DECAY INSTABILITY OF PARALLEL ALFVEN WAVES IN COLLISIONLESS PLASMAS: A TEST PROBLEM

P.L. Sulem
Observatoire de la Côte d'Azur

LANDAU FLUID MODELS involve heuristic closures based on fitting with linear kinetic theory. They ARE TO BE VALIDATED by comparison with kinetic simulations.

Possible benchmarks: Instabilities of MHD waves and their nonlinear developments.

In particular, ***parallel-propagating circularly-polarized Alfvén waves***:

- exact solutions of MHD, Hall-MHD, Landau fluids
- require simplest Landau fluid description:
  - gyrotropic heat flux tensors
  - lowest order FLR contribution to pressure tensors

DECAY INSTABILITY PROVIDES A SHARP TEST.

Main types of wave instabilities [Hollweg, JGR 99 (A2), 23431 (1994)]:

k_0 : pump wavenumber

k : wavenumber of the density perturbation

Modulational instability : $k < k_0$ (REQUIRES DISPERSION)

Decay instability: $k_0 > 0$

Beat instability: $k \approx k_0$

MODULATIONAL INSTABILITY: Generalization of the usual long-wave modulational instability of small amplitude dispersive wave that

extends up to $k = 0$ (slowly modulates the pump amplitude)

is amenable to an analytic description (envelope equations) $(*)$

is captured by long-wave asymptotics (KDNLS equation) $(*)$

is well reproduced by Landau fluids [Passot & Sulem, PoP 10, 3906 (2003)]

for transverse perturbations, leads to Alfvén wave filamentation [Passot & Sulem, PoP 10, 3914 (2003)]

$(*)$ For a discussion in the fluid (Hall MHD) context, see Champeaux, Laveder, Passot & Sulem,, NPG 6, 169 (1999).

Decay instability

No dispersion (MHD): Jayanti & Hollweg, JGR 98 (A8) 13247 (1993).

Pump wave (assumed ambient field along the z-direction):

$$b_\pm = b_x \pm i b_y \propto e^{\pm i(k_0 z - \omega_0 t)}$$
$$v_\pm = v_x \pm i v_y \propto e^{\pm i(k_0 z - \omega_0 t)}$$

with ω_0 and k_0 related by the Alfvén wave dispersion relation.

Equations linearized about Alfvén wave have PERIODICALLY VARYING COEFFICIENTS. General framework: FLOQUET theorem.
Here the problem strongly simplifies.

Density perturbation: $\delta\rho = \overline{\rho} e^{i(kz - \omega t)} + \mathrm{c.c.}$

The (linearized) equations then prescribe the functional form of the perturbations of the other variables.

δv_z has the same form as $\delta\rho$

Magnetic field perturbations $\delta b_{\pm} = \delta b_x \pm i\delta b_y$ (sideband waves)

δb_+ combination of $e^{i(k_+ z - \omega_+ t)}$ and $e^{-i(k_- z - \omega_-^* t)}$ with the resonance condition

$$\omega_{\pm} = \omega_0 \pm \omega \qquad k_{\pm} = k_0 \pm k$$

The result of the analysis is the dispersion relation that prescribes the stability/instability condition.
See also Webb, Zakharian, Brio & Zank, J. Plasma Phys. 66, 167 (2001)

Physically, a forward propagating Alfvén wave DECAYS into a forward propagating magnetosonic wave and a backward propagating Alfvén wave.
(Forward propagating sideband wave has negligible power)

Decay instability is of particular interest in the solar wind, since it can provide the backward propagating Alfvén wave needed for MHD turbulence.

Del Zanna, Velli & Londrillo, AA 367, 705 (2001): simulations in 1, 2 and 3 D for a large amplitude pump: very strong backscattered daughter.

In the presence of dispersion (Hall-MHD)

Hoshino & Goldstein, Phys. Fluids B 1 (7), 1405 (1989).

(do not consider the beat instability)

For a low-frequency pump,

for $\beta < 1$, RHP and LHP Alfvén waves are unstable for decay instability
for $\beta < 1$, LHP Alfvén waves are unstable for modulational instability
for $\beta > 1$, RHP Alfvén waves are unstable for modulational instability

Furthermore (see Hollweg 1994),
for $\beta > 1$, beat instability can occurs for a
LHP wave whose amplitude is not too large
RHP wave whose amplitude is large enough.

Decay instability: A forward propagating Alfvén wave decays into a forward propagating sound wave and a backward propagating Alfvén wave

Beat instability: Interaction between forward propagating upper side band and backward propagating lower side band. Primary produces outward propagating waves. May provide a mechanism for direct transfer. Survives with no dispersion.

Jayanti & Hollweg, JGR 98 (A11) 19049 (1993)

