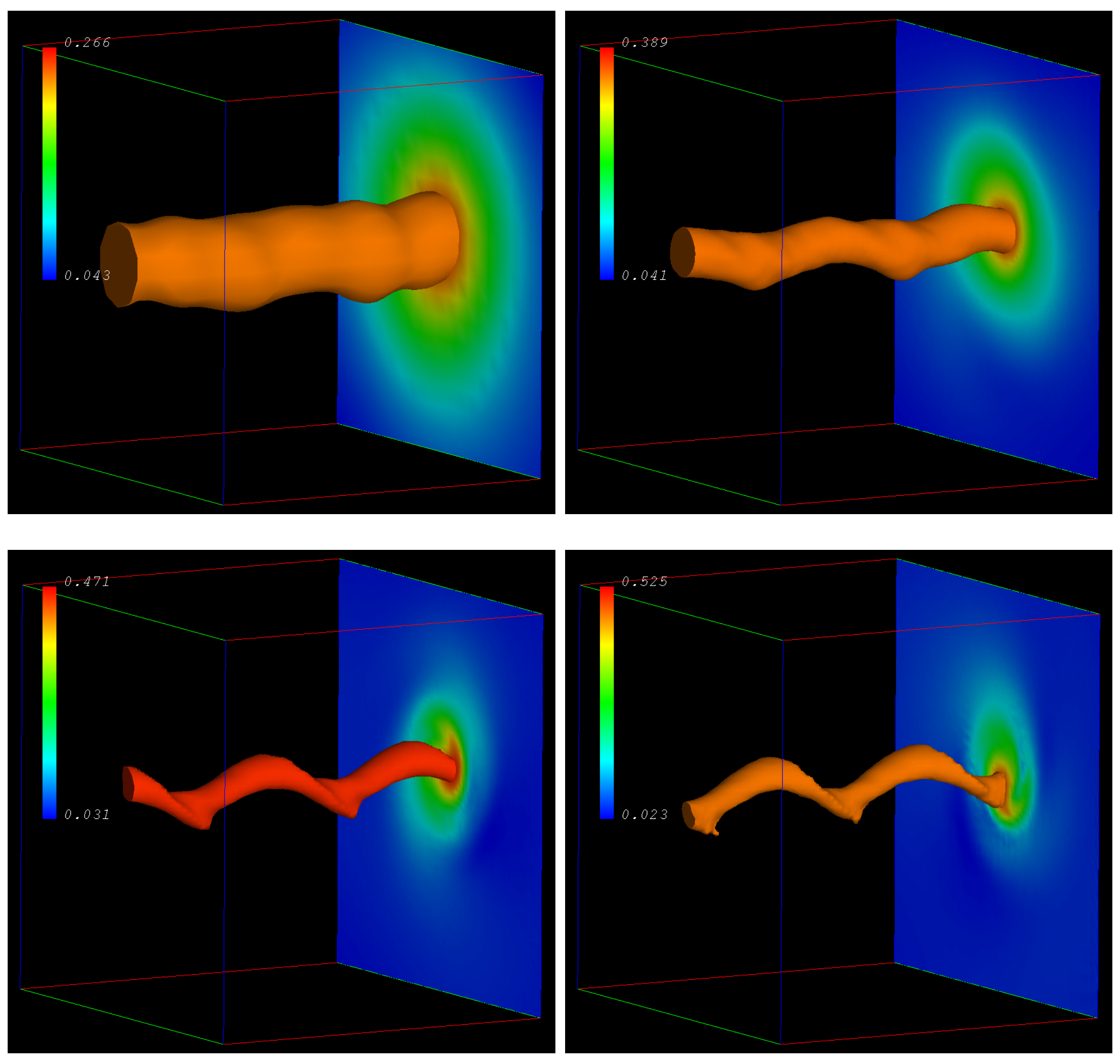

Figure 4: AMR simulation of filamentation: isosurface and transverse section of the magnetic amplitude $|\mathrm{b}_\perp|$ at t=631, t=653, t=665, t=667.5. Colors represent the magnitude of $|\mathrm{b}_\perp|$.

Growth of magnetic field and plasma velocity

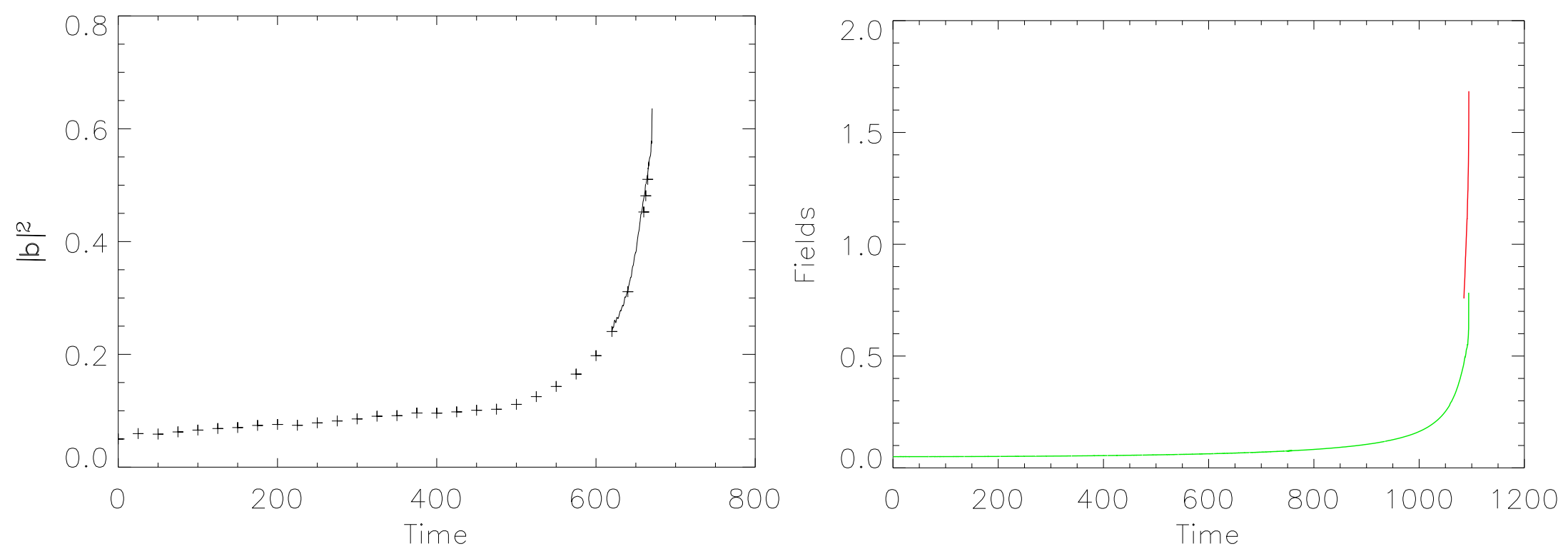

Figure 5: Left : maximum of the magnetic amplitude $|\mathrm{b}_\perp|$ (crosses: spectral simulation; continuous line: AMR restart from spectral simulation); Right : maxima of magnetic field amplitude $|\mathrm{b}_\perp|$ (green) and of plasma longitudinal velocity u_x (red) in a fully AMR simulation

Counter-propagating streams into the filament : plasma acceleration

A strong two-stream jet forms in the interior of tubes for the plasma velocity and the current. The filamentation process leads then to plasma acceleration, the plasma velocity becoming **superalfvenic** (see Fig. 5).