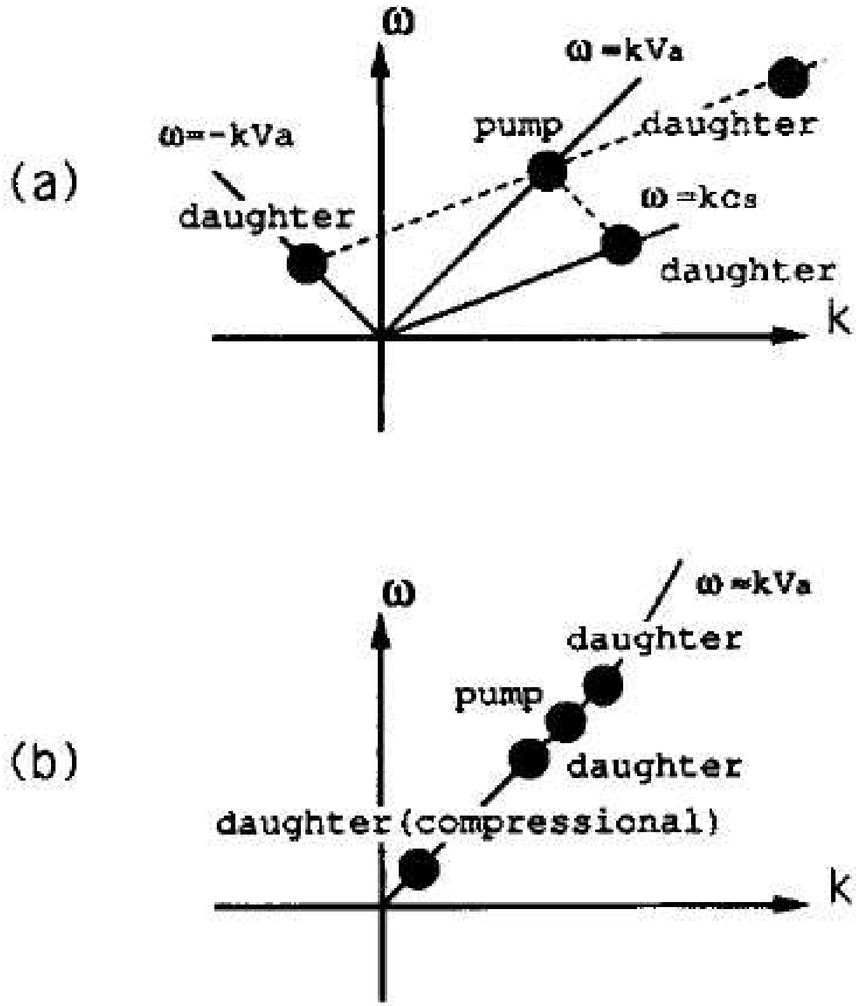


FIG. 1. Schematic representation of the wave resonance conditions for (a) decay instability and (b) modulational instability.

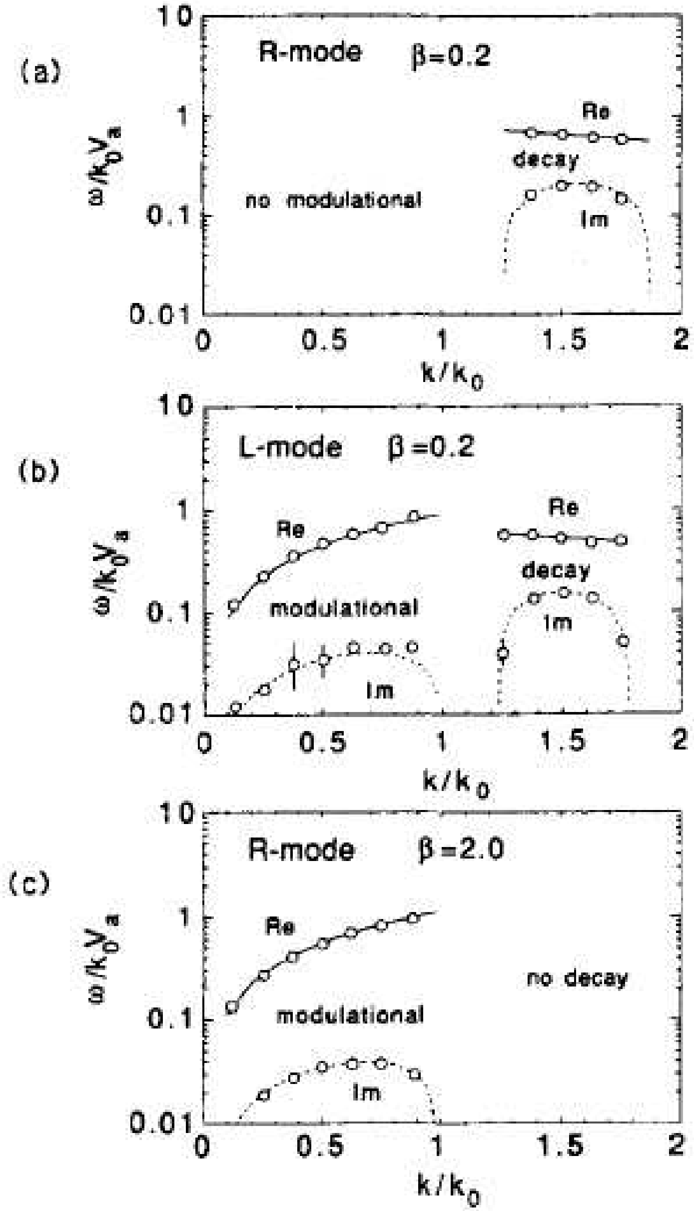
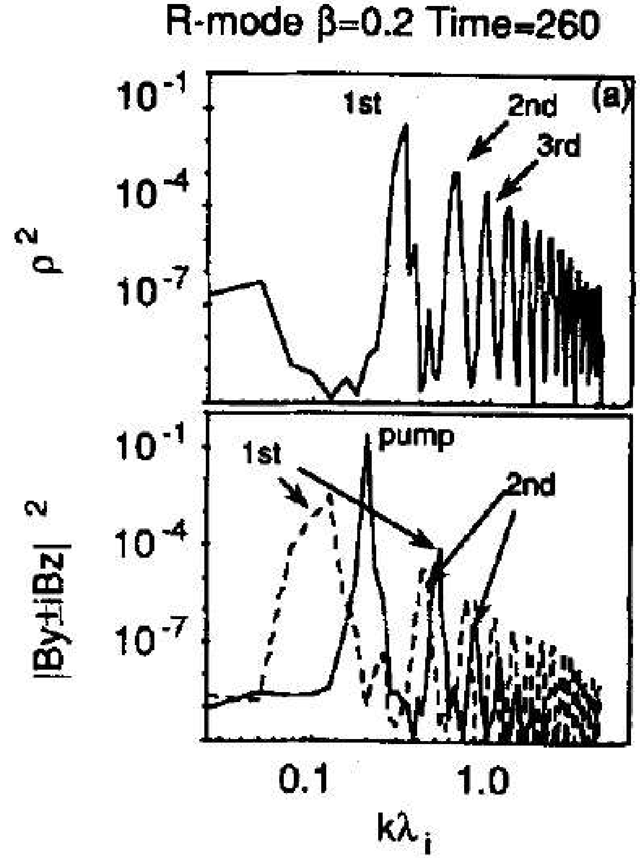