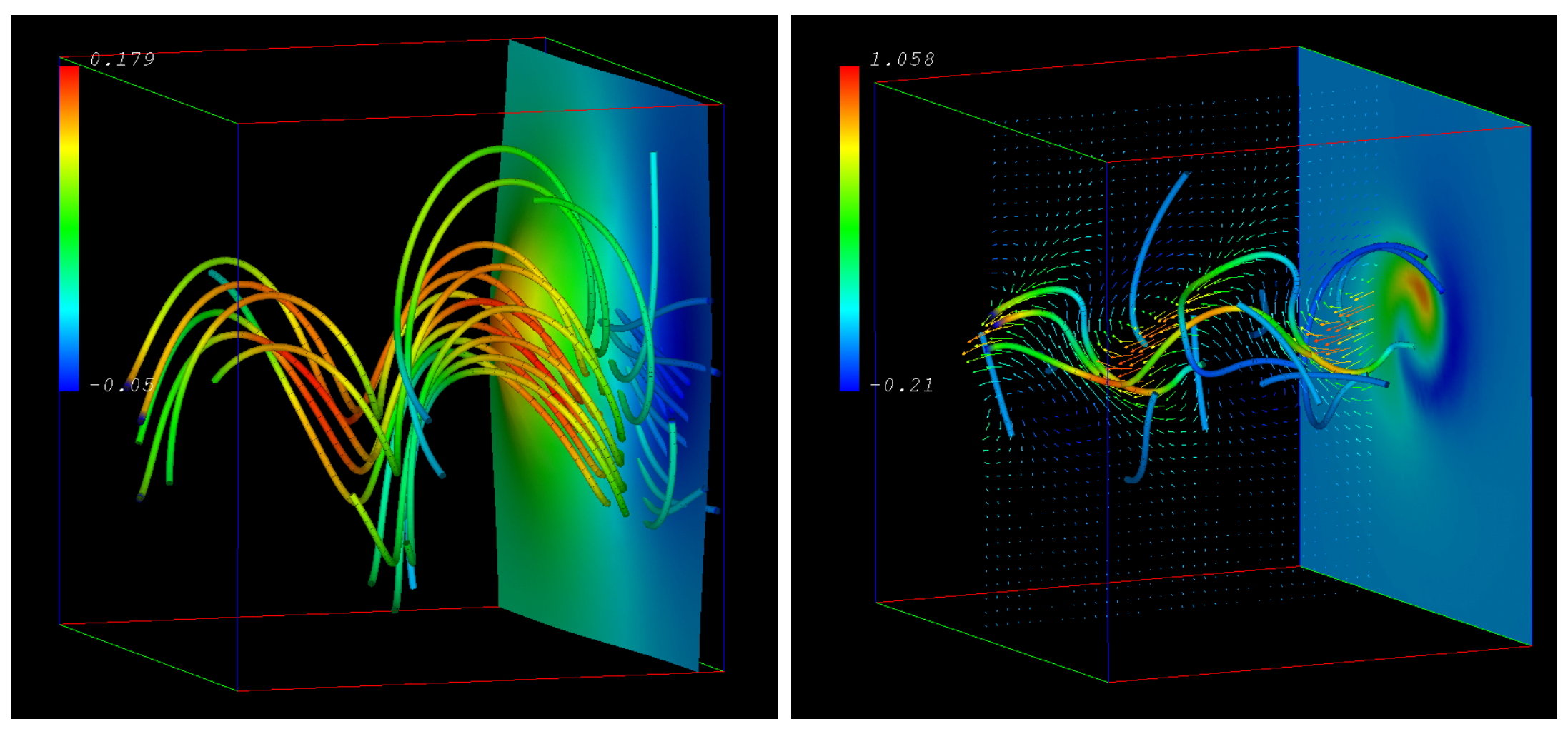

Figure 6: AMR simulation of filamentation: streamlines of the plasma velocity u; transverse cut for its longitudinal component u_x; field u on a plane cut (arrows). Colors represent u_x. Left (t=631) : the flow is still Alfvénic. Right (t=665): Afvenicity is destroyed and two counter-propagating streams develop.

Plasma acceleration. Longitudinal steepening of plasma tubes

The magnetic filaments are considerably distorted: they **steepen** on the back side and eventually break in the longitudinal direction. This process is related to the development of intense plasma jets inside the filaments in counter-propagating directions and with a helical symmetry. They compress the plasma on a helicoidal sheet steepened on the inflow side.