FIG. 4. The linear growth rate and wave frequency from both theory (solid and dashed lines indicate wave frequency and growth rate, respectively) and simulation (open circles) for cases (a), (b), and (c). The simulation results are computed from Fig. 3 during the interval $50 < t < 300$.

Decay instability



Strong peak at $k_0 = 0.2$ (solid line) in the lower panel: pump wave.
Strong peak with $k_d = 0.3$: daughter compressional (ion acoustic like) wave.
Second and third peaks with $k \approx k_0 - k_d$ (dashed line) and $k \approx k_0 + k_d$ (solid line) in the lower panel are daughter waves.
Higher harmonics at $k \approx 0.6 = 2k_d$, $k \approx 0.9 = 3k_d$, $\cdots$ for compressional waves and $k \approx 0.2 \pm 0.6 = k_0 \pm 2k_d$, $k \approx 0.2 \pm 0.9 = k_0 \pm 3k_d$, $\cdots$ for transverse waves.

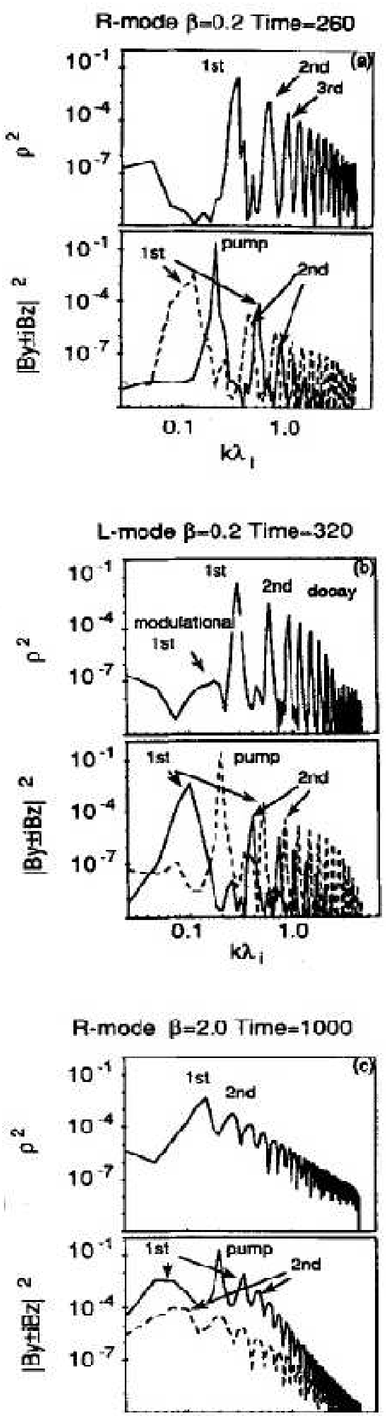


FIG. 5. The power spectra in k space for density and magnetic fields for positive helicity (positive propagating RHP mode or negative propagating LHP mode, solid line) and negative helicity (negative propagating RHP mode or positive propagating LHP mode, dashed line) for cases (a), (b), and (c).

Decay instability: from linear to nonlinear stage

⋆ The power in the fundamental compressional wave is larger than in the transverse wave.
⋆ Steepening of the compressional wave (ion acoustic wave) to a SHOCK LIKE STRUCTURE (not subject to dispersion).
⋆ Excitation of higher harmonic waves: initiation of a DIRECT CASCADE of energy to small scales.
⋆ High harmonic transverse Alfvén-like waves do not satisfy the normal dispersion relation: slower transfer from the fundamental to higher harmonics.
⋆ Excitation of a $k = 0$ parallel velocity mode that subsequently contributes to the coupling of high harmonic waves: broadband turbulence in which individual harmonic peaks loose their distinct identity.
⋆ Instability saturation: nonlinear couplings affect the dispersion relation. Shift in frequency (originating from a modification of plasma medium by the excited daughter waves) that destroys the wave resonance condition, then stabilizing the wave coupling.

Retain kinetic effects:

⋆ hybrid simulations [Vasquez, JGR 100 (A2) 1779 (1995)]
⋆ Landau fluids [Passot &Sulem, NPG 11, 609 (2004)]
⋆ Drift-kinetic analysis at scales large enough for dispersion to be negligible [Inhester, JGR 95 (A7) 10525 (1990)].

In the non dispersive limit, kinetic effects (Landau damping) decrease the rate of the decay instability and increase the wavenumber range over which the instability exists.