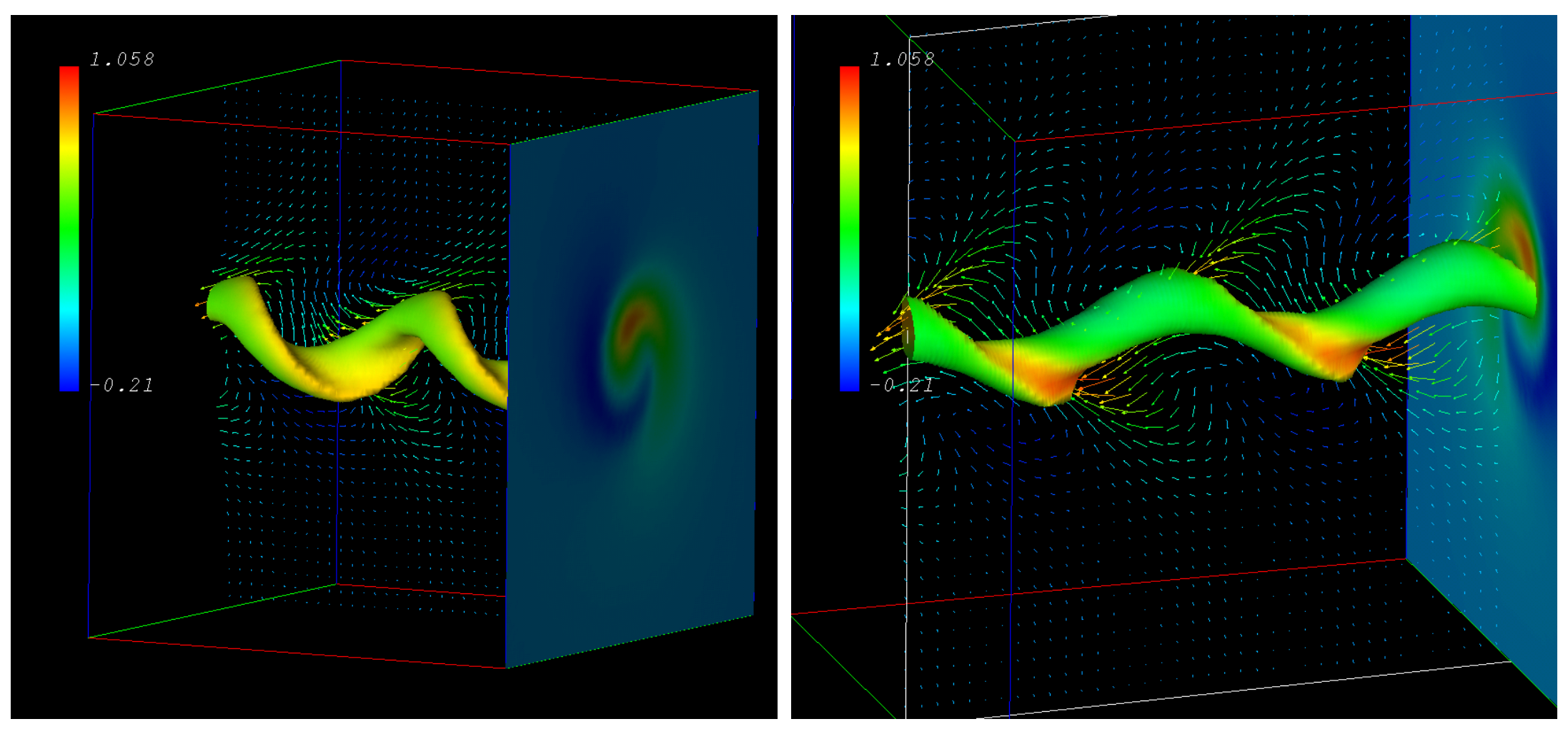

Figure 7: AMR simulation of filamentation(t=665): isosurfaces of the plasma velocity magnitude $|\mathrm{u}|$ (left) and of the magnetic field magnitude $|\mathrm{b}|$ (right); transverse cut for the longitudinal velocity u_x; plasma velocity field u on a plane cut (arrows). Colors represent u_x. Plasma tubes undergo steepening.

AMR + CWENO: saturation of filamentation

AMR alone without the use of an appropriate scheme cannot capture the saturation of filamentation. The formation of strong shocks requires a shock-capturing procedure. Simulations using a CWENO scheme show that **the magnetic amplitude eventually saturates**, the plasma **remains superalfvenic** and the magnetic tubes are longitudinally broken.