Reproduced with Landau fluids [Bugnon, Passot & Sulem, NPG 11, 609 (2004)]

Landau fluid model for parallel propagation:

(Ambient field and propagation along the x-direction).

$$\partial_t \rho + \partial_x(\rho u_x) = 0$$

$$\partial_t u_x + u_x \partial_x u_x + \frac{\beta_{\|p}}{2M_a^2}\frac{1}{\rho}\partial_x(p_{xx} + \pi_{xx}) + \frac{1}{M_a^2}\frac{1}{\rho}\partial_x \frac{|b|^2}{2} = 0$$

$$\partial_t u_y + u_x \partial_x u_y + \frac{\beta_{\|p}}{2M_a^2}\frac{1}{\rho}\partial_x(p_{xy} + \pi_{xy}) - \frac{b_x}{M_a^2}\frac{1}{\rho}\partial_x b_y = 0$$

$$\partial_t u_z + u_x \partial_x u_z + \frac{\beta_{\|p}}{2M_a^2}\frac{1}{\rho}\partial_x(p_{xz} + \pi_{xz}) - \frac{b_x}{M_a^2}\frac{1}{\rho}\partial_x b_z = 0$$

$$\partial_t b_y = \partial_x(b_x u_y - u_x b_y + \frac{R_p b_x}{M_a}\frac{\partial_x b_z}{\rho} - \frac{\beta_{\|p}}{2}\frac{R_p}{M_a}\frac{1}{\rho}\partial_x p_{e,xz})$$

$$\partial_t b_z = \partial_x(b_x u_z - u_x b_z - \frac{R_p b_x}{M_a}\frac{\partial_x b_y}{\rho} + \frac{\beta_{\|p}}{2}\frac{R_p}{M_a}\frac{1}{\rho}\partial_x p_{e,xy})$$

The gyrotropic components of the pressure tensor are given by $p_{ij} = \sum_r p_{r,ij}$ with

$$p_{r,xx} = p_{\perp r} + (p_{\| r} - p_{\perp r})\frac{b_x^2}{|b|^2}$$

$$p_{r,xy} = (p_{\| r} - p_{\perp r})\frac{b_x b_y}{|b|^2}$$

$$p_{r,xz} = (p_{\| r} - p_{\perp r})\frac{b_x b_z}{|b|^2}$$

FLR corrections in the pressure tensor (leading order retained):

$$\pi_{xx} = 0$$

$$\pi_{xy} = M_a R_p (p_{\perp p} - 2p_{\| p})\partial_x u_z$$

$$\pi_{xz} = -M_a R_p (p_{\perp p} - 2p_{\| p})\partial_x u_y$$

$$\partial_t p_{\|r} + \partial_x(u_x p_{\|r}) + 2\frac{p_{\|r}}{|b|^2}(b_x^2 \partial_x u_x + b_x b_y \partial_x u_y + b_x b_z \partial_x u_z)$$

$$+\partial_x(\frac{b_x}{|b|} q_{\|r}) - 2q_{\perp r}\partial_x \frac{b_x}{|b|} = 0$$

$$\partial_t p_{\perp r} + \partial_x(u_x p_{\perp r}) + p_{\perp r}\partial_x u_x - \frac{p_{\perp r}}{|b|^2}(b_x^2 \partial_x u_x + b_x b_y \partial_x u_y + b_x b_z \partial_x u_z)$$

$$+\partial_x(\frac{b_x}{|b|} q_{\perp r}) + q_{\perp r}\partial_x \frac{b_x}{|b|} = 0$$

$$(\partial_t + u_x\partial_x + \frac{1}{(\frac{u_0}{v_{th,r}})}\frac{1}{\sqrt{\frac{8}{\pi}(1-\frac{3\pi}{8})}}\mathcal{H}\partial_x)q_{\|r} = \frac{1}{(\frac{u_0^2}{v_{th,r}^2})(1-\frac{3\pi}{8})}\partial_x(\frac{p_{\|r}}{\rho})$$

$$(\partial_t + u_x\partial_x - \frac{1}{(\frac{u_0}{v_{th,r}})}\sqrt{\frac{\pi}{2}}\mathcal{H}\partial_x)q_{\perp r} = -\frac{1}{(\frac{u_0^2}{v_{th,r}^2})}\partial_x[\frac{p_{\perp r}}{\rho} + (\frac{p_{\perp r}^{(0)}}{p_{\|r}^{(0)}} - 1)\frac{p_{\perp r}^{(0)}}{p_0}|b|]$$

$\mathcal{H}$ denotes the Hilbert transform (in Fourier space: $i \ \mathrm{sgn}\ k$)

$v_{thr} = \sqrt{T_{\|r}^{(0)}/m_r}$ is the thermal velocity of the particles of species r

$M_a = u_0/v_A$ is the Alfvénic Mach number (here taken equal to 1)

$\beta_{\|p} = 8\pi p_0/B_0^2$, related to $\beta = 8\pi P_0/B_0^2$ by $\beta = \beta_{\|p}(1 + T_e^{(0)}/T_p^{(0)})$

$R_p = \dfrac{v_A}{\Omega_p L_0}$ (where $\Omega_p = \dfrac{eB_0}{m_p c}$ is the proton gyrofrequency)

(ratio of the proton inertial length to the reference length scale L_0).

Nondispersive Landau fluid simulations

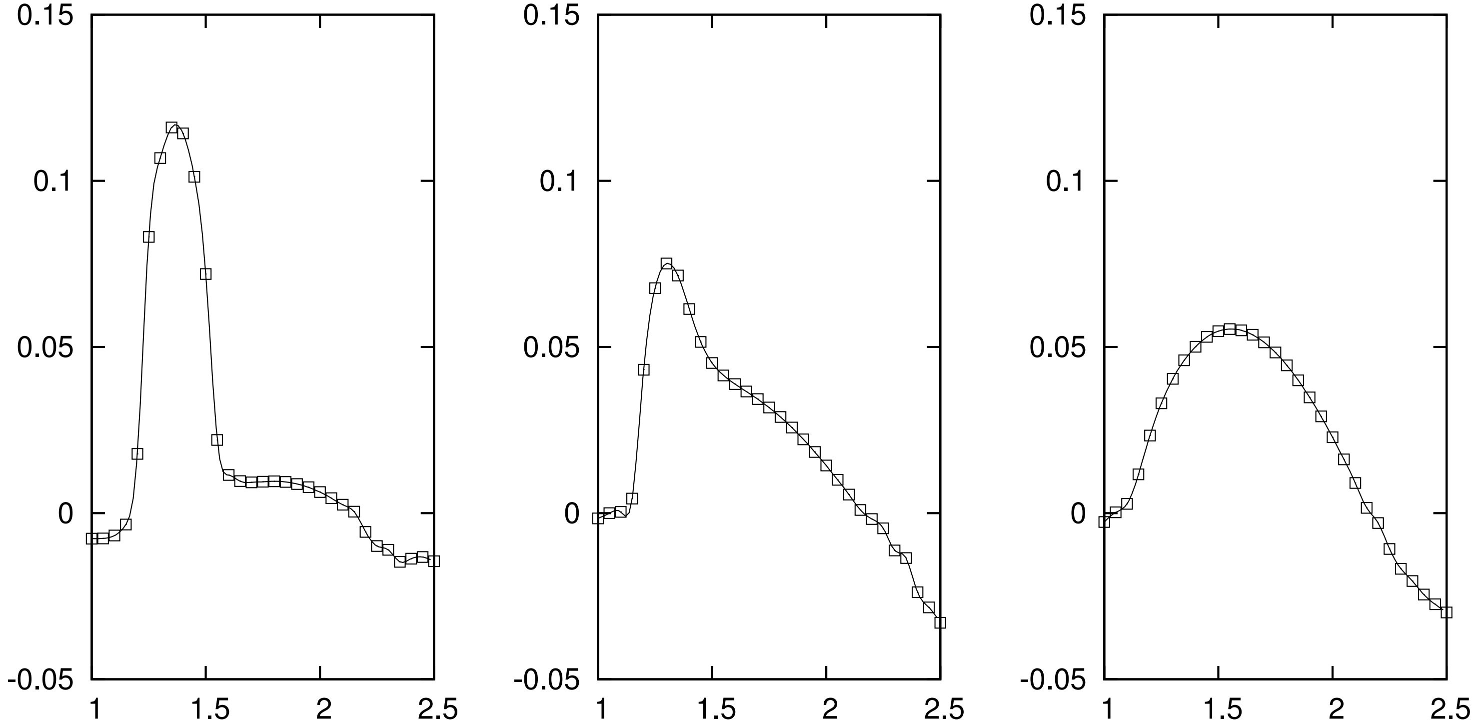


Figure 1: Growth rates of the density modes (whose wavenumber is normalized by the pump wavenumber) resulting from the decay instability of a non dispersive Alfvén wave of normalized amplitude $b_0 = 0.447$, propagating in a plasma with $\beta = 1.2$ and isotropic equilibrium temperatures of the electrons and ions in a ratio $T_e^{(0)}/T_p^{(0)} = 33$ (left), $T_e^{(0)}/T_p^{(0)} = 5$ (middle) and $T_e^{(0)}/T_p^{(0)} = 1$ (right) [INCREASING IMPORTANCE OF KINETIC EFFECTS].

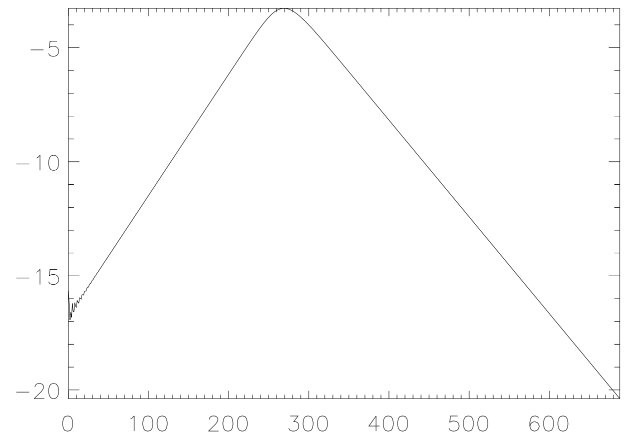


Figure 2: Time evolution of the amplitude of the most unstable density mode k=1.5 in lin-log scales, for $T_e^{(0)}/T_p^{(0)} = 1$.

Saturation by Landau damping and not by wave coupling (no other mode grows while the density mode decays).

Variation of the parallel and perpendicular temperatures of the protons and the electrons.