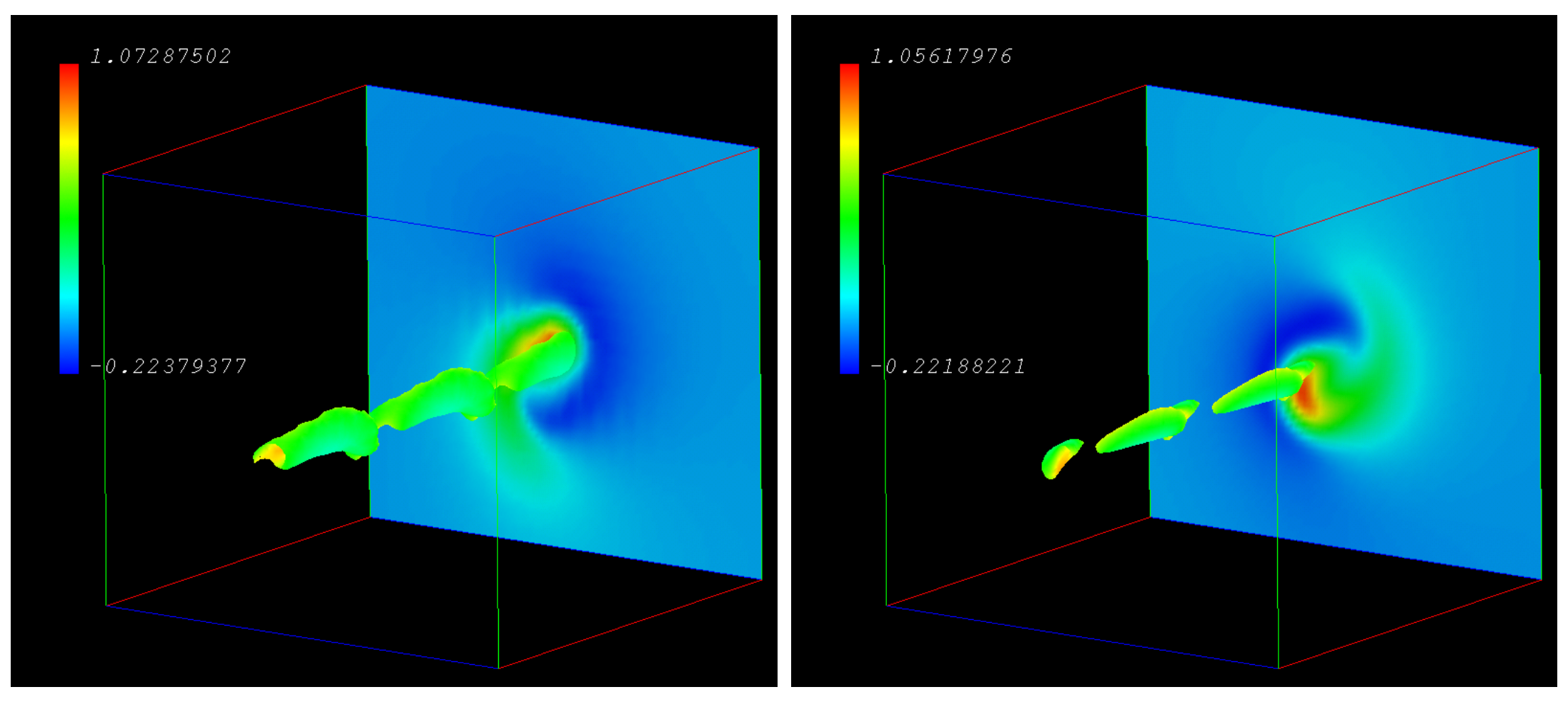

Figure 8: AMR simulation of filamentation with CWENO: saturation stage, with filaments breaking and stopping of magnetic amplitude growth .

Conclusions and perspectives

- Alfvén-wave filamentation is a phenomenon susceptible of generating magnetic filaments in some astrophysical plasmas.
- While spectral simulations do not allow in some cases to conclude about the saturation of filaments, AMR simulation can reveal that magnetic amplitude saturates and the system evolves toward gradient singularities. Moreover, strong plasma acceleration takes place.[4]
- Is this a generic mechanism for the destruction of flux tubes in magnetized plasmas?
- Is plasma acceleration and small-scale formation generic in Hall-MHD?

References

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