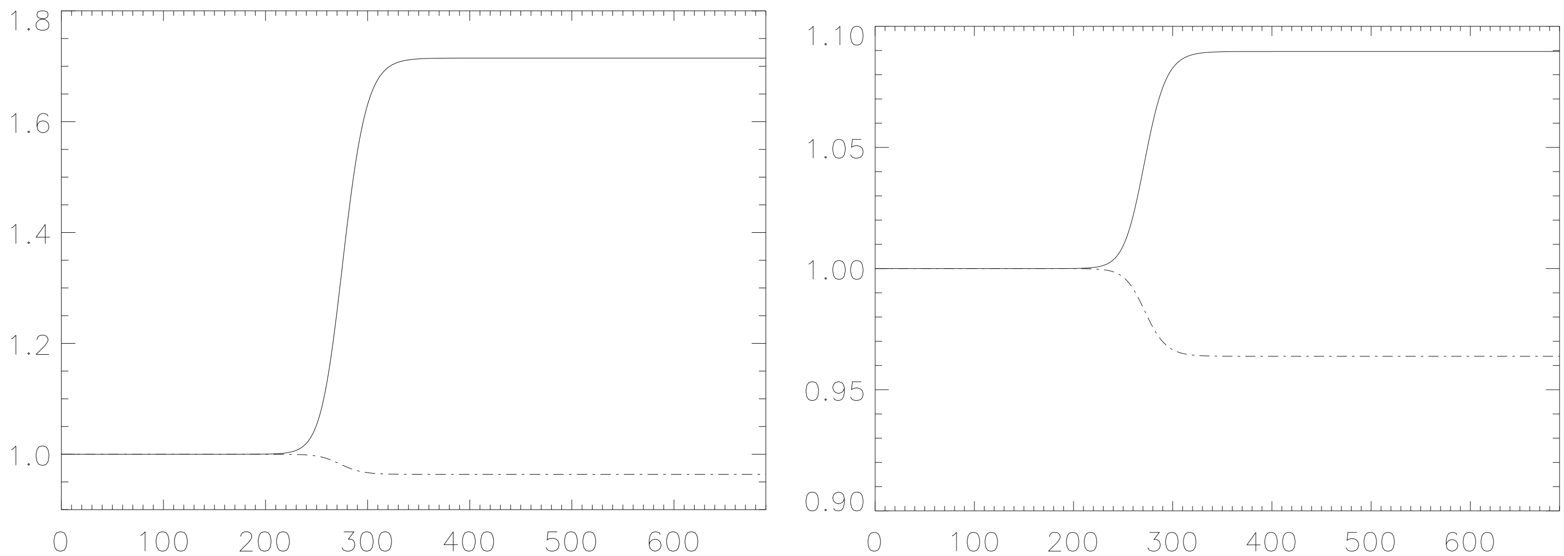


Figure 3: Time evolution of parallel (solid lines) and transverse (dashed-dotted lines) mean temperatures of the ions (left) and the electrons

Significant parallel heating of the ions (temperature growths of about 75 %). Electron heating non-negligible (temperature growths of about 8 %) ($\neq$ modulational instability where parallel electron temperature decreases).

Note that this “heating” survives when Landau damping is suppressed.

Effect of the dispersion:

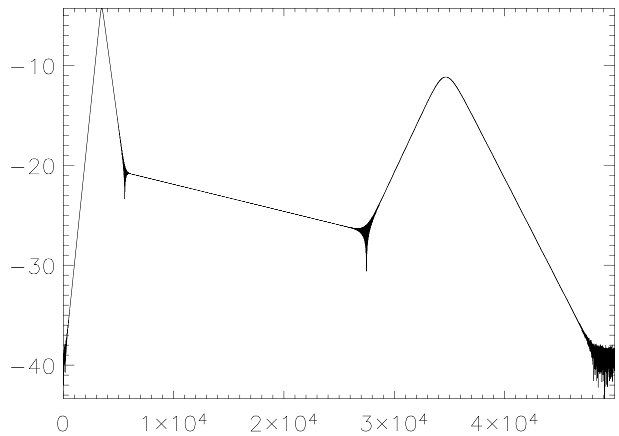
Hybrid simulations [Vasquez, JGR 100 (A2) 1779 (1995)]

Landau fluid simulations:

$R_p = 1$, $\beta = 0.42$, $T_p^{(0)} = T_e^{(0)}$

Forward propagating RHP pump

(amplitude 0.1, wavenumber $k_0 = 4 \times 2\pi/D = 0.64$ with $D = 6.25 \times 2\pi$)



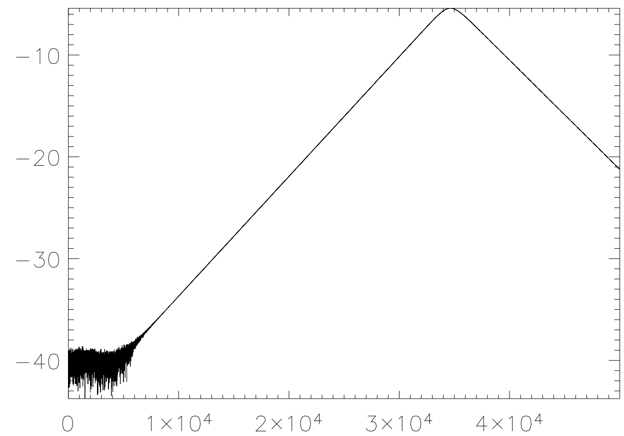


Figure 4: Time evolution of the amplitude of the density modes $m = 6$ (left) and $m = 3$ (right) in lin-log scales, for a right-hand polarized Alfvén wave of amplitude $b_0 = 0.1$, $k_0 = 0.64$, in a plasma with $R_p = 1$, $\beta = 0.42$ and equal electron and ion equilibrium temperatures.

Decay instability makes the density mode $m = 6$ to be the most unstable at short time.
Saturation due to Landau damping.
After a while mode $m = 3$ starts growing (induces a second increase of $m = 6$ as an harmonics of $m = 3$).
Further dynamics corresponds to an INVERSE CASCADE involving the successive amplification of the $m = 2$ BACKWARD and $m = 1$ FORWARD propagating Alfvén modes.

Pump with larger amplitude (for comparison with Vasquez)
RHP wave with amplitude: 0.5, wavenumber $k_0 = 0.408$ (normalized with the ion inertial length) i.e. $m_0 = 8$; $\beta = 0.45$.

$T_e^{(0)}/T_p^{(0)}$	most unstable mode	growth rate in units of Ω_p
44.	13	0.087
2.75	12	0.069
1.	12	0.059
0.36	13	0.056
0.	13	0.056

For a polytropic gas ($p \propto \rho^{5/3}$), growth rate is 0.092.

When $T_e^{(0)}/T_p^{(0)} = 44$, dynamics close to a fluid regime.
Generation of many harmonics.

$T_e^{(0)} = 0$, electrons remain cold (consistent with hybrid codes)
Inverse cascade where the excitation is transfered to larger and larger scales (up to $m = 1$), while the direction of the wave propagation switches at each step of the cascade, with a simultaneous increase of the ion parallel temperature.

Larger value of β: $\beta = 5$ with $b_0 = 0.5$, $k_0 = 0.408$ and $R_p = 0.1$

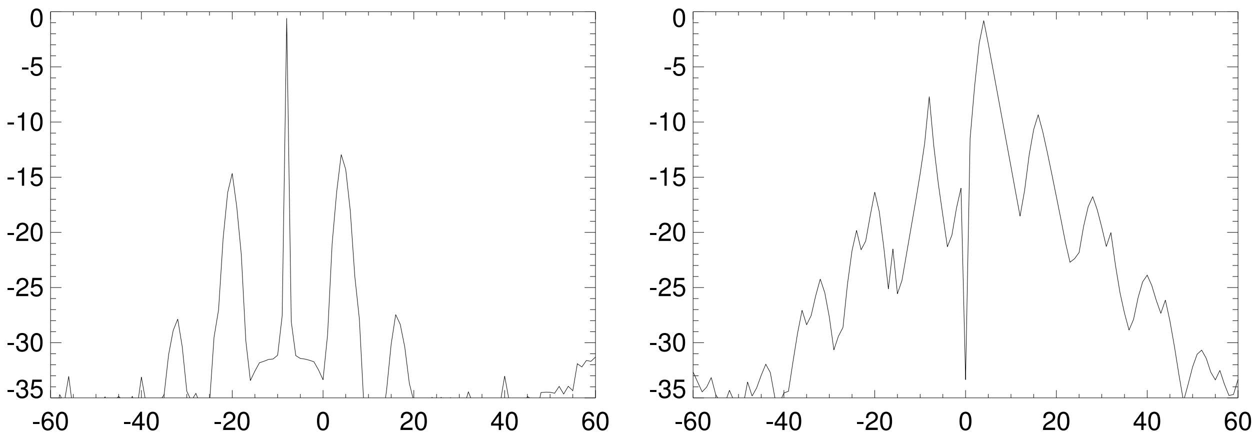


Figure 5: Spectral density (versus the wavenumber index) in lin-log10 scale for the (complex) quantity $b_+ = b_x + ib_y$ (the wavenumber sign indicating a helicity and thus, after the polarization is specified, the propagation direction) at time $t = 2000$ (left) and $t = 3700$ (right) belonging respectively to the linear and nonlinear phases for the instability of a right-hand polarized wave with amplitude $b_0 = 0.5$, $k_0 = 0.408$ in a plasma with $\beta = 5$, $R_p = 0.1$ and $T_i^{(0)}/T_e^{(0)} = 1.5$.

Decay instability, while fluid theory predicts modulational instability.

For comparison: MODULATIONAL INSTABILITY [LHP pump ($b_0 = 0.3$, $k_0 = 0.408 = 8 \times 2\pi/D$) in a plasma with $\beta = 1.5$, $R_p = 1$, and $T_i^{(0)}/T_e^{(0)} = 0.5$]. FOR THESE PARAMETERS, FLUID THEORY ONLY GIVES A BEAT INSTABILITY.

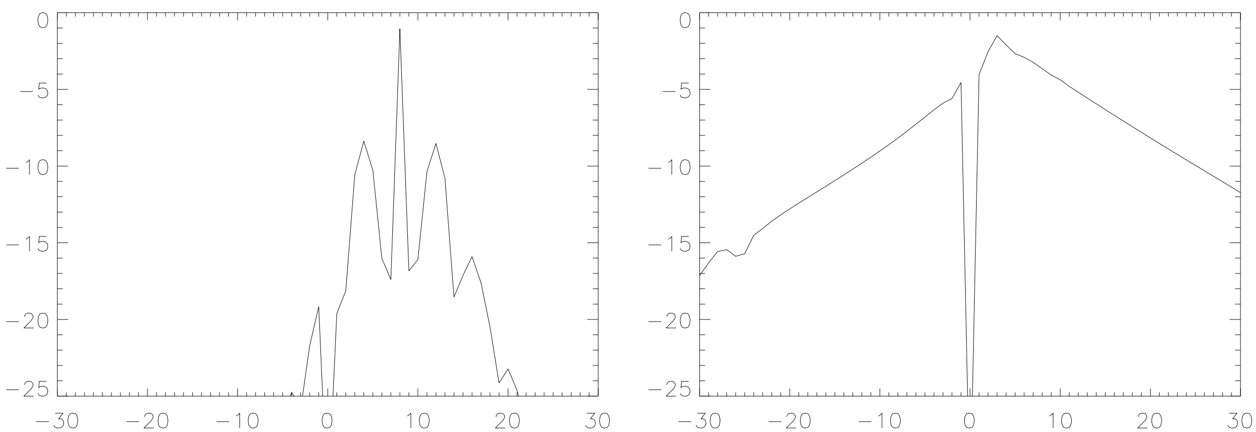


Figure 6: Spectral density (versus the wavenumber index) in lin-log10 scale for the transverse magnetic field $b_+ = b_x + ib_y$ at time $t = 2000$ (left) and $t = 3700$ (right) belonging respectively to the linear and nonlinear phases for the instability.

Development of modes $m = 4$ and $m = 12$ with similar growth rates (≈ 0.014). By $t = 2600$, $m = 4$ dominates; by $t = 3700$, $m = 3$ emerges, while an exponential spectrum develops at small scale. By $t = 4700$, $m = 2$ is dominant. A $t = 7700$, $m = 1$ is significantly excited and, in physical space, the Alfvén wave displays a NONLINEAR STRUCTURE OCCUPYING ALL THE COMPUTATIONAL BOX.

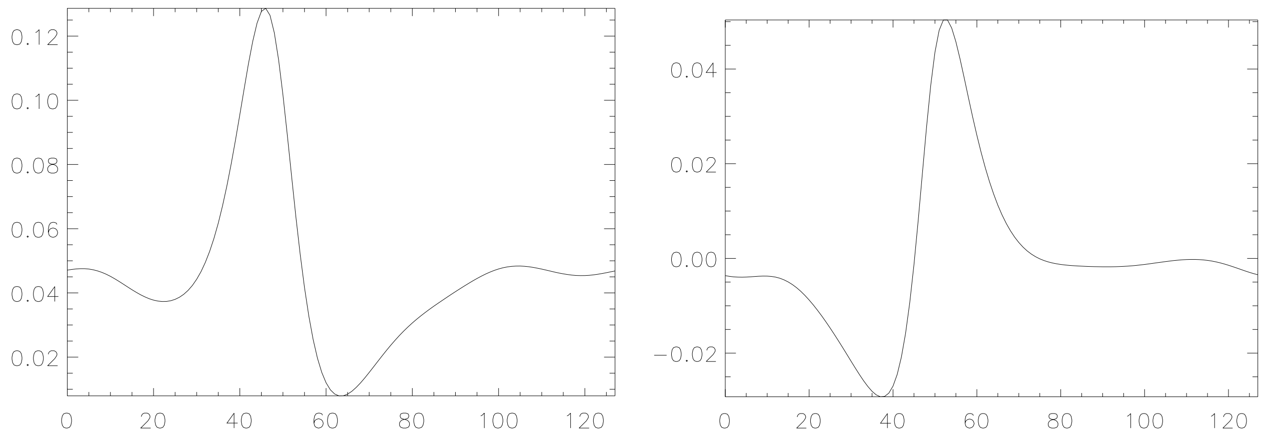


Figure 7: Profile of $|b_+|^2$ (left) and $(\rho - 1)$ (right) at $t = 3700$ in the conditions of Fig. 7. The labels on the abscissa axis refer to the collocation point indices.

Note the anticorrelation between $|b_+|^2$ and $\rho - 1$.

Parallel and transverse electron pressures (not shown) are proportional to the density fluctuations, which justifies the description of the electrons as an isothermal fluid.

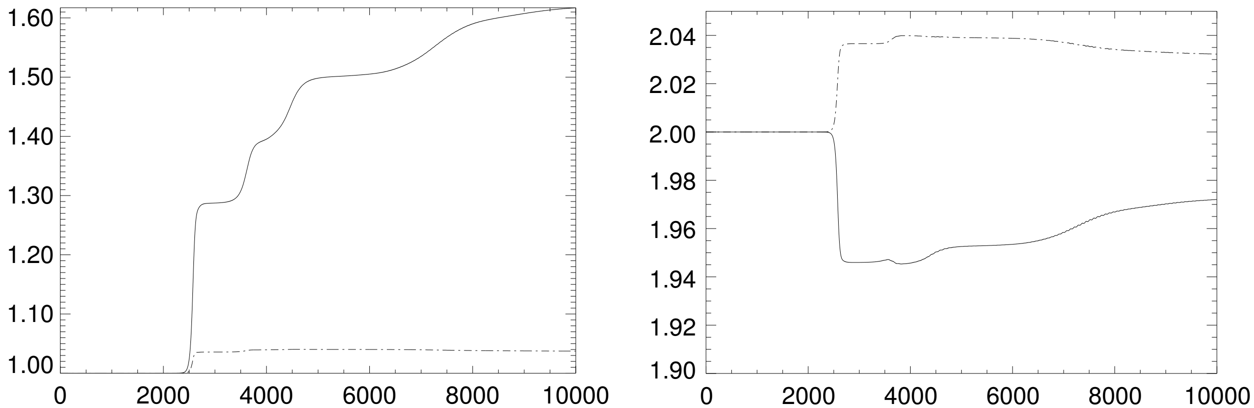


Figure 8: Time evolution of parallel (solid lines) and transverse (dashed-dotted lines) of the ion (left) and electron (right) mean temperatures.

Note the DECREASE of electron parallel temperature.
This contrast with decay instability for which the parallel electron temperature increases (possibly slightly).

When INCREASING THE AMPLITUDE to $b_0 = 0.8$: Landau fluid still displays a dominant MODULATIONAL INSTABILITY (as predicted by the fluid description), WHILE VASQUEZ HYBRID SIMULATION GIVES A DECAY INSTABILITY.

Landau fluid display a significant perpendicular heating of the electrons (more than 35%).

Two possible reasons for this DISCREPANCY:

The assumption of isothermal electrons used in the hybrid simulation is not valid.

The pump amplitude is too strong for the present Landau fluid model, leading to overestimated FLR corrections.

SUGGESTED PROBLEM:

At moderate β, in a regime where the decay instability dominates,

study the TRANSITION BETWEEN THE FLUID REGIME (direct energy cascade)
 that occurs when $T_e^{(0)}/T_p^{(0)}$ is large
to the KINETIC REGIME (inverse energy cascade)
 that develops when $T_e^{(0)}$ and $T_p^{(0)}$ are comparable.

Is there an intermediate regime where both cascades coexist